Azimuthal AVO Inversion (AVOZI) in Full Elastic Property Determination (FEPD) of Fractured Reservoirs (HTI media)

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Summary

Azimuthal AVO inversion (AVOZI) for full elastic properties determination (FEPD) in horizontal transverse isotropic (HTI) media (fractured reservoirs) is introduced. The theoretical background in AVOZ inversion for elastic properties as an extension from isotropic media has been described. It includes the definitions for inverted anisotropic seismic attributes and corresponding anisotropic parameters. Two methodologies, one using isotropic assumption and the other using anisotropic assumption, to realize AVO extraction and inversion, are suggested. Finally, their advantages and disadvantages are discussed.

Introduction

Azimuthal AVO inversion is considered as a technique in seismic inversion to obtain anisotropic information of fractured reservoirs. Current applications of this technique are mainly for determining the orientations and density of fractures in carbonate reservoirs (Perez, et al., 1999, Robert et al., 1999, Gary and Head, 2000). The inverted fracture orientation and density are of reflectivity types. In this study, an attempt is made to determine full elastic properties of HTI medium through azimuthal AVO inversion. The advantages of using this method are 1) reservoir interval information can be determined; 2) the results can be directly related to rock properties; and 3) it provides the possibility to obtain the information of fluid content in rock. In this study, the definitions of anisotropic elastic properties and their forms in association with seismic inversion are given. The methodologies to implement the inversion and their pros and cons are analyzed.

Theoretical Background

Figure 1 shows the coordinates, velocities and elastic properties in horizontal transverse isotropic medium. Figure 1a and 1b illustrate the velocities and the elastic moduli required for describing the elastic properties of HTI medium, respectively Notice that the signs '||' and '⊥' represent the principal directions parallel and perpendicular to the fast velocity direction. For fractured carbonate reservoirs, they are the directions parallel and perpendicular to the fractures. For horizontal layered shale, they are the direction parallel and perpendicular to the bedding. Anisotropic properties in these principal directions are of interest because they provide the information required for describing anisotropic properties of a reservoir. Figure 1c shows the seismic elastic properties or attributes expected to be obtained from 3D seismic, which are derived from Fig. 1a and 1b by introducing density.

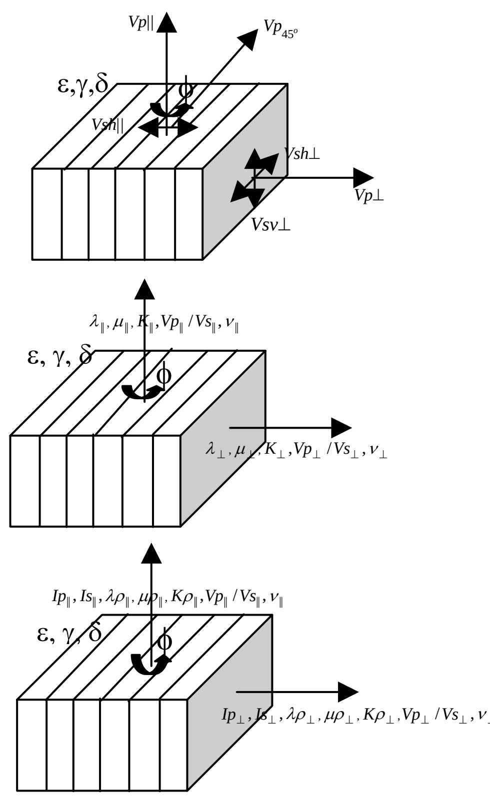

Fig. 1 Anisotropic elastic properties of HTI medium in velocities, moduli and seismic attributes

The following section gives the mathematical form of the rock physics background for anisotropic rock properties and their seismic version. First, stiffness matrix for TI medium, which relates stresses and strains is given in a traditional form (Eq. 1) and an explicit form (Sheriff, 1991) (Eq. 2)

$$\begin{bmatrix}\sigma_{zz}\\ \sigma_{yy}\\ \sigma_{xx}\\ \sigma_{xz}\\ \sigma_{yz}\\ \sigma_{xy}\end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & c_{13} & & & \\ c_{12} & c_{11} & c_{13} & & & \\ c_{13} & c_{13} & c_{33} & & & \\ & & & c_{44} & & \\ & & & & c_{44} & \\ & & & & & c_{66}=(c_{11}-c_{12})/2 \end{bmatrix} \begin{bmatrix}\varepsilon_{zz}\\ \varepsilon_{yy}\\ \varepsilon_{xx}\\ \varepsilon_{xz}\\ \varepsilon_{yz}\\ \varepsilon_{xy}\end{bmatrix} \quad (1)$$

$$\begin{bmatrix}\sigma_{zz}\\ \sigma_{yy}\\ \sigma_{xx}\\ \sigma_{xz}\\ \sigma_{yz}\\ \sigma_{xy}\end{bmatrix} = \begin{bmatrix} \lambda_\perp + 2\mu_\perp & \lambda_\perp & \lambda_\parallel & & & \\ \lambda_\perp & \lambda_\perp + 2\mu_\perp & \lambda_\parallel & & & \\ \lambda_\parallel & \lambda_\parallel & \lambda_\parallel + 2\mu_\parallel & & & \\ & & & \mu_\parallel & & \\ & & & & \mu_\parallel & \\ & & & & & \mu_\perp \end{bmatrix} \begin{bmatrix}\varepsilon_{zz}\\ \varepsilon_{yy}\\ \varepsilon_{xx}\\ \varepsilon_{xz}\\ \varepsilon_{yz}\\ \varepsilon_{xy}\end{bmatrix} \quad (2)$$

where λ is Lame's parameter and μ is shear modulus. The relationships between these elastic properties and phase velocities are

$$
\begin{aligned}
c_{11} &= \rho Vp^2{}_{\perp} \\
c_{12} &= c_{11} - 2\rho Vsh^2{}_{\perp} \\
c_{33} &= \rho Vp^2{}_{\parallel} \\
c_{44} &= \rho Vsh^2{}_{\parallel} \\
c_{13} &= -c_{44} + \\
&\sqrt{4\rho^2 Vp^4(45^o) - 2\rho Vp^2(45^o)(c_{11}+c_{33}+2c_{44}) + (c_{11}+c_{44})(c_{33}+c_{44})} \\
c_{66} &= \rho Vsh^2{}_{\perp}
\end{aligned}
\tag{3}
$$

The anisotropic parameters and their weakly anisotropic approximations (Thomsen, 1986) have the forms of

$$
\begin{aligned}
\varepsilon &= \frac{c_{11}-c_{33}}{2c_{33}} = \frac{Vp_{\parallel}{}^2 - Vp_{\perp}{}^2}{2Vp_{\perp}{}^2} \approx \frac{Vp_{\parallel} - Vp_{\perp}}{Vp_{\perp}} \\
\varepsilon_{45^o} &\approx \frac{Vp_{45^o} - Vp_{\perp}}{Vp_{\perp}} \\
\gamma &= \frac{c_{66}-c_{44}}{2c_{44}} = \frac{Vs_{\parallel}{}^2 - Vs_{\perp}{}^2}{2Vs_{\parallel}{}^2} \approx \frac{Vs_{\parallel} - Vs_{\perp}}{Vs_{\perp}} \\
\delta &= \frac{(c_{13}+c_{44})^2 - (c_{33}+c_{44})^2}{2c_{33}(c_{33}-c_{44})} \approx 4\varepsilon_{45^o} - \varepsilon
\end{aligned}
\tag{4}
$$

The seismic elastic properties as inverted seismic attributes, P-impedances $Ip_{\parallel}$, $Ip_{\perp}$, S-impedances $Is_{\parallel}$, $Is_{\perp}$, and moduli $\lambda\rho_{\parallel}$, $\lambda\rho_{\perp}$, and $\mu\rho_{\parallel}$, $\mu\rho_{\perp}$ (see Figure 1c) and their relationships are

$$
\begin{aligned}
\lambda_{\parallel}\rho &= Ip_{\parallel}{}^2 - 2Is_{\parallel}{}^2 \\
\mu_{\parallel}\rho &= Is_{\parallel}{}^2
\end{aligned}
$$

and

$$
\begin{aligned}
\lambda_{\perp}\rho &= Ip_{\perp}{}^2 - 2Is_{\perp}{}^2 \\
\mu_{\perp}\rho &= Is_{\perp}{}^2
\end{aligned}
\tag{5}
$$

The anisotropic parameters expressed by impedances and inverted seismic moduli can thus be expressed as

$$
\begin{aligned}
\varepsilon &= \frac{Ip_{\parallel}{}^2 - Ip_{\perp}{}^2}{2Ip_{\perp}{}^2} \\
\gamma &= \frac{Is_{\parallel}{}^2 - Is_{\perp}{}^2}{2Is_{\perp}{}^2}
\end{aligned}
\tag{6}
$$

and

$$
\begin{aligned}
\varepsilon &= \frac{\lambda_{\parallel}\rho - \lambda_{\perp}\rho}{(\lambda_{\perp}\rho + 2\mu_{\perp}\rho)} + 2\frac{\mu_{\parallel}\rho - \mu_{\perp}\rho}{(\lambda_{\perp}\rho + 2\mu_{\perp}\rho)} = \varepsilon_{\lambda\rho} + 2\varepsilon_{\mu\rho} \\
\gamma &= \frac{\mu_{\parallel}\rho - \mu_{\perp}\rho}{2\mu_{\perp}\rho} \\
\varepsilon_{45^o} &= \varepsilon_{45^o\lambda\rho} + 2\varepsilon_{45^o\mu\rho} \\
\delta &\approx 4\varepsilon_{45^o} - \varepsilon
\end{aligned}
\tag{7}
$$

The P-wave reflection coefficient for HTI medium has been given by Ruger (1996) as

$$
\begin{aligned}
R_{pp}(\phi,\theta) &= R_{pp-iso}(\phi,\theta) + R_{pp-ani}(\phi,\theta) \\
R_{pp-iso}(\phi,\theta) &\approx \frac{1}{2}\left(\frac{\Delta Ip}{Ip}\right)\left(1+\tan^2(\theta)\right) - 4\left(\frac{\alpha}{\beta}\right)^2\left(\frac{\Delta Is}{Is}\right)\sin^2(\theta) \\
R_{pp-ani}(\phi,\theta) &\approx \left\{\left[\frac{\Delta\delta^v}{2} + 4\left(\frac{\beta}{\alpha}\right)^2\Delta\gamma\right]\cos^2(\phi)\right\}\sin^2(\theta) \\
&+\left\{\left[\frac{\Delta\varepsilon^v}{2}\right]\cos^4(\phi) + \left[\frac{\Delta\delta^v}{2}\right]\sin^2(\phi)\cos^2(\phi)\right\}\sin^2(\theta)\tan^2(\theta)
\end{aligned}
\tag{8}
$$

where $R_{pp\ -iso}$ is the approximation of Zoeppritz equation by Fatti at. al. (1994), and $Vp_{\perp}$ and $Vs_{\perp}$ are represented by α and β. It can be seen that this is an extension of isotropic case. As an example, the reflectivities for an HTI medium is shown in Figure 2.

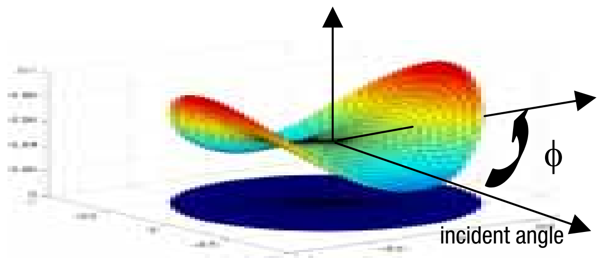

Fig. 2 Azimuthal reflectivity variation

The reflection coefficients in the principal directions $\phi = 0^o$ and 90^o ('||' and '$\perp$') are

$$
\begin{aligned}
&R_{pp-ani}(\phi = 0^o, \theta) \approx R_{pp-iso}(\theta) + \\
&\left[\frac{\Delta\delta^v}{2} + \left(\frac{2\beta}{\alpha}\right)^2\Delta\gamma^v\right]\sin^2(\theta) + \frac{\Delta\varepsilon^v}{2}\sin^2(\theta)\tan^2(\theta)
\end{aligned}
\tag{9}
$$

and

$$
R_{pp}(\phi = 90^o, \theta) = R_{pp-iso}(\theta) \tag{10}
$$

AVOZ Extraction and Inversion

To achieve the success of azimuthal AVO inversion for determining full elastic properties, three important issues need to be resolved. They are fracture orientations; AVO extraction; and AVO inversion. All of these issues are affected by seismic data quality, fold, and shot and receiver distribution in a 3D seismic survey.

The technique often used in fracture orientation determination, is equal angle range stacking to produce azimuthally dependent CDP gathers for AVO extraction. The errors introduced in this process are significant because of the averaging within given azimuth range. More accurate amplitude information is required for determining elastic properties and anisotropic parameters in

AVOZ inversion. As the reflectivities shown in Figure 2, to obtain fitting to the amplitude at all offset and azimuths is the necessary step. The same technique has been suggested by Skoyles et al. (1999).

The unknowns in Eq. 8 include fracture orientations, Rp and Rs in the plane perpendicular to fractures, and three anisotropic parameters. The simplification of Eq. 8, using techniques such as dropping high order terms, or using empirical relationship to merge anisotropic parameters, can not solve all these unknows. In fact, we have an overdetermined case. This case could be solved by least squares method using the amplitude information at all azimuths and offsets. Rewrite Eq. 9 as

$$\begin{aligned} R_{pp}(\phi_i,\theta_i) &= R_{pp-iso}(\phi_i,\theta_i) + R_{pp-ani}(\phi_i,\theta_i) \\ R_{pp-iso}(\phi_i,\theta_i) &\approx \frac{1}{2}R_{p\_iso}A(\theta_i) - 4\left(\frac{\alpha}{\beta}\right)^2 R_{s\_iso}B(\theta_i) \\ R_{pp-ani}(\phi_i,\theta_i) &\approx \left[\frac{\Delta\delta^v}{2} + 4\left(\frac{\beta}{\alpha}\right)^2\Delta\gamma^v\right]C(\phi_i,\theta_i) \\ &\quad + \left[\frac{\Delta\varepsilon^v}{2}\right]D(\phi_i,\theta_i) + \left[\frac{\Delta\delta^v}{2}\right]E(\phi_i,\theta_i) \end{aligned} \tag{11}$$

where *i* represents amplitude at given offset and azimuth. To solve for the attributes Rp_iso, Rs_iso, $\Delta\varepsilon$, $\Delta\gamma$, and $\Delta\delta$, this equation is further simplified to

$$\mathbf{R} = \mathbf{GX} \tag{12}$$

where

$$R = \begin{bmatrix} R_{pp1} \\ R_{pp2} \\ \vdots \\ \vdots \\ \vdots \\ R_{ppi} \\ \vdots \\ R_{ppn} \end{bmatrix}$$

the accurate amplitude information at given azimuth, surface

$$G = \begin{bmatrix} A(\theta_1) & B(\theta_1) & C(\phi_1,\theta_1) & D(\phi_1,\theta_1) & E(\phi_1,\theta_1) \\ A(\theta_2) & B(\theta_2) & C(\phi_2,\theta_2) & D(\phi_2,\theta_2) & E(\phi_2,\theta_2) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ A(\theta_i) & B(\theta_i) & C(\phi_i,\theta_i) & D(\phi_i,\theta_i) & E(\phi_i,\theta_i) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ A(\theta_n) & B(\theta_n) & C(\phi_n,\theta_n) & D(\phi_n,\theta_n) & E(\phi_n,\theta_n) \end{bmatrix}$$

and

$$X = \begin{bmatrix} \frac{1}{2}R_{p\_iso} \\ 4\left(\frac{\beta}{\alpha}\right)^2 R_{s\_iso} \\ \frac{\Delta\delta^v}{2} + 4\left(\frac{\beta}{\alpha}\right)^2\Delta\gamma^v \\ \frac{\Delta\varepsilon^v}{2} \\ \frac{\Delta\delta^v}{2} \end{bmatrix}$$

The seismic atributes thus can be solved by

$$X = (G^{\mathrm{T}}G)^{-1}G^{\mathrm{T}}R \tag{13}$$

After AVOZ extraction, the inversion for the elastic properties as described in Eqs.. 5 to7 is the final step. It is expected that the final results will include fracture orientations, fracture density and the interval properties of a reservoir.

Conclusions

The theoretical background and methodologies for azimuthal AVO inversion for determining Full Elastic Properties in HTI medium have been discussed. The definitions of anisotropic properties in seismic version have been given in Fig. 1c and Eqs. 5 to 7. The theoretical background, combined with practical considerations, establish a basis for AVOZ extraction and full elastic property inversion.

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