# Errors related to using near/far offset stacks in AVO Inversion

*Michael Burianyk, Jon Downton, John Logel*


## Summary

Performing AVO inversion on near and far offset stacked data instead of the original prestack data introduces bias and greater uncertainty into the AVO parameter estimates. The parameter bias is introduced by averaging data that changes in nonlinear fashion as function of offset. In creating the near and far offset stacks the effective offset range is reduced. This makes the matrix inversion less stable, leading to greater amplification of noise and more unreliable results.


## Introduction

Due to economic and philosophical considerations, AVO inversion (Shuey, 1985, Fatti et al, 1994, Smith and Gidlow, 1987) is being performed on near and far offset stacked data rather than the actual prestack data. The argument is that for large data volumes it is not economically feasible to output all the prestack data for subsequent AVO analysis. As a substitute, near and far offset stacks are archived for later AVO analysis. Connolly, (1999) shows an AVO processing sequence where near and far offset stacks are inverted for elastic parameters. Based on noise studies, Cambois, (1998) argues that near and far offset stacks are superior to AVO attribute stacks, such as the fluid stack.

This paper shows that performing AVO inversion on near and far offset stacked data, rather than the prestack data will result in biased AVO estimates with greater uncertainty than would have been the case if the actual prestack gathers were inverted. This is shown theoretically using perturbation theory and demonstrated with a numerical example.

## Theory

### AVO Inversion

Elastic parameters may be inverted for using a linearized approximation of the Zoeppritz equation such as Aki and Richards (1980)

$$r(\theta)=\frac{1}{2}\left(1-4\gamma^2\sin^2\theta\right)\frac{\Delta\rho}{\rho}+\frac{1}{2}\frac{\Delta\alpha}{\alpha}\frac{1}{\cos^2\theta}-4\gamma^2\left(\frac{\Delta\beta}{\beta}\right)\sin^2\theta, \qquad (1)$$

Where $\alpha$, $\beta$, $\rho$, $\gamma$ respectively are the average p-wave velocity, s-wave velocity, density and the ratio of S-velocity to P-velocity across the interface. The variable $\theta$ is the average angle of incidence and $\Delta\alpha$, $\Delta\beta$, $\Delta\rho$ are the change in p-wave velocity, s-wave velocity and density

Equation 1 may be solved for using linear inverse techniques. The linear inverse problem is written as

$$\mathbf{Gm}=\mathbf{d}, \qquad (2)$$

where **G** is the Jacobian of the problem, **m** the unknown parameter vector and **d** the input data vector. Equation (1) may be written in these terms by treating only the density, compressional and shear velocity reflectivity coefficients as unknowns. The average angle of incidence $\theta$ and $\gamma$ are considered to be known as part of the initial model. Since the data is recorded as a function of offset, raytracing must be done to transform the functional dependence of the data from offset to average angle of incidence. The Jacobian, **G,** is defined by the initial model ($\gamma$ ratio and the average angle of incidence $\theta$) and the acquisition geometry. The unknown parameter vector **m** represents the reflectivity coefficients to be solved for and the data vector **d** contains the data recorded as a function of offset.

In practice, the 3 parameter AVO inversion problem is ill-conditioned (Lines, 2000), so generally an approximation of equation (1) is used, solving for just two parameters. Equation (14) of Shuey, 1985 and equation (4) of Fatti et al, 1994 are examples of this. In the two term approximation, the linear constraints forming equation (2) are

$$\begin{bmatrix} f(\theta_1,\gamma_1) & g(\theta_1,\gamma_1) \\ \vdots & \vdots \\ f(\theta_K,\gamma_K) & g(\theta_K,\gamma_K) \end{bmatrix}\begin{bmatrix} m_1 \\ m_2 \end{bmatrix}=\begin{bmatrix} d_1 \\ \vdots \\ d_K \end{bmatrix}. \qquad (3)$$

In the case of the Fatti equation $f(\theta,\gamma)=1/2\sec^2\theta$, $g(\theta,\gamma)=-4\sin^2\theta$, $m_1$ is the P-impedance reflectivity and m2 is the S-impedance reflectivity. The variable $d_i$ is the observed amplitude for the $i^{th}$ offset or angle (i=1,…K). This system of equations may be solved in a least squares fashion

$$\mathbf{m}=\left[\mathbf{G}^T\mathbf{G}\right]^{-1}\mathbf{G}^T\mathbf{d}. \qquad (4)$$

Smith and Gidlow (1987) showed that this can be efficiently implemented as weighted stacks.

**Near / Far angle approximation**

To facilitate the mathematics and the subsequent analysis of results, this paper will work with near and far angle stacks instead of offset stacks. Further it is assumed that the data is sampled with an even angle increment $\Delta\theta$, such that $\theta_i = i\Delta\theta$ where (i=1,…K). Equation (2) may be solved after this transformation. Further, equation (2) may also be solved in an approximate sense using only the near and far angle stacks as input. In this case, equation (2) has just two rows.

$$\begin{bmatrix} f(\theta_{K/4},\gamma_{K/4}) & g(\theta_{K/4},\gamma_{K/4}) \\ f(\theta_{3K/4},\gamma_{3K/4}) & g(\theta_{3K/4},\gamma_{3K/4}) \end{bmatrix}\begin{bmatrix} m_1 \\ m_2 \end{bmatrix} = \begin{bmatrix} d_N \\ d_F \end{bmatrix}, \tag{5}$$

The near angle stack $d_{N,}$ and far angle stack $d_F$, are defined as

$$d_N = \frac{2}{K}\sum_{i=1}^{K/2} d_i, \quad \text{and} \quad d_F = \frac{2}{K}\sum_{i=K/2+1}^{K} d_i. \tag{6}$$

The angle used in the calculation of the Jacobian ($f(\theta,\gamma)=\sec^2\theta$, $g(\theta,\gamma)=-8\sin^2\theta$) is chosen as the average angle within the near and far angle stacks respectively. Equation (5) may be written in matrix form

$$\overline{\mathbf{G}}\mathbf{m} = \overline{\mathbf{d}}, \tag{7}$$

writing a bar over the variable to differentiate equation (5) from (3). Solving equation (5) by least squares leads to

$$\mathbf{m} = \left[\overline{\mathbf{G}}^{\mathbf{T}}\overline{\mathbf{G}}\right]^{-1}\overline{\mathbf{G}}^{\mathbf{T}}\overline{\mathbf{d}}. \tag{8}$$

yielding estimates of the elastic parameters. The next section will show that the estimates given by equation (8) are biased and are less reliable than those given by equation (4).

**Errors introduced by the near / far angle approximation**

To understand the errors introduced by using just using the near and far angles in the AVO inversion it will be shown that equation (3) is equivalent to equation (5) under a series of approximations. The validity of these approximations influence the size of error introduced by solving equation (8) instead of equation (4). The errors associated with solving the AVO inversion using only near and far offset data (equation 7) may be understood by studying the approximations used in transforming equation (3) to (5).

Divide the rows of equation (3) into two subsets: those rows whose data point would sum to the near angle stack and those rows whose data points would sum to the far angle stack. Expand $f(\theta,\gamma)$ and $g(\theta,\gamma)$ as a Taylor series around the near angle or far angle as appropriate, keeping only the first order terms. Equation (2) becomes

$$\left[\overline{\mathbf{G}} + \delta\mathbf{G}\right]\mathbf{m} = \mathbf{d}, \tag{9}$$

which may be solved in a least squares sense. Due to symmetries in the geometry this simplifies to

$$\left[\overline{\mathbf{G}}^T\overline{\mathbf{G}} + \delta\mathbf{G}^T\delta\mathbf{G}\right]\mathbf{m} = \left[\overline{\mathbf{G}}^T + \delta\mathbf{G}^T\right]\mathbf{d}. \tag{10}$$

The data vector can also be written in terms of perturbations from the near and far angle average, $\mathbf{d} = \overline{\mathbf{d}} + \delta\mathbf{d}$ so that equation (10) becomes

$$\left[\overline{\mathbf{G}}^T\overline{\mathbf{G}} + \delta\mathbf{G}^T\delta\mathbf{G}\right]\mathbf{m} = \left[\overline{\mathbf{G}}^T + \delta\mathbf{G}^T\right]\left(\overline{\mathbf{d}} + \delta\mathbf{d}\right). \tag{11}$$

If the data changes in a linear function with angle then $\overline{\mathbf{G}}^T\delta\mathbf{d}$ =0 and equation (11) simplifies to

$$\begin{bmatrix} \overline{\mathbf{G}}^T\overline{\mathbf{G}} \\ \delta\mathbf{G}^T\delta\mathbf{G} \end{bmatrix}\mathbf{m} = \begin{bmatrix} \overline{\mathbf{G}}^T\overline{\mathbf{d}} \\ \delta\mathbf{G}^T\delta\mathbf{d} \end{bmatrix}. \tag{12}$$

The first row in equation (12) is equivalent to equation (8). Performing the AVO inverse problem on the near and far angles stacks (equation 8) ignores the contribution of the second row in equation (12). Solving equation (12) by least squares results in

$$\mathbf{m} = \left[\overline{\mathbf{G}}^T\overline{\mathbf{G}} + \delta\mathbf{G}^T\delta\mathbf{G}\right]^{-1}\overline{\mathbf{G}}^T\overline{\mathbf{d}} + \left[\overline{\mathbf{G}}^T\overline{\mathbf{G}} + \delta\mathbf{G}^T\delta\mathbf{G}\right]^{-1}\delta\mathbf{G}^T\delta\mathbf{d}. \tag{13}$$

Equation 8 ignores the contribution of the second term in equation (13) introducing bias into the parameter estimate.

The term $\delta\mathbf{G}^T\delta\mathbf{G}$ adds extra energy to the diagonal of the inverse in equation (13). This helps stabilize the matrix inverse and the AVO inversion resulting in more reliable AVO estimate in the presence of noise. The physical significance of this extra term is to account for the greater range of angles going into equation (3). The act of creating the angle stacks reduces the effective angle range, making the inversion less stable and the results more sensitive to noise.

If the assumption used moving from equation (11) to (12) is not made then the least squares inverse is more complex.

$$\mathbf{m} = \left[\overline{\mathbf{G}}^T\overline{\mathbf{G}} + \delta\mathbf{G}^T\delta\mathbf{G}\right]^{-1}\overline{\mathbf{G}}^T\overline{\mathbf{d}} + \left[\overline{\mathbf{G}}^T\overline{\mathbf{G}} + \delta\mathbf{G}^T\delta\mathbf{G}\right]^{-1}\delta\mathbf{G}^T\delta\mathbf{d} + \left[\overline{\mathbf{G}}^T\overline{\mathbf{G}} + \delta\mathbf{G}^T\delta\mathbf{G}\right]^{-1}\overline{\mathbf{G}}^T\delta\mathbf{d}. \quad (14)$$

The assumption that the data changes as linear function of angle is suspect when large angles (greater than 20 degrees) are considered. Equation (14) is to a first order approximation, equivalent to equation (4). In solving the problem using only near and far angle stacks (equation 8) two forms of error have been introduced. There is an error in Jacobian that occurs because the exact offset/angle positions are approximated by the average position of the near and far angle stacks. This is the factor $\delta\mathbf{G}^T\delta\mathbf{G}$. The net effect of ignoring this term is to make the matrix inversion less stable amplifying the noise. The second error is ignoring the 2nd and 3rd terms in equation (14). This introduces bias into the parameter estimates.

**Discussion**

Figure 1 illustrates the cause and impact of the bias. Averaging a nonlinear function will introduce a slight difference between the instantaneous amplitude and the average value. This difference shifts the trend of linear regression introducing an error in both the zero intercept and slope.

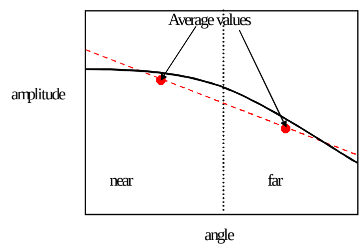


Figure 1

The effect of ignoring the $\delta\mathbf{G}^T\delta\mathbf{G}$ .term in the Jacobian can be seen by analyzing the unit covariance matrix. The unit covariance matrix can be studied to determine the standard deviation of the parameter estimates (Downton, 2000). The unit covariance matrix is only a function of geometry of the problem, that is the angle distribution, and $\gamma$, and so can be studied without knowing the actual data. The unit covariance matrix indicates how noise will be amplified in the final solution.

Assuming $\gamma$=0.5 the unit covariance matrix was calculated for two geometries. In one it was assumed that there were 10 angles recorded from 0 to 27 degrees in 3 degree increments. In the second case it was assumed that there were just two angles recorded at 6 and 21 degrees. This would be the case if we calculated angle stacks based on the geometry of the first case. The unit variance for the S-impedance reflectivity, for the two term Fatti inversion (equation 4, Fatti et al, 1994) is 6.35 in the first case and 8.50 in the second case. The variance (uncertainty) of the S-impedance parameter estimate has increased nearly 25% by approximating the geometry of the problem by only the near and far angles.

**Conclusions**

Thus performing AVO inversion on near and far offset stacked data instead of the original prestack data introduces bias and greater uncertainty into the AVO parameter estimates. If AVO inversion is to be performed on data, it is better to do it on the original geometry rather than some approximation such as near and far angle stacks. If the main argument for doing AVO inversion on near and far angle stacks is save money on output costs, we are trading error for dollars. This seems like a poor trade-off for data that we have spent millions of dollars to acquire.

**References**

Aki, K., and Richards, P.,G., 1980, Quantitative seismology: theory and methods: W.H. Freeman and Co.
Cambois, G., 1998, AVO attributes and noise – pitfalls of crossplotting: EAGE 60th Conference and Technical Exhibition
Connolly, P. 1999, Elastic Impedance: The Leading Edge, Vol. 18, No. 4, p 438-452
Downton J., Lines, L.R., Goodway, W., Xu, Y, Li, Y, 2000, Predicting the statistical properties of the fluid stack: 70th annual international SEG meeting, expanded abstracts
Fatti, J.,L., Smith, G.,C., Vail, P.,J., Strauss, P.,J., and Levitt, P., R., 1994, Detection of gas in sandstone reservoirs using AVO analysis: A 3-D seismic case history using the Geostack technique: Geophysics, 59, 1362-1376
Lines, L.,R., 2000, Density contrast is difficult to determine from AVO; Canadian Journal of Exploration Geophysics, In press
Smith, G.,C., and Gidlow, P.,M., 1987, Weighted stacking for rock property estimation and detection of gas: Geophysical Prospecting, 35, 993-1014
Shuey, R.,T., 1985, a simplification of the Zoeppritz equations: Geophysics, 50, 609-614