3D thermodynamic automata modeling of fluid flow in porous media

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3D thermodynamic automaton is constructed by adding probability rules for collision and propagation and by using cubic model. A series of numerical modeling experiments of fluid flow in porous media are carried out. The results are in agreement with measurements, which suggest it will help investigate the physical features of porous media and supplement some laboratory works in the future. First, the equilibrium state is tested to check the validity of the model. Barometric formula due to gravity is obtained by applying body force to particles in the model. Later on, more realistic experiments such as Darcy flow, diffusion are modeled. The match between the modeling and the laboratory measurements shows that these 3D thermodynamic automaton simulations are a successful approach to simulate the fluid flow.

Introduction

The simulation of multiphase flow in porous media is of great importance, especially in petroleum reservoir engineering. The ability of lattice gas model developed from cellular automata to simulate a variety of physical process causes the appeal for physics research on fluid flow in porous media. Since Frisch et al.(1986) first showed that particles containing discrete mass and low velocity in the limit of large lattice size will evolve according to Navier-Stokes equation, many advances have been made. However, most of these models did not incorporate thermal effects (Hardy, et. 1976; Wolfram, 1986) and those that did apply temperature (Chopard, and Droz, 1987; Chen, et al., 1989) were severely constrained by discrete particle velocities. Also triangular lattice for 2D and face-centered-hypercubic lattice for 3D modeling had to be used in order to maintain isotropy. A 2D thermodynamic automaton was constructed by Udey et al.(1998), and then developed by D.Yang (1999) to simulate fluid flow and diffusion in porous media. In their papers, thermodynamic automaton model differs from these standard lattice gas models in that the momentum of a particle in a lattice is a continuous variable and that the discrete particle motion is related to the particles continuous momentum through a probabilistic propagation rule. As a consequence, the lattice is only used for bookkeeping purpose; its shape is unimportant to the simulation result. Thus it is easier to develop this model to 3 dimensional model without necessarily using face-centered-hypercubic lattice. In this paper, 3-Dimensional cubic thermodynamic automaton model is constructed based on 2D model Udey et al. (1998), the collision and propagation rule as well as boundary conditions are all extended to 3D.

Method

A porous medium is introduced by allowing a probability of collision for a fluid-solid at each lattice site. Permeability is adjusted by changing the probability. Heterogeneity is applied by changing the probability in space. Randomly assigning the particles' moving direction, and giving them new momentum according to the temperature of the boundary, we implement thermo-boundary.

Thermal equilibrium state is obtained by first initializing all particles with the same velocity, after a number of time steps, the system will evolved into equilibrium configuration, that is, all the particles obey non-relativistic Maxwell-Boltzmann distribution at lower particle speeds, and relativistic Boltzmann distribution for particle speeds close to the speed of light.

Adding a pump to each particle, which acts as a body force, adding six thermodynamic boundaries (D.Yang et al. 1997), the distribution of the number of gas, can be observed as Barometric formula in the sealed box.

Poiseuille flow is modeled by adding a small pump on each particle, applying periodic boundary condition at left and right boundaries. Darcy flow is modeled under similar conditions to the Poiseuille flow except introducing permeability into lattice.

Diffusion (Dongsong yang, 1997) is simulated by assigning two kinds of particles into lattice; the tracing particles are assigned in the middle. No pump is added and periodic boundary is set to left and right boundaries.

Examples

The first application is a simulation of thermal equilibrium state to check the validity of the 3D model. The simulation results after 1000 iterations are show in Figure1 and 2. The simulation result in Figure 1 is checked against the non-relativistic Maxwell-Boltzmann distribution. It is found that the result is consistent with the theoretical predictions. Figure 2 illustrates simulation results for m=1.0, v=0.1 and 0.2 respectively. The plots of ln(P(E) vs. E are straight lines as predicted as well. However at higher energy (v=0.2), scattered points were observed. Such is reasonable because only a few particles reach these high energies, while a large number of particles are required for a valid statistical analysis.

Figure 3 and 4 clearly show exponential curves sliced from XY and XZ plane respectively. The results are in agreement with Barometric formula $N(z)=N_o e^{-mgz}/KT$.

Figure 5 and 6 illustrate simulation results of Poiseuille flow and Darcy flow respectively. The results exactly match the laboratory experiments in that it shows a straight line for Darcy flow (same flow rates everywhere) and a parabolic curve for Poiseuille flow. The interesting point in figure 6 is that the flow rate at the middle is bigger than the rest; it is a well-known phenomenon of Poiseuille flow for a cubic model.

Figure 7 describes diffusion process in the tube. Compared the result with the analytical solution, a close agreement is found. Both theoretical prediction and simulation result obey Bell function. Diffusion in homogeneous and heterogeneous porous media is modeled and compared with diffusion in a tube. The results are shown in figure 8. It is noted that the diffusion in a tube is faster than in a homogeneous medium and that diffusion in a homogeneous medium is faster than in a heterogeneous medium in most cases. However, this later conclusion really depends on the structure of the porous medium.

Conclusions

The model study shows that the thermal equilibrium state, Barometric formula, Poiseuille flow, Darcy flow and diffusion were successfully simulated by the 3D thermodynamic automaton model. The Simulation results were consistent with the theoretical predictions, thus providing support for the validity of the model.

References

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Figure 1

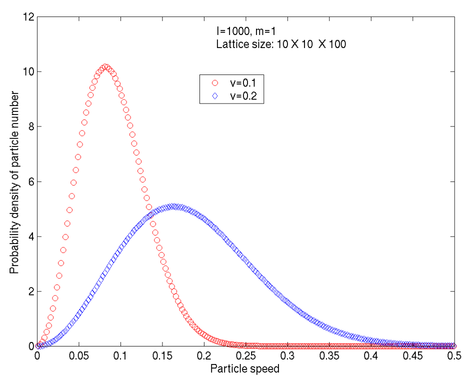

Figure 1 The speed distribution of particles after 1000 iterations

Figure 2

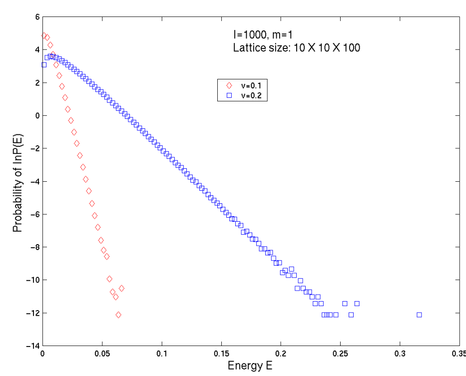

Figure 2 Corresponding plots of ln(P(E)) versus Energy E for the particle speed distribution

Figure 3

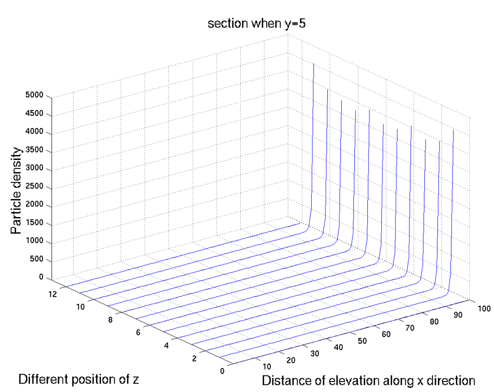

Figure 3 Distribution of particle number in XZ plane, Gravity is added along x

Figure 4

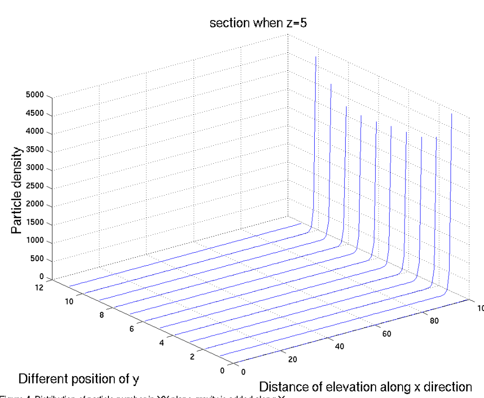

Figure 4 Distribution of particle number in XY plane,gravity is added along X

Figure 5

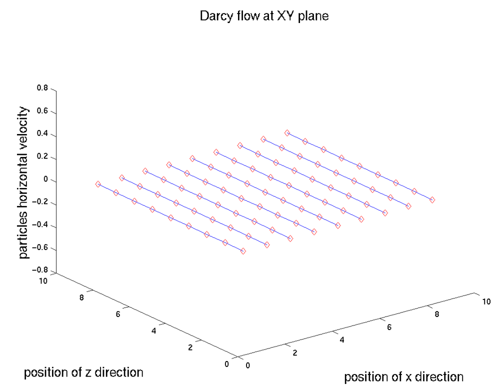

Figure 5 3D thermodynamic autmaton simulation of Darcy flow, averge along Y

Figure 6

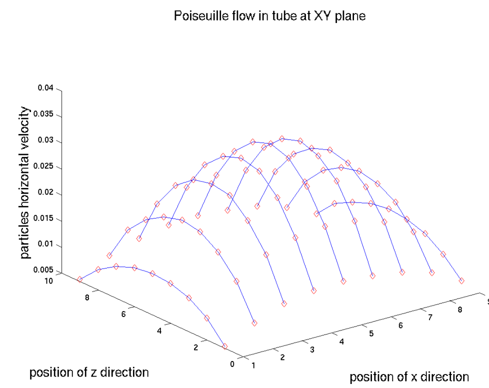

Figure 6 3D thermodynamic automaton simulation of poiseuille flow in a cubic model

Figure 7

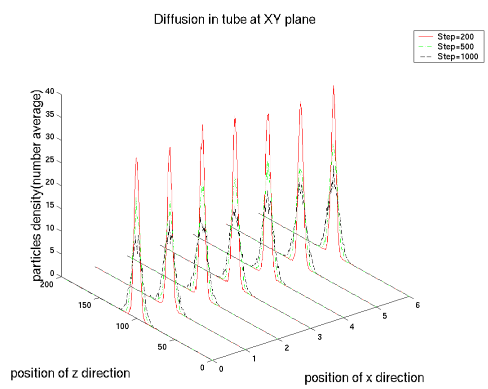

Figure 7 3D simulation of diffusion in a cubic tube model with different time steps

Figure 8

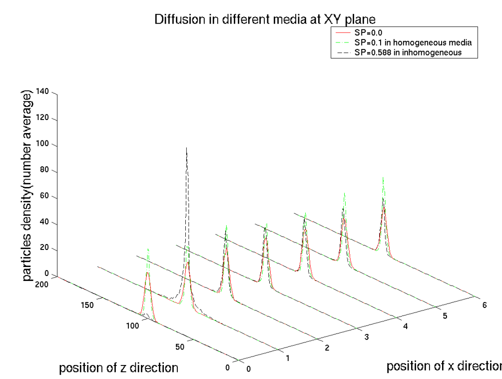

Figure 8 The comparision of 3D simulation of diffusion process in different media