Pathlength, *pathtime*, and "*qvel*": the essential variables for along-raypath aggregated velocities and velocity heterogeneity

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Summary

Commencing from a point disturbance at $r_A(x_A,y_A,z_A)$ in an isotropic heterogeneous lossless medium with velocity field $v(r(x,y,z))$, rays find their path-arc incrementals $d\zeta(r)$ and thus their paths $\zeta(r)$ by the dictates of Fermat's Principle (or Snell's law). Along any such raypath, *pathtime* t and *pathlength* l are accumulated while pointwise constrained by the incremental relationship $dt = S(\zeta(r))d\zeta$, where $s(\zeta(r))$ symbolizes local *slowness* at pathpoints. A third essential accumulation variable q links to the others with incrementals $dq = v(\zeta(r))d\zeta$. The q variable clarifies how, in seismic propagation, the detail of the frequency (density) distribution for *along-raypath velocity* combines with the detail of its paired frequency distribution for *along-raypath slowness*. The suggested name for variable q is *qvel*.

For a single ray progressing through the medium from point A to B, the variable triplet (t_{AB}, q_{AB}, l_{AB}) yields *time average*, *pathmean*, and *rms* velocities along the path, also a quantitative *along-path velocity heterogeneity*. For a ray bundle with event encounters that manifest as coherent wavefront across an area-array of B point sensors, the set of (t_{AB}, q_{AB}, l_{AB}) event triplets combined with sensor locations encapsulates information that, in principle, can yield an approximating velocity field for the intercepted front. The variable triplets are significant for seismic event discrimination and potentially also for imaging. Their relevance extends to spectral components of energy paths in media generally.

Introduction

Geological media, as is well known, can have great complexity and strong local variability of detail. Values of medium parameters encountered along the seismic propagation paths can thus range quite significantly, among them the values of local or pointwise propagation velocities. Yet quite simple velocity models do sometimes yield remarkably good migration images. Why this robustness? Do we fully understand and appreciate the fundamentals?

Much of the robustness comes from the additivity attribute of *pathlength*, *pathtime*, and a third variable *qvel* . Values of these variables are accumulated along the event raypaths, but those values are identical to values summed along so-called canonical paths. Those canonical representation paths have incremental path-arcs along which velocities have been monotonically ordered, e.g. from smallest to largest (c.f. Vetter, 1987). The analytical basis for such alternate representations is simply that definite integrals can be arbitrarily partitioned and parts arbitrarily reordered without affecting values of the integrals. Through the canonical path we can clarify how *pathlength*, *pathtime*, and *qvel* link to the *along-path slowness* and *along-path velocity* frequency (density) distributions.

I introduce and explore these concepts, using the tractable 1D ramp velocity field for examples (constant velocity gradient medium). I show then how those three variables, together with the aggregation velocities which they encapsulate, contribute to seismic event discrimination and potentially also to medium imaging.

Slowness and velocity distributions

Length (*distance*) and *time*, among others, have in the collective experience of mankind become recognized as independent accumulation or additivity variables through which we can ponder and comprehend some of the phenomena manifesting in the universe. It is then a moot point whether one or the other of the ratios, *slowness = time/length* or *velocity = length/time*, is in any way more meaningful or significant than the other. In the context of ray and energy propagation paths in seismics, it turns out that the frequency distributions of *along-path slowness* and *along-path velocity* both contribute equally to the essential slowness//velocity aggregation descriptors. The *slowness* distribution yields $S_{PATH\text{-}MEAN}$ as its mean or first moment. This is more familiar to the seismics community in its reciprocal form, as v_{TA} , the *time average velocity* along the path or simply "*average velocity"* (Sheriff,1984). The *along-path velocity* distribution yields as its mean or first moment the *path-mean velocity* v_{PM} . The so-called root-mean-squared velocity evaluated along the path is the geometric mean of the two velocities, $v_{RMS} = (v_{PM}v_{TA})^{1/2} = (v_{PM}/S_{PATHMEAN})^{1/2}$. The distributions and velocity//slowness aggregations are readily understood when exemplified, say for raypaths in a ramp velocity field.

Accumulated *pathtime* t along a raypath is the integral of slowness-weighted path incrementals $dt = S(\zeta(r))d\zeta$. Equally significant is the accumulation variable of velocity-weighted path incrementals $dq = v(\zeta(r))d\zeta = v^2(\zeta(r))dt$. While the integral or summation expression has been variously used (e.g. Duerbaum 1954, Slotnick 1959, Robinson 1983, Vetter 1987, …), it has not apparently been suitably named or explicitly symbolized. I propose it be designated by the relatively uncommitted letter symbol q, and that it be named as *qvel*. That name conveys the *velocity* context and it sounds out the aggregation expression, $q = (v_{PM})(l_{PATH})$. To appease concerns of language purists, as precedent for NOT-*qu*…. , I point to *qwerty* (the standard English language keyboard) as a well-suited word for its particular context (Oxford 1996).

The variable triplet (t_{AB}, q_{AB}, l_{AB}) for accumulations along a raypath between points A and B yields the three aggregation velocities $v_{TA} = l_{AB}/t_{AB}$, $v_{RMS} = (q_{AB}/t_{AB})^{1/2}$, $v_{PM} = q_{AB}/l_{AB}$. The ratio $v_{RMS}/v_{TA} = (q_{AB}\ t_{AB})^{1/2}/l_{AB}$ can be taken as a measure of along-path velocity heterogeneity (c.f. Al-Chalabi 1974, Vetter 1987). Visualize the velocity heterogeneity (or effect of v_{RMS} versus v_{TA}) as an *rms*-stretched *pathlength* $l_{RMS(AB)} = (v_{RMS}\ t_{AB})$, longer than actual pathlength $l_{AB} = (v_{TA}\ t_{AB}) > |r_{AB}|$.

For certain contexts or applications (e.g. when segment A to B is part of a longer path, say the down-path to a reflector) we might want to complement the (t_{AB}, q_{AB}, l_{AB}) triplet with the straight-line length $|r_{AB}|$; the complement to earlier velocities would be a defined apparent velocity $v_{APNT} = |r_{AB}|/t_{AB}$.

In principle, one can contemplate higher moment aggregation variables for the along-path slowness and velocity distributions. There is little prospect at this time that such aggregators could be meaningfully linked to information in seismic data or to velocity field approximations.

Event (*t, q, l*) and (*t, q*) representation plots
Clearly, in ray-tracing routines we can complement overall *pathtime* to sensors with overall *pathlength* and *qvel*, whatever the event-specific rays (transmissions, transmissions plus p-p primary reflections, p-s converted modes, p-p plus refractions, diverse multiples, …). Just as any particular coherent event front triggers sensor responses in accordance with shot-to-sensor elapsed time, so can we visualize event-associated *pathlength* and *qvel* accumulations also “pulsed” at those instants. The sensor patch specific k-event triplets (t_{ABk}, q_{ABk}, l_{ABk}), {k=1,2, …}, for individual rays or for sets of triplets in ray bundles, can be displayed in a (t, q, l)-coordinate space. Such representations can yield valuable insights re likely cause or category of the k-indexed different events.

Displays even in the simpler (t, q)-coordinate space give almost as much information, in particular when k-indexed events are annotated with applicable *velocity-heterogeneity* along the path. The relationships $v_{RMS}^{\ 2} = (q/t)$ and $l_{RMS}^{\ 2} = (q\ t)$ yield straight-line and hyperbola overlay grids for the (t, q)-coordinate space, with v_{RMS} and l_{RMS} values respectively constant along the overlay grid loci. The plotted locations of k-indexed events will be well spread out, showing values of and the significant differences between respective *rms-velocities*_cum_*rms-pathlengths*. Different event categories occupy distinctly different zones in the (t, q)-space.

Suppose further that in ray-tracing routines, for each of the k-indexed events, we were complementing *pathtime* values pertaining to all m-indexed medium segments, with like detail for *pathlength* and *qvel*. That would yield along-path information (t_{ABkm}, q_{ABkm}, l_{ABkm}), {m=1,2, …}. Again, all this could be displayed in the (t, q, l)-coordinate space, or else almost as completely but much more compactly in the (t, q)-coordinate space. The v_{RMS} and l_{RMS} grid overlays would facilitate again interpretation. If k-indexed event raypath segments had constant velocities, the m-segment slopes in (t, q)-representations would detail the interval velocities, and the associated pathlengths could be easily determined through the applicable Δt and Δq values in the plots. All such detail can help visualizing and associating (t_{ABkm}, q_{ABkm}, l_{ABkm}) information with a raypath and its segments.

Event-specific (*t, q, l*)-data
The challenge presents itself to explore further how event-front “(t, q, l)-data” can be extracted from shot records, and how then such information can be constructively used. Were they available, due to additivity one might try to re-bin “(t, q, l)-data” to event segments of the actual paths, say using insights on likely event type and other prior information. In particular, for not-multipled events, one would try to partition at the reflector, spot its position, and obtain thereby also the significant down- and up-path velocities, thus also *straight-lines apparent velocity* $v_{APNT} = (|r_{DWN}|+|r_{UP}|)/(t_{DWN}+t_{UP})$. An interesting point here is that we would have tripled data and relationships over those from pathtime picks alone, bringing thereby strong constraints to the path and velocity field decomposition task.

The *rms* velocities for event-specific (t, q, l) from paths through heterogeneous media link to “*moveout”* sensed at the front points along specific directions, in some particular cases possibly along all directions within some plane. *Directionally sensed velocities,* whether along directions that yield *rms-velocity* values or others, encompass and generalize *normal moveout velocities*.

Conclusions

For nearly half a century we have seen and used variants of expressions like $v_{RMS} = ((\Sigma_k\ v_k^2 \Delta t_k) / t)^{1/2}$ in context of velocity analysis and others (Duerbaum 1954, Dix 1955, …). Now, having re-examined the relevant fundamentals, having clearly discerned along every raypath the accumulating role of what I call the *qvel* variable, this fuller sense of the expression is captured in the simpler relationship $v_{RMS} = (q / t)^{1/2} = ((\Sigma_k \Delta q_k / \Sigma_k \Delta t_k)^{1/2}$ when path is segmented), or in definition/property-linked variants like $q = (v_{PM}\ l_{PATH}) = (v_{RMS}^{\ 2}\ t) = (v_{RMS}\ l_{RMS})$. q and t (plus l also) are essential variables whereas v_{RMS} is but one of the relevant along-path aggregated velocities. I have detailed how *qvel*, when paired with pathtime, can help discriminate event types through their *rms-pathlength*_cum_*rms-velocity* regimes. The *qvel* variable manifests also as the significant accumulator for geometrical-spreading-linked amplitude scaling along rays, plus in some other non-trivial circumstances.

Concerning issues on the broader scale, it is fairly clear that the (*t, q, l*) triplets are *necessary* and, to first order, *sufficient* for quantifying along-path velocity heterogeneity. The information is encapsulated in the geometrical-temporal detail of event fronts. When values of triplets for any particular event can be determined, such information can contribute to effective medium imaging.

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