An Edge-Preserving Algorithm for 2D Seismic Wavefield Inversion

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Summary

The reconstruction of material properties from sampled wavefields entails the solution of an ill-posed problem. In order to compute a stable and unique solution, the problem must be regularized. The most common regularization method, the damped least squares method, attempts to find a solution that exhibits a certain degree of smoothness. In this paper we present a method to retrieve high resolution images of discontinuities in the acoustic potential. The method incorporates a regularization term that enhances edges in the recovered image. In the absence of discontinuities, the regularization term defaults to a simple smoothing operator.

The method can be utilized to estimate high resolution images in VSP and cross-well tomographic experiments.

Introduction

In image reconstruction, the original object cannot be directly measured but must be recovered from observed data that are likely distorted or noisy. This problem is generally ill-posed. A regularization term is added to the cost function to stabilize the problem.

There are regularization approaches that constrain the solution of the problem to be smooth (Constable et al., 1987; de Groot-Hedlin and Constable, 1990). These methods have been used successfully in electromagnetic and magnetotelluric problems. This type of regularization works well in image restoration, but tends to blur the image. In contrast, the edge-preserving regularization method permits one to estimate the boundaries in material properties, and attempts to retrieve a sharp image of the discontinuities. This technique has been applied in geophysics (Sacchi and Ulrych, 1995), medical imaging (Blanc-Feraud et al., 1995; Charbonnier et al., 1997) and astronomy (Geman and Yang, 1995).

The edge-preserving regularization works by weighting the smoothing operator. Parameters are selected to choose a threshold value that distinguishes between small discontinuities caused by noise, and large discontinuities, or edges (Geman and Reynolds, 1992; Charbonnier et al., 1997). In this way, the discontinuities at every point in the image are evaluated, and the appropriate level of smoothing, if any, is applied.

Theory

Under the assumption of weak scattering, we can represent the measured wavefield using the Born approximation (Miller et al., 1987). The observed or scattered data in the seismic experiment, *d*, are linearly related to the acoustic potential, *f*, by:

$$d = Lf + \eta \, .$$

In this expression, *L* is the Born operator, η is Gaussian noise, and the acoustic potential *f(x)* is given by:

$$f(x) = \frac{1}{c^2(x)} - \frac{1}{c_0^2(x)} \, .$$

The known background velocity is $c_0(x)$, and the unknown velocity of the medium is *c(x)*.

Instead of introducing smoothing terms into the cost function, we choose a regularization term that highlights the presence of discontinuities:

$$J(f) = \left\| d - Lf \right\|^2 + \lambda^2 \sum_k \Phi\left[(D_x f)_k\right] + \lambda^2 \sum_k \Phi\left[(D_z f)_k\right].$$

The first term is the misfit function, the following ones are the regularization terms, and λ^2 is the weighting parameter. The operators D_x and D_z are given by:

$$(D_x f)_{i,j} = \frac{f_{i,j+1} - f_{i,j}}{\delta} \quad \text{and} \quad (D_z f)_{i,j} = \frac{f_{i+1,j} - f_{i,j}}{\delta} \, ,$$

where δ is a scaling parameter. The non-linear function, $\Phi(t)$, is chosen as:

$$\Phi(t) = \frac{t^2}{1+t^2}.$$

The solution to the cost function is:

$$f^{n+1} = \left[L^T L + \lambda^2 D_x^T B_x^{n+1} D_x + \lambda^2 D_z^T B_z^{n+1} D_z\right]^{-1} L^T d \ ,$$

where

$$trace\, B_x^{n+1} = \sum_k \frac{1}{\left[1+(D_x f^n)_k^2\right]^2} \quad \text{and} \quad trace\, B_z^{n+1} = \sum_k \frac{1}{\left[1+(D_z f^n)_k^2\right]^2}.$$

The weighting functions B_x^{n+1} and B_z^{n+1} mark the locations of the discontinuities and turn the smoothing operator off at these locations. The scaling parameter, δ, sets the sensitivity of the auxiliary variables to discontinuities, and therefore, controls the amount of smoothing applied. The algorithm makes use of iteratively re-weighted least squares to solve the non-linear problem.

Examples

To explore the effect of the weighting and scaling parameters, λ^2 and δ, we invert a 1D velocity model. Synthetic data with a signal to noise ratio of 3 are used. Figure 1 illustrates the influence of varying values of the scaling parameter. When δ is too small, all discontinuities qualify as edges and no smoothing is permitted. Conversely, when δ is too large, the smoothing operator receives full weight at all positions of the image, and the regularization term acts as a smoothing operator. When the scaling parameter is chosen correctly, a combination of smoothing and edge-preservation is applied.

In order to test the algorithm we prepare the following 2D synthetic test. The original perturbation and geometry are portrayed in Figure 2A. Synthetic data with a signal to noise ratio (SNR) of 8 are used (Figure 2B). The damped least squares solution (Figure 2C) is found using a conjugate gradient algorithm. The edge-preserving algorithm is applied to the model, and the weighting functions B_x^{n+1} and B_z^{n+1} of the last iteration are portrayed in Figures 3A and 3B respectively. The red indicates where the smoothing operator is on, and the blue marks the regions where it is turned off. The final solution is seen in Figure 3C.

Conclusions

In this paper, we propose an edge-preserving algorithm that can be used to reconstruct acoustic profiles from scattered wavefields. The 2D example illustrates that the algorithm is capable of preserving sharp discontinuities while still producing a piecewise smooth solution. Though a VSP example is shown, the technique is applicable in other image reconstruction problems.

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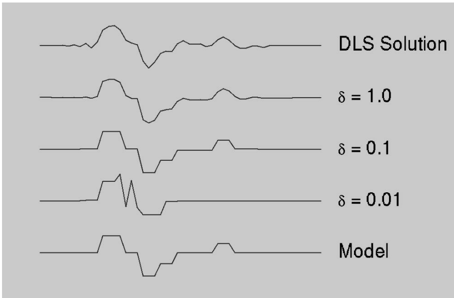

Fig. 1. The initial model, solutions for 3 values of δ, and the damped least squares solution (DLS). In all solutions, the weighting parameter, λ^2, is held constant.

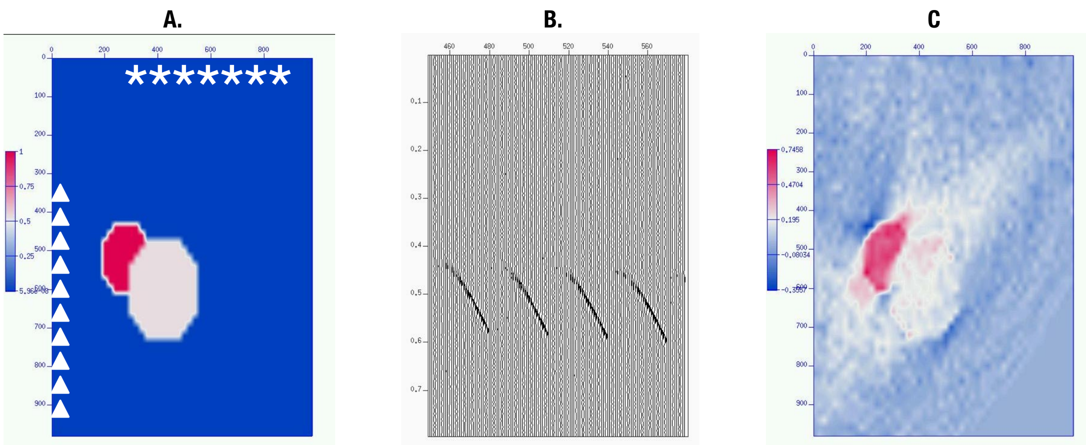

Fig. 2. A. The model of acoustic potential. B. The synthetic data (SNR=8). C. The damped least squares solution.

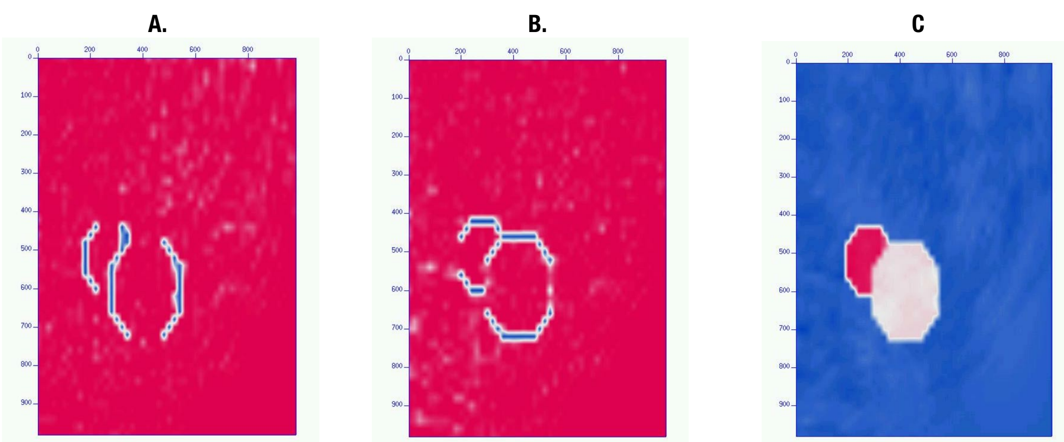

Fig. 3. The weighting functions A. B_x^{n+1} and B. B_z^{n+1} applied to the smoothing operator at the last iteration. Red indicates that the smoothing operator is on, while blue indicates that it is off. C. The final image recovered by the edge-preserving algorithm.