Reconstruction of incomplete seismic data using a minimum weighted norm least-squares algorithm

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Summary
In seismic data processing, we often need to interpolate missing source and receiver positions. The interpolation/resampling problem can be posed as an inverse problem where from inadequate and incomplete data, one attempts to recover the Fourier transform of the seismic wavefield. The estimated Fourier spectrum is used to reconstruct the wavefield. The Fourier transform can be inverted using a minimum weighted norm least squares algorithm.

Introduction
The interpolation problem can be posed as an inverse problem. One can try to estimate the Fourier transform of the regularly sampled data from irregularly acquired data. In this case, a least-squares minimization can provide a solution to the problem when the data are band-limited (Cary, 1997; Duijndam et al., 1998, Duijndam et al., 2000). Least squares solutions with zero-order quadratic regularization (damping) exhibit poor performance when facing the problem of interpolating large gaps of data. This is why we have modified the least-squares approach to include a weighted norm regularization term that helps to model the spectrum of the unknown signal. Cabrera and Parks (1991) have proposed a similar approach to interpolate time series. In their approaches, the missing samples are inverted rather than the coefficients of the discrete Fourier transform.

Theory
We define the discrete 2-D (e.g., source and receiver) inverse Fourier transformation for any source and receiver pair location \((x_S, x_R)\) as

\[ u(x_S, x_R, \omega) = \frac{1}{MN} \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} U(k_S(m), k_R(n), \omega)e^{j k_S(m)x_S}e^{j k_R(n)x_R}, \]  

where \(x_S\) and \(x_R\) are the spatial variables along source and receiver coordinates, \(k_S\) and \(k_R\) are the corresponding wave-numbers, and \(\omega\) is the temporal frequency. Equation (1) gives rise to a linear system equations

\[ u = FU \]  

where the \(u\) and \(U\) denote the known data and unknown coefficients of the DFT, respectively. A unique solution of \(U\) can be obtained by minimizing the following expression:

\[ \| u \|_Q + \| FU - u \|_2 \]  

where \(\| u \|_Q = \sum_k \left\{ \frac{U_k^* U_k}{Q_k} \right\} \) is a weighted DFT-domain norm (Cabrera and Parks, 1991) and the weighting function \(Q\) has the same spectral support as \(U_k\). The solution of equation (3) can be shown to take the form:

\[ \hat{U} = (F^H F + Q^{-1})F^H u. \]  

The elements of \(Q\) are computed from the amplitude spectrum of \(U\). Ideally, one should know the amplitude spectrum of the data. Unfortunately, \(U\) is the unknown of our problem. The latter can be overcome by defining \(Q\) in terms of the DFT of the irregularly sampled data \(F^H u\) and smoothing the result to attenuate the artifacts introduced by the irregularity of \(u\) (Ning and Nikias, 1990). The scheme can be summarized as follows:
1. Start with an initial $\hat{U}$

2. Compute $Q = S(\hat{U}^*\hat{U})$, where $S$ is a smoothing filter

3. Solve $\hat{U} = (F^H F + Q^{-1})F^H u$ using Conjugate Gradients

4. Iterate until convergence.

**Example**

The 2-D reconstruction is illustrated on synthetic shot gathers. Figure (1) shows 20 shot gathers with the shot #2, #3, and #11 removed. The reconstruction was carried along shot and receiver direction using the algorithm described above. The reconstruction is shown in Figure (2). The missing shots were completely reconstructed.

**Conclusions**

We have presented a method to interpolate pre-stack data. In this method, a frequency domain weighting function is used in the 2-D minimum norm least squares extrapolation. The algorithm generally gives a better reconstruction than the method of minimum norm least square without weighting. It is efficient since the algorithm is carried out in Fourier domain and, in general, is not restricted to the narrow-band case.

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**References**

Figure 1: Synthetic shot gathers. Shot #2, #3, and #11 were removed.

Figure 2: Reconstructed shot gathers from Fig. (1). The missing shots are completely reconstructed using minimum weighted norm least squares algorithm.