From stacking to interval velocities in a media with non-horizontal interfaces and inhomogeneous layers (explicit formulae approach)

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Introduction

We can divide reflected travel-time investigations into two parts. Into the first category, we may place a large number of studies, which deal with numerical methods for travel-time inversion (Goldin, Hubral and Krey, Al-Chalabi, Shah, Chernyak, Gritsenko and others). This approach requires numerical ray tracing. For the laterally varying layered media, within this approach, we cannot derive direct analytical relationships between interval and stacking velocities as we can for the horizontally homogeneous media. That means that we cannot obtain explicit general quantitative conclusions about how laterally varying boundaries (that is, boundaries with structures) influence stacking velocities.

Into the second category, we may place general investigations of direct analytical relationship between subsurface parameters (reflected boundaries and interval velocities) and stacking velocities. Most of these investigations were of laterally homogeneous layered medium [Bolshih, Taner and Koehler, Puzirjov].

Let us ask several questions: What happens if the boundaries or (and) interval velocities vary laterally? What lateral changes of the normal incident time $t_0$ and stacking velocities $V_{stack}$ can we expect? Do we need to take into account lateral velocity changes? Can we go from stacking velocities to the interval ones through the Dix formula and then to time-to-depth transformation? What are the restrictions for using this formula? What are the necessary corrections and when do we have to make them?

To answer these questions, the perturbation method was used to derive explicit formulas of stacking velocities and normal incident time for a laterally varying layered velocity model. The main question that I try to answer is: “What happens with the normal incident time $t_0$ and stacking (NMO) velocity when boundaries and interval velocities start to deviate from constant values?”

Theory

In this paper, for simplicity sake, I consider 2-D media but the same scheme can be applied to 3-D media as well. Let us consider layered media with the boundaries $z = F_k(x)$ and layered velocities $v_k(x)$, $k=1, 2, \ldots, m$. Let $\Upsilon(S, R)$ be the reflected wave raypath from the source $S(X-l/2,0)$ to the receiver $R(X+l/2,0)$ where $X$ is the surface midpoint and $l$ is the distance between $S$ and $R$, Fig.1; $P_k(\xi_k, F_k(\xi_k))$ –intersection points of the downgoing path, $Q_k(\eta_k, F_k(\eta_k))$ –intersection points of the upgoing path, $k = 1,2,\ldots,n$, $n$ – number of intersecting layers. Then for time $t(S, R)$, from the source $S$ to the receiver $R$, we can write

$$t(S,R) = \sum_{k=1}^{n} [t_k(\xi_{k-1}, \xi_{k}) + t_k(\eta_{k-1}, \eta_{k})] = T(S,R,\xi_1,\xi_2,\ldots,\xi_n,\eta_1,\eta_2,\ldots,\eta_n). \quad (1)$$

Fig. 1. Ray from the source $S$ to the receiver $S$ and intersection points

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Here $t(\xi_{k-1}, \xi_k)$ denotes traveltime from $P_{k-1}$ to $P_k$, $\tau(\eta_{k-1}, \eta_k)$ - traveltime from $Q_{k-1}$ to $Q_k$. Let $x_k$ be the $x$-projection of the segment $P_{k-1}P_k$, $x_k = \xi_{k-1} - \xi_k$; $y_k$ - $x$-projection of the segment $Q_{k-1}Q_k$, $y_k = \eta_{k-1} - \eta_k$. Then

$$
\xi_k = X - l/2 + \sum_{i=1}^{k} x_i,
\eta_k = X + l/2 - \sum_{i=1}^{k} y_i
$$

(2)

and

$$
l = \sum_{k=1}^{n} (x_k + y_k)
$$

(3)

Let us substitute (2) into equation (1). Then function $T$ depends on variables $x_1, x_2, ..., x_n, y_1, y_2, ..., y_n$. According to Fermat's principle $(x_1, x_2, ..., x_n, y_1, y_2, ..., y_n)$ is a stationary point of function $T$ under condition (3). If $L$ - Lagrangian function then $x_1, x_2, ..., x_n, y_1, y_2, ..., y_n$ and Langrangian multiplier $\lambda$ satisfie equation (3) and the system

$$
\frac{\partial T}{\partial x_k} = \sum \frac{\partial t_i}{\partial x_k} = \lambda, \quad \frac{\partial T}{\partial y_k} = \sum \frac{\partial \tau_j}{\partial y_k} = \lambda, \quad k=1,2,\ldots, m.
$$

(4)

Thus, the problem of ray path and time calculation is reduced to solving the nonlinear system of equations (3), (4). Let us consider equations (4) as equations with unknown $x_k, y_1, y_2, ..., y_n$ with the parameter $\lambda$. If we find $x_k, y_k$ as functions of a single variable $\lambda$ and substitute into right sides of (1), (3), we will get parametric representation of the traveltime $t(X,l)$:

$$
t = t(X,l,\lambda), \quad l = l(X,l,\lambda), \quad \lambda - \text{parameter.}
$$

(5)

Accurate solution of these equations can be obtained only for laterally homogeneous media [Bolshii, Taner and Koehler]. To find approximate solution for laterally inhomogeneous media, we can use perturbation approach. For this, with the original velocity model with the interval velocities

$$
v_k = v_k(x), \quad k=1, 2, ..., m
$$

we consider an auxiliary model with the layered velocities depending on small undimensional parameter $\varepsilon$:

$$
v_k(x,\varepsilon) = v_k(X) + \varepsilon u_k(x), \quad k=1, 2, ..., m
$$

(6)

Here $v_k(X)$ are constants (for the fixed midpoint $X$). We assume that interval velocity variation is small as compared to the average value of this velocity between the points $\xi_{k-1}$ and $\xi_k$. Then in (3) we can consider $u_k(x)$ to be approximately the same value as $v_k(X)$ and $\varepsilon$ to be a small dimensionless coefficient.

We can also use a small dimensionless coefficient $\mu$ to describe boundaries:

$$
F_k(x,\varepsilon) = F_k(X) + \mu G_k(x), \quad k=1, 2, ..., m
$$

(8)

Perturbation theory allows us to acquire the explicit formula of the travel time $t_k$ in the inhomogeneous $k$-th layer. This explicit presentation has the form of power series of $\varepsilon$:

$$
t_k(\xi_{k-1}, \xi_k) = t_k(0)(\xi_{k-1}, \xi_k) + t_k(1)(\xi_{k-1}, \xi_k)\varepsilon + t_k(2)(\xi_{k-1}, \xi_k)\varepsilon^2 + ...
$$

After that, we can write the system (4) - (5) as

$$
\frac{\partial T}{\partial x_k(\varepsilon,\mu)} = \sum \frac{\partial t_i}{\partial x_k(\varepsilon,\mu)} = \lambda, \quad \frac{\partial T}{\partial y_k(\varepsilon,\mu)} = \sum \frac{\partial \tau_j}{\partial y_k(\varepsilon,\mu)} = \lambda, \quad k=1,2,\ldots,m.
$$

This system has a solution that can be written as

$$
x_k = x_k^{(0)}(\lambda,\mu) + x_k^{(1)}(\lambda,\mu)\varepsilon + x_k^{(2)}(\lambda,\mu)\varepsilon^2 + ..., \quad y_k = y_k^{(0)}(\lambda,\mu) + y_k^{(1)}(\lambda,\mu)\varepsilon + y_k^{(2)}(\lambda,\mu)\varepsilon^2 + ...
$$

(9)

After finding the coefficients $x_k^{(1)}, x_k^{(0)}$, we substitute (9) into (5), solve the second equation for $\lambda$ and replace $\lambda$ in the first equation. This allows us to obtain an explicit formula for the time as a function of the midpoint $X$ and the offset $l$ in the form of power series of $l$:

$$
t^2(X,l) = c_0(X) + c_1(X) l^2 + c_2(X) l^4 + ...
$$

(10)

Here $c_0(X) = t_0^2(X)$, $c_1(X) = 1/V_{stack}^2$, where $t_0(X)$ is the normal incident time at the CDP point $X$ and $V_{stack}$ is the stacking velocity at the same point. For a medium with dipping boundaries, the approximate formula for normal incident time $t_0$ can be written as

$$
t_0(X) = 2 \sum_{k=1}^{n} \left( h_k/v_k \right) - \sum_{k=1}^{n} h_k \sum_{i=k}^{n} (1/v_i - 1/v_{i+1}) F'(x_i)^2
$$

(11)
Here \( v_k \) is the velocity in the \( k \)-th layer, \( h_k \) - the thickness of the layer at the point \( x = X \). The first term provides the travel time along the vertical ray in the model with horizontal layers. The second term gives the correction, defining the influence of the non-horizontal reflecting and transmitting interfaces. Practically, the second term defines the influence on the time \( t_o \) which is caused by the bias of the central ray from the vertical one.

For the stacking velocity we obtain:

\[
\frac{1}{V_{\text{stack}}^2} = \frac{1}{V_{\text{rms}}^2} \left[ 1 + \sum_{i=1}^{n-1} \left( \frac{1}{v_k} - \frac{1}{v_{k+1}} \right) F_k''(x) a_k \right] \tag{12}
\]

For the coefficient \( a_k \) we have

\[
a_k = \frac{\left( \sum_{i=k+1}^{n} h_i v_i \right)^2}{\left( \sum_{i=1}^{n} h_i v_i \right)}
\]

Formulae (11), (12), obtained for the normal incident time \( t_o \) and stacking velocity \( V_{\text{stack}} \), allow us to answer the questions written in the introduction.

**Medium with the curvilinear waterbottom line**

Let us consider a relatively simple but important marine case with a curvilinear waterbottom. It has been empirically noted that the lateral behavior of stacking velocities in marine seismic often shows a relationship with the structure of the waterbottom. Work that I have published in the Russian literature (Blias 1981, 1984, 1987) shows how a structured water bottom will change seismically measured stacking velocities with respect to the RMS velocities (Fig. 2). The difference between RMS and stacking velocity from deep boundaries can reach 30% and more. Therefore, when the Dix formula is used to obtain interval velocities, we get large errors since this formula assumes that our picked stacking velocities are close to the values of the RMS velocities.

To determine realistic interval velocities (and eventually realistic depths) we need to make corrections to the Dix formula. The main influence on lateral velocity changes is due to the structural curvature of the waterbottom. This suggests that the corrected Dix formula must include the second derivative of the water bottom horizon. Formula (12), connecting interval and stacking velocities, contains the second derivative of the transmission boundaries with some coefficients.

For a simple example, we can consider a 2-D case with a curvilinear boundary defined by the function \( F(x) \), which we will take to be the water bottom. If \( h_k \) is the thickness of the \( k \)-th layer and \( v_k \) is the velocity in this layer, then approximate formulae that connect stacking velocity \( V_{\text{stack}} \) with the interval velocity are

\[
\frac{1}{V_{\text{stack}}^2} = \frac{1}{V_{\text{rms}}^2} \left[ 1 + (1/v_1 - 1/v_2) F''(x) A_3 \right]
\]
Here \( F''(x) \) is the second derivative of the water bottom. If \( F''(x) = 0 \) (a flat water bottom) then we simply have the stacking velocity equivalent to RMS velocity. Any difference between RMS and stacking velocities will then depend on the expression \( (1/v_1 - 1/v_2) F''(x) A_n \).

A similar formula for the general case of a 3D medium with curvilinear boundaries and laterally varying interval velocities has been obtained. The next step to solve the problem is to invert these formulae in order to obtain the true interval velocities.

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