Two Generalized-screen operators for seismic imaging in complex media

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CSEG Geophysics 2002

Abstract

For a recording aperture of dimensions M by N, the cost of migration and modeling in heterogeneous media is proportional to $MN[M+N]$. To reduce this cost, we derive two efficient operators. One reduces the number of significant values in the extrapolation matrix by desampling the velocity field. The resulting cost is proportional to $mnMN\log_2(mn)$, where m and n are the dimensions of the desampled velocity field. The second operator restricts the number of unique velocities by rounding the velocity field to, for example, the nearest 10, 100 or 1000 m/s. This operator is implemented by a windowed superposition of stationary phase shifts. The cost of the second operator is proportional to $pMN\log(MN)$, where p is the number of unique velocities. Using the Marmousi seismic model, we find the operator based on velocity rounding returns greater accuracy for equivalent cost.

Theory

For one temporal frequency ω, a pseudo-differential operator solution (see Stein, 1993, p. 231) to the Helmholtz equation is (Grimbergen et al., 1998; Margrave and Ferguson, 1999; Le Rousseau and de Hoop, 2001)

$$\psi(x,y,z)=\int_{-\infty}^{\infty}\varphi(k_x,k_y,0)\exp\left(izk_z(x,y,k_x,k_y)-ik_x x-ik_y y\right)dk_x dk_y\ , \tag{1}$$

where x and y are spatial coordinates with Fourier duals k_x and k_y, z is depth, and k_z is vertical slowness

$$k_z(x,y,k_x,k_y)=\sqrt{\left(\frac{\omega}{v(x,y)}\right)^2-k_x^2-k_y^2}\ . \tag{2}$$

We will refer to $\exp(izk_z)$ as the *extrapolation symbol*. Velocity v varies in x and y, and spectrum φ is the 2D Fourier transform of the monochromatic wavefield obtained at surface $(x',y',0)$. Equation (1) extrapolates φ from surface $(x',y',0)$ to surface (x,y,z) simultaneously with a Fourier transform from (x,y) to (k_x,k_y). Because the domain change occurs without the benefit of the fast Fourier transform, in digital form, the computational effort required for extrapolation is proportional to $MN[M+N]$, where M and N are the dimensions of the recording aperture.

Introduce a background velocity $\tilde{v}\neq v$ so that

$$k_z(x,y,k_x,k_y)=\tilde{k}_z(x,y,k_x,k_y)\sqrt{1+\frac{\left(\frac{\omega}{v(x,y)}\right)^2-\left(\frac{\omega}{\tilde{v}(x,y)}\right)^2}{\tilde{k}_z(x,y,k_x,k_y)}}\ , \tag{3}$$

where

$$\tilde{k}_z(x,y,k_x,k_y)=\sqrt{\left(\frac{\omega}{\tilde{v}(x,y)}\right)^2-k_x^2-k_y^2}\ . \tag{4}$$

Substituting the first order expansion of equation (3) for k_z, equation (1) becomes

$$\psi(x,y,z)=\tilde{\psi}(x,y,z)\exp\left(iz\left[\frac{\omega}{v(x,y)}-\frac{\omega}{\tilde{v}(x,y)}\right]\right), \tag{5}$$

where

$$\tilde{\psi}(x,y,z)=\int_{-\infty}^{\infty}\varphi(k_x,k_y,0)\exp\left(iz\tilde{k}_z(x,y,k_x,k_y)-ik_x x-ik_y y\right)dk_x dk_y . \tag{6}$$

For $\tilde{v}=\bar{v}$, where $\bar{v}$ is constant, equation (5) is equivalent to the first-order generalized-screen operator (Wu, 1992), and the split-step Fourier operator of Stoffa et al. (1990). When $\tilde{v}=v$, equation (5) is equivalent to equation (1). Careful selection of $\tilde{v}$ intermediate to v and $\bar{v}$ provides operators more accurate than the first-order generalized-screen operator, and less costly to implement than equation (1). For later comparison, Figure 1b is a snapshot of a wavefield modeled using equation (1) and the velocity field of the Marmousi model (Bourgeois et al., 1991). A source, excited at $t=0$ at the location indicated by the star on Figure 1a, propagates through the velocity field until $t=0.95$ seconds where the field is imaged.

Desample the velocity field

The Fourier transform of $\tilde{\psi}$ (equation 6) is

$$\tilde{\varphi}(k_x',k_y',z)=\int_{-\infty}^{\infty}\varphi(k_x,k_y,0)A(k_x,k_y,k_x',k_y',z)dk_x dk_y , \tag{7}$$

where

$$A(k_x,k_y,k_x',k_y',z)=\int_{-\infty}^{\infty}\exp\left(iz\tilde{k}_z(x,y,k_x,k_y)\right)\exp\left(ix[k_x'-k_x]+iy[k_y'-k_y]\right)dxdy . \tag{8}$$

When $\tilde{v}=v$, the cost of implementing equation (7) is proportional to $MN[M+N]$ plus the cost of computing A, $(MN)^2\log_2(MN)$, plus the cost of inverse Fourier transform back to the space domain, $MN\log_2(MN)$. If $\tilde{v}$ is chosen, however, such that

$$\tilde{v}(x,y)=f(x,y)*v(x,y), \tag{9}$$

where f is a high-cut filter, the number of discrete samples in the extrapolation symbol is reduced from M by N, to $m<M$ by $n<N$ (Wapenaar, 1992; Margrave and Ferguson, 1999). The cost of approximating equation (6) using equation (7) is proportional to $mnMN\log_2(mn)$ to compute A, plus $MN(m+n)$ to apply A, plus Fourier transform to space coordinates $MN\log_2(MN)$. On a cost only basis, the use of equation (7) is justified when $M+N>mn\log_2(mn)+\log_2(MN)+m+n$. For example, when $M=1024$, $N=1$, $m=16$, and $n=1$, cost is reduced one order of magnitude.

Figure 2a is a snapshot of a wavefield similar to Figure 1b, but with the velocity field desampled to $m=16$. As a measure of accuracy, the autocorrelation of Figure 1b and the crosscorrelation of Figures 2a and 1b are computed, and the difference between their amplitudes squared along zero lag are plotted in Figure 3b. When compared to the result (not shown) using the first-order generalized-screen operator (Figure 3a), error is reduced between 4000 and 9000m, and the sum of the amplitudes is an order of magnitude less.

Reduce the number of unique velocities

For $\tilde{v}=\lambda\times\mathrm{round}(v/\lambda)$, where $\lambda\leq\min(v)$, the number of unique velocities p in $\tilde{v}$ is

$$p = \frac{\max(\tilde{v}) - \min(\tilde{v})}{\lambda}. \quad (10)$$

Analogous to the phase-shift-plus-interpolation (PSPI) method of Gazdag and Sguazzero (1984), for each unique velocity $\tilde{v}_j, 1 \le j \le p$, a window Ω_j is constructed so that it takes on a value of 1 where $\tilde{v}$ has the value $\tilde{v}_j$ and is zero otherwise. Similar to Margrave and Ferguson (1999), equation (6) becomes:

$$\tilde{\psi}(x, y, z) = \sum_{j=1}^{n} \Omega_j \int_{-\infty}^{\infty} \varphi(k_x, k_y, 0) \exp\left(iz\tilde{k}_{zj}(k_x, k_y) - ik_x x - ik_y y\right) dk_x dk_y \quad (11)$$

where

$$\tilde{k}_{zj}(k_x, k_y) = \sqrt{\left(\frac{\omega}{v_j}\right) - k_x^2 - k_y^2}. \quad (12)$$

Equation (11) constructs $\tilde{\psi}$ using constant-velocity phase-shift to extrapolate the input spectrum for each $\tilde{v}_j$, windows Ω_j are applied, and the resulting wavefields are superimposed. Unlike PSPI, no interpolation is used. The cost of implementing equation (5) using equation (11) is proportional to p times the cost $MN \log_2(MN)$ of the 2D FFT. Implementation of equation (11) is justified when $M + N > p \log_2(MN)$. When $M = 1024$, $N = 1$ and $p = 16$, cost relative to equation (1) is reduced one order of magnitude.

Figure 2b is a snapshot of a wavefield similar to Figures 1b and 2a, but with $p \approx 16$ achieved by rounding the velocity field to the nearest 150 m/s. The cost of computing Figure 2b is similar to the cost of computing figure 2a. The resulting zero-lag squared-error, Figure 3c, shows a significant error reduction everywhere along the model. To approach the same level of accuracy using equation (17) (Figure 3d), $m = 64$ is required with a five-fold increase in cost.

Conclusions

To reduce the cost of migration and modeling, two efficient operators are derived. The first desamples the velocity field to reduce the number of significant values in the extrapolation matrix, and the resulting cost is proportional to $mnMN \log_2(mn)$, where *m* and *n* are the dimensions of the desampled velocity field. The second operator restricts the number of unique values in the velocity field by rounding them at a cost proportional to $pMN \log(MN)$, where *p* is the number of unique velocities. Using the Marmousi seismic model, the operator based on velocity rounding is found to be more accurate for equivalent cost.

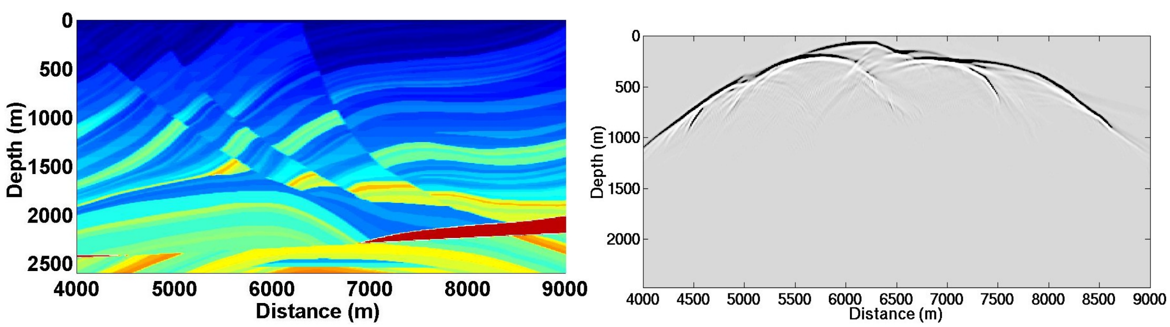

Figure 1a. Marmousi velocity model. The star marks source location. b) Snapshot of a wavefield modeled using equation (1) and the velocity in (a) imaged at $t = 0.95$ s.

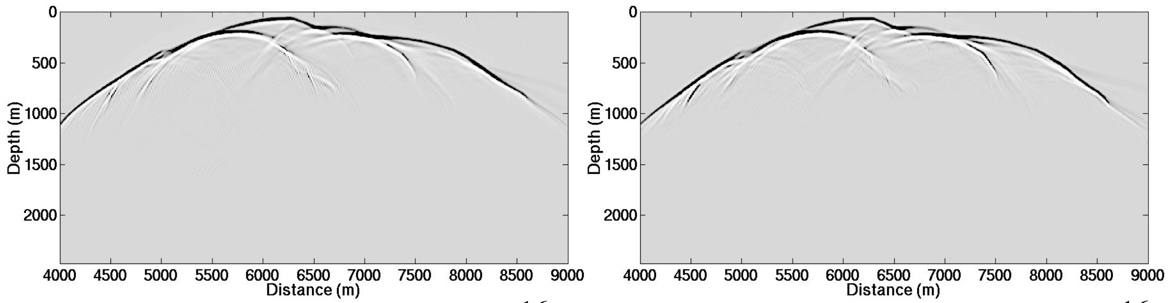

Figure 2a. Snapshot of a wavefield using equation (7) with $m = 16$. *b) Snapshot of a wavefield using equation (11) with* $p = 16$.

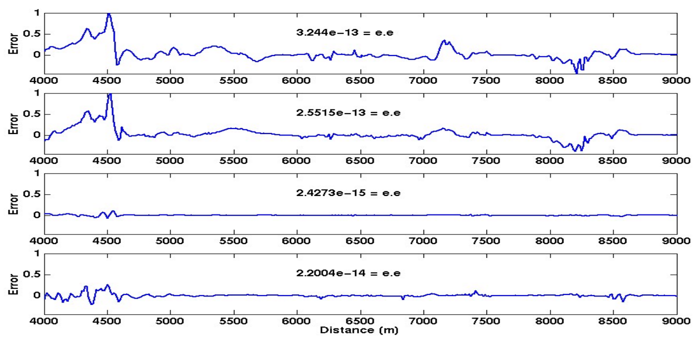

Figure 3. Amplitude-squared errors. a) Difference along zero lag of the autocorrelation of Figure 1b, and the crosscorrelation of Figure 1b and a wavefield (not shown) generated using the first-order generalized-screen. b) Difference using crosscorrelation of Figures 1b and 2a (desampled velocity). c) Difference using crosscorrelation of Figures 1b and 2b (rounded velocity). d) Difference using crosscorrelation of Figure 1b and a wavefield (desampled velocity, not shown) with $m = 64$.

References

Bourgeois, A., Bourget, M., Lailly, P., Poulet, M., Ricarte, P., and Versteeg, R., 1991, Marmousi, model and data: in The Marmousi experience, EAGE, 5 - 9.

Gazdag, J., and Squazerro, P., 1984, Migration of seismic data by phase shift plus interpolation: Geophysics, **49**, 124 - 131.

Grimbergen, J. L. T., Dessing, F. J., and Wapenaar, C. P. A., 1998, Modal expansion of one-way operators in laterally varying media: Geophysics, **63**, 995-1005.

Le Rousseau, J. H., and de Hoop, M. V., 2001, Modeling and imaging with the scalar generalized-screen algorithms in isotropic media: Geophysics, **66**, 1551-1568.

Margrave, G. F., and Ferguson, R. J., 1999, Wavefield extrapolation by nonstationary phase shift: Geophysics, **64**, 1067-1078.

Stoffa, P. L., Fokkema, J. T., de Luna Freire, R. M., and Kessinger, W. P., 1990, Split-step Fourier migration: Geophysics, **55**, 410-421.

Wu, R. S., 1992, Scattered field calculation in heterogeneous media using a phase-screen propagator: 62nd Annual Internat. Mtg., Soc. Expl. Geophys., Expanded Abstracts, 1289 - 1292.