Elastic-wave AVO methods

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Summary

For more than 3 decades, industry has known that shear seismic waves (S-waves) contain different information about rock properties than do standard compressional seismic waves (P-waves). Efforts to record PS converted-waves, even on the seabed, have attracted industry attention with increasing success. Separate efforts to analyse conventional pre-stack P-wave gathers for the S-wave information contained or "missing" within them, have had more success and popularity as a function of significantly lower recording cost and relatively simpler analysis. These methods termed P-wave AVO are a logical and quantifiable petrophysical extension into pre-stack seismic data from the confusing and simplistic interpretation of stacked amplitudes. Since the mid 90's new AVO inversion methods have been gradually succeeding in the exploration and development of gas pools within both clastic and carbonate plays. This paper compares various P-wave and PS converted wave AVO methods to estimate normal incidence P and S reflectivity (Rp and Rs) and extends the AVO equations for the potential to combine P-wave and PS converted wave AVO for a more robust method to extract elastic parameters such as Lamé parameters λ and μ, or LambdaRho $\lambda\rho$ and MuRho $\mu\rho$. These various methods will be applied to surface and walkaway VSP "AVO gather" data examples and verified to sonic and dipole log measurements. The presentation will also attempt to understand the value that this additional shear information has in exploration seismic applications.

Angle dependent reflectivity equations and approximations for AVO seismic or VSP analysis.

P-wave; A common starting point for AVO analysis is the linearized 3 term approximation to the Knott-Zoeppritz equations given by Aki and Richards (1980) shown below (equations 1, 7 and 8). These form the starting point for further 2 term approximations as the number of unknowns should not exceed the measurable CMP gather quantities (intercept and gradient) to ensure robust unambiguous parameter estimates, especially in the presence of noise (Cambois 2000). The basic linearized Aki and Richards assumptions are small fractional velocity and density changes with 2nd order terms ignored and θp < 10 degrees of critical.

$$\mathrm{Rpp}(\theta) = \frac{1}{2}\left(\frac{\Delta \mathrm{Vp}}{\mathrm{Vp}} + \frac{\Delta\rho}{\rho}\right) - 2\frac{\mathrm{Vs}^2}{\mathrm{Vp}^2}\sin^2\theta_p\left(2\frac{\Delta \mathrm{Vs}}{\mathrm{Vs}} + \frac{\Delta\rho}{\rho}\right) + \frac{1}{2}\tan^2\theta_p\frac{\Delta \mathrm{Vp}}{\mathrm{Vp}} \quad (1)$$

where: Vp, Vs velocities, ρ density, are averaged across an interface and angle θp (or just θ) is the average of incident and transmitted P-wave. $\Delta Vp/Vp$, $\Delta Vs/Vs$, $\Delta\rho/\rho$ are fractional changes in Vp, Vs and ρ across an interface and are equivalent to $\Delta \ln Vp$, $\Delta \ln Vs$ and $\Delta \ln\rho$. The common industry method considered here extracts Rp(0) and Rs(0) "seismic traces" through "weighted stacking" of CMP gather data (Gidlow et. al.1992, Fatti et. al.1994) which can be inverted into $\lambda\rho$ and $\mu\rho$. Starting from the Aki & Richards equation above with some algebraic manipulation gives;

$$\mathrm{Rpp}(\theta) = (1+\tan^2\theta)\frac{\Delta \mathrm{Ip}}{2\mathrm{Ip}} - 8\left(\frac{\mathrm{Vs}}{\mathrm{Vp}}\right)^2\sin^2\theta\frac{\Delta \mathrm{Is}}{2\mathrm{Is}} - \left(\frac{1}{2}\tan^2\theta - 2\left(\frac{\mathrm{Vs}}{\mathrm{Vp}}\right)^2\sin^2\theta\right)\frac{\Delta\rho}{\rho} \quad (2)$$

with $\frac{\Delta \mathrm{Ip}}{2\mathrm{Ip}} = \mathrm{Rpp}(0) = \frac{1}{2}\left(\frac{\Delta \mathrm{Vp}}{\mathrm{Vp}} + \frac{\Delta\rho}{\rho}\right)$ $\frac{\Delta \mathrm{Is}}{2\mathrm{Is}} = \mathrm{Rss}(0) = \frac{1}{2}\left(\frac{\Delta \mathrm{Vs}}{\mathrm{Vs}} + \frac{\Delta\rho}{\rho}\right)$

Equation (2) the "Geogain Equation" from Gidlow et. al. 1992, is solved in a least squares sense to extract Rp and Rs by assuming the 3rd term cancels for small density contrasts ($\Delta\rho/\rho$), as well as for small angles i.e. $\tan^2\theta p = \sin^2\theta p$, and $(Vs/Vp) = 1/2$. If the density contrast is not small then this third "error" term can be significant at large angles and more importantly inconsistent for varying rock properties by being dependent on both angle and Vp/Vs ratio. For Vp/Vs ratios < 2 the error between this 2 term equation and the exact Aki & Richards 3 term curve is small and evenly distributed, but still angle dependent. However for ratios > 2.5 or 3, this error increases with increasing angle and because the error term is strongly dependent on both angle and Vp/Vs ratio this restricts the useable range of angles.

In practice, however, this equation is very useful and can be used to fairly large angles as $\Delta\rho/\rho$ has the smallest variation compared to $\Delta Vp/Vp$ and $\Delta Vs/Vs$, seen in Gardner's (1974) relationship as $\Delta\rho/\rho \approx (\Delta Vp/Vp)/4$. If the angle range is restricted to a commonly quoted 30° due to the ignored error term, then the catch-22 problem is that the same large angle restriction reduces the separability of sin and tan curves involved in the remaining first two terms of equation (2). If this angle restriction can be reduced then more robust and independent estimates of $\Delta Ip/Ip$ and $\Delta Is/Is$ are possible in theory. However a different approximation with no angle dependent error can be obtained by substituting the following relationships into the original Aki & Richards equation (1) above;

$$\frac{\Delta \mathrm{Is}}{2\mathrm{Is}} = \frac{1}{4}\left(\frac{\Delta\mu}{\mu} + \frac{\Delta\rho}{\rho}\right), \quad \frac{\Delta \mathrm{Vp}}{2\mathrm{Vp}} = \frac{1}{4}\left(\frac{\Delta(\lambda+2\mu)}{(\lambda+2\mu)} - \frac{\Delta\rho}{\rho}\right) \text{ and } \frac{\Delta \mathrm{Vs}}{2\mathrm{Vs}} = \frac{1}{4}\left(\frac{\Delta\mu}{\mu} - \frac{\Delta\rho}{\rho}\right) \quad (3)$$

$$\Rightarrow \mathrm{Rpp}(\theta) = \frac{\Delta\rho}{2\rho} + (1+\tan^2\theta)\frac{\Delta \mathrm{Vp}}{2\mathrm{Vp}} - 4\left(\frac{\mathrm{Vs}}{\mathrm{Vp}}\right)^2\sin^2\theta\frac{\Delta\mu}{2\mu} \quad (4)$$

Now the approximation in this equation (4) (Goodway, 1997), is to drop the small zero offset term $\Delta\rho/2\rho$ (again from Gardner's relationship $\Delta\rho/2\rho \approx (\Delta Vp/Vp)/8$). Note that this error is independent of angle unlike the Gidlow et. al. 2 term approximations and hence allows a more accurate fit to the exact Aki & Richards AVO curve at large angles, but has a "bulk" $\Delta\rho/2\rho$ scalar shift at all angles. These 2 term approximations are compared to the Aki and Richards 3 term equation as graphed against incident angle in figure 1 below. Yet further interesting reformulations of the original Aki & Richards AVO equation can be obtained in terms of linear $\cos 2\theta$ only and Lamé moduli, density terms using the relationships in equation (3) above, as;

$$\mathrm{R}(\theta) = \frac{1}{2(1+\cos 2\theta)}\frac{\Delta(\lambda+2\mu)}{(\lambda+2\mu)} - (1-\cos 2\theta)\left(\frac{\mathrm{Vs}}{\mathrm{Vp}}\right)^2\frac{\Delta\mu}{\mu} + \frac{\cos 2\theta}{2(1+\cos 2\theta)}\frac{\Delta\rho}{\rho} \quad (5)$$

Note in equation (5) there is no density influence on R(θ) at 45° i.e. R(θ) is just a function of moduli as a ratio that is almost $\Delta(Vp/Vs)^2/(Vp/Vs)^2$. This concurs with Hilterman's work on far offset reflectivity (course notes on AVO accompanying the SEG DISC 2001). Finally a useful quadratic equation in terms of cos2θ:

$$\mathrm{R}(\theta)2(1+\cos 2\theta)=\frac{\Delta\lambda}{(\lambda+2\mu)}+\cos 2\theta\left(\frac{\Delta\rho}{\rho}+2\cos 2\theta\,\frac{\Delta\mu}{(\lambda+2\mu)}\right)\quad(6)$$

shows a similar though more interesting insight, to equation (5) above. The insight here is that at θ = 45° the reflectivity R(45) is just a scaled version of the change in the basic fluid sensitive indicator Δλ as;

$$2\mathrm{R}(45)=\frac{\Delta\lambda}{(\lambda+2\mu)}\qquad \text{where } \lambda+2\mu=\mathrm{Ip.Vp}\ (\text{product of averaged P impedance and interval velocity})$$

PS and S-wave; The Aki and Richards approximations to Zoeppritz equations for Rps(θ) and Rss(θ) are shown below with the same assumptions as the P-wave equations. These form the starting point for yet further approximations and combinations of PS and SS reflectivity equations used to extract Rs(0) that will be considered.

$$\mathrm{Rps}(\theta)=\frac{\boldsymbol{-p}\mathrm{Vp}}{2\cos\theta\mathrm{s}}\left[\left(1-2\mathrm{Vs}^2\boldsymbol{p}^2+2\mathrm{Vs}^2\frac{\cos\theta\mathrm{p}}{\mathrm{Vp}}\frac{\cos\theta\mathrm{s}}{\mathrm{Vs}}\right)\frac{\Delta\rho}{\rho}-\left(4\mathrm{Vs}^2\boldsymbol{p}^2-4\mathrm{Vs}^2\frac{\cos\theta\mathrm{p}}{\mathrm{Vp}}\frac{\cos\theta\mathrm{s}}{\mathrm{Vs}}\right)\frac{\Delta\mathrm{Vs}}{\mathrm{Vs}}\right]\quad(7)$$

$$\mathrm{Rss}(\theta)=-\frac{1}{2}\left(1-4\mathrm{Vs}^2\boldsymbol{p}^2\right)\frac{\Delta\rho}{\rho}-\left(\frac{1}{2\cos^2\theta\mathrm{s}}-4\mathrm{Vs}^2\boldsymbol{p}^2\right)\frac{\Delta\mathrm{Vs}}{\mathrm{Vs}}\quad(8)$$

where $\boldsymbol{p}$ is the constant ray parameter sin θs/Vs or sin θp/Vp, approximated to the average ray parameter.
Substituting equation (8) into equation (7) gives;

$$\mathrm{Rps}(\theta)=\frac{\boldsymbol{-p}\mathrm{Vp}}{2\cos\theta\mathrm{s}}\left[\left(2\mathrm{Vs}\cos\theta\mathrm{s}\frac{\cos\theta\mathrm{p}}{\mathrm{Vp}}\right)\left(\frac{\Delta\rho}{\rho}+\frac{2\Delta\mathrm{Vs}}{\mathrm{Vs}}\right)-\mathrm{Rss}+\frac{\Delta\rho}{2\rho}-\frac{1}{2\cos^2\theta\mathrm{s}}\frac{\Delta\mathrm{Vs}}{\mathrm{Vs}}\right]\quad(9)$$

Next by assuming cosθs = 1, $\cos^2\theta s = 1$, sinθs =0 gives;

$$\mathrm{Rps}(\theta)\approx\boldsymbol{-p}\mathrm{Vs}\left[\left(\frac{2\Delta\mathrm{Vs}}{\mathrm{Vs}}+\frac{\Delta\rho}{\rho}\right)\cos\theta\mathrm{p}+\frac{\mathrm{Vp}}{2\mathrm{Vs}}\frac{\Delta\rho}{\rho}\right]\quad(10)$$

This approximation for Rps(θ) (equation 10) can be seen to be the same as the Stewart et. al. (1997) approximation;
Rps(θ) ≈ 4(Vs/Vp) sinθp Rss(0), but without the cosθp = 1 assumption.
Rearranging the Stewart et. al. approximation;
=> Rss(0) ≈ (Vp/4Vs) cscθp Rps(θ)   (11)
Similarly equation (10) can be rearranged to give an Rss(0) estimate by scaling Rps(θ) by assuming [(Vp/2Vs)- cosθp] Δρ/ρ is zero for small Δρ/ρ and θp as well as Vp/Vs ≈ 2;
=> Rss(0) ≈ Rps(θ) /sinθs 4cosθp = (Vp/2Vs) csc2θp Rps(θ)   (12)
A further improvement for an Rs(0) estimate is shown below (Larsen et. al. 1999) by substituting the relationships in equation 3, into the Rps AVO Aki & Richards equation (7) given above;

$$\mathrm{Rps}(\theta)=-\tan\theta\mathrm{s}\left[\frac{\mathrm{Vp}}{\mathrm{Vs}}\left(\frac{1}{2}+\sin^2\theta\mathrm{s}\right)-\cos\theta\mathrm{s}\cos\theta\mathrm{p}\right]\frac{\Delta\rho}{\rho}-\left(\frac{\mathrm{Vp}}{\mathrm{Vs}}\tan\theta\mathrm{s}\sin^2\theta\mathrm{s}-\sin\theta\mathrm{s}\cos\theta\mathrm{p}\right)4\mathrm{Rss}(0)$$

$$\text{dropping the }\frac{\Delta\rho}{\rho}\text{ term assuming small density contrasts}\Rightarrow\mathrm{Rps}(\theta)\approx-\left(\frac{\mathrm{Vp}}{\mathrm{Vs}}\tan\theta\mathrm{s}\sin^2\theta\mathrm{s}-\sin\theta\mathrm{s}\cos\theta\mathrm{p}\right)4\mathrm{Rss}(0)$$

$$\Rightarrow\mathrm{Rss}(0)\approx\frac{-\mathrm{Rps}(\theta)}{4\sin\theta\mathrm{s}\left(\tan\theta\mathrm{s}\sin\theta\mathrm{p}-\cos\theta\mathrm{p}\right)}\quad(13)$$

These 3 approximations, the Stewart equation 11 (labeled Rss(0)'Stewart') and equation 12 (labeled Rss(0)'Goodway') as well as the new equation 13 (labeled 'new Rss(0)'), are compared below in figure 2, using the same model layer parameters used by Stewart et. al. (1997) for predicting Rss(0) by averaged summation or stacking. In conclusion the 'Goodway' equation (12) is a slightly better approximation than the 'Stewart' equation (11), while the new equation (13) has the best estimate of Rss(0)

Fig. 1, P-P refelctivity vs angle; 3 term Aki & Richard's compared to 2 term approximations Gidlow et al '92, Goodway '97

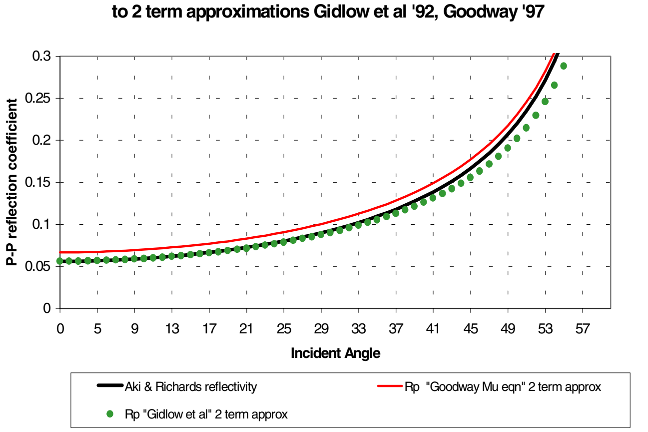
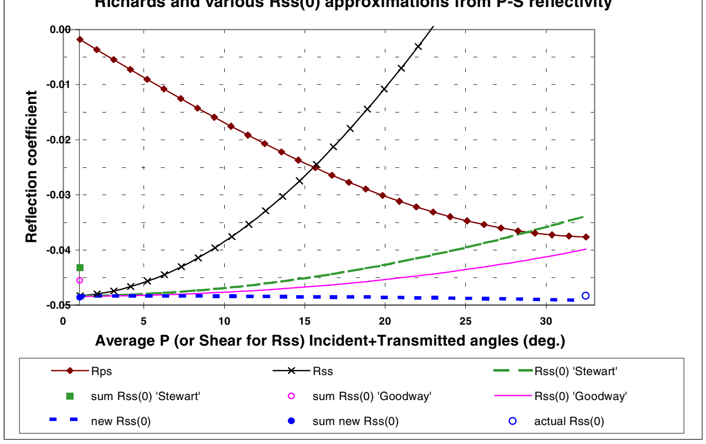


Fig. 2, S-S and P-S reflectivity vs P or S angles (deg.) from Aki & Richards and various Rss(0) approximations from P-S reflectivity



Combined PP and PS AVO inversion to obtain accurate estimates of Lamé moduli and density reflectivity.
The following combined P-wave and converted wave inversion method follows the various weighted stacking approaches of Smith et. al. 1987, Stewart 1991, Gidlow et. al. 1992 and Larsen et. al. 1999, to invert the Aki and Richards (1980) approximations to Zoeppritz equations for Rpp(θ) alone or joint Rpp(θ) and Rps(θ), so as to obtain estimates of P- and S-wave reflectivity i.e. Rp(0) and Rs(0). However by contrast to these previously published methods, the derivations that follow are exact and do not rely on dropping terms in fractional density changes or empirical (Gardner et. al. 1974) relationships between density and velocity (Vp) or impedance (Ip).
First rewriting the Aki and Richards Rpp (equation 1) in terms of Lamé parameters using the substitutions in equation (3) above;

$$\mathrm{Rpp}(\theta)=\frac{1}{4}(1+\tan^2\theta\mathrm{p})\frac{\Delta(\lambda+2\mu)}{(\lambda+2\mu)}-2\sin^2\theta\mathrm{p}\left(\frac{\mathrm{Vs}}{\mathrm{Vp}}\right)^2\frac{\Delta\mu}{\mu}+\frac{1}{4}(1-\tan^2\theta\mathrm{p})\frac{\Delta\rho}{\rho}\qquad(14)$$

Next rewriting the Rps equation 7, in Lamé parameter terms using the ΔVs/Vs substitution equation (3) above;

$$\mathrm{Rps}(\theta)=-\frac{\mathrm{Vp}}{\mathrm{Vs}}\frac{\tan\theta\mathrm{s}}{2}\frac{\Delta\rho}{\rho}+\frac{\mathrm{Vs}}{\mathrm{Vp}}\left(\sin^2\theta\mathrm{p}\tan\theta\mathrm{s}-\sin\theta\mathrm{p}\cos\theta\mathrm{p}\right)\frac{\Delta\mu}{\mu}\qquad(15)$$

From this Rps result either the density reflectivity Δρ/ρ or the shear or rigidity reflectivity Δμ/μ terms can be replaced in the Lamé formulated Rpp equation (14).

1) Replacing density reflectivity Δρ/ρ.

Rearranging the Rps equation (15);

$$\Rightarrow\frac{\Delta\rho}{\rho}=2\left(\frac{\mathrm{Vs}}{\mathrm{Vp}}\right)^2\left(\sin^2\theta\mathrm{p}-\frac{\sin\theta\mathrm{p}\cos\theta\mathrm{p}}{\tan\theta\mathrm{s}}\right)\frac{\Delta\mu}{\mu}-2\frac{\mathrm{Vs}}{\mathrm{Vp}}\frac{\mathrm{Rps}(\theta)}{\tan\theta\mathrm{s}}$$

replacing $\frac{\Delta\rho}{\rho}$ in Rpp equation (14)

$$\Rightarrow\mathrm{Rpp}(\theta)+\frac{1}{2}\frac{\mathrm{Vs}}{\mathrm{Vp}}\frac{(1-\tan^2\theta\mathrm{p})}{\tan\theta\mathrm{s}}\mathrm{Rps}(\theta)=\frac{1}{4}(1+\tan^2\theta\mathrm{p})\frac{\Delta(\lambda+2\mu)}{(\lambda+2\mu)}-\left(\frac{\mathrm{Vs}}{\mathrm{Vp}}\right)^2\sin\theta\mathrm{p}\left[\left(\frac{\sin\theta\mathrm{p}}{2}(3+\tan^2\theta\mathrm{p})+\frac{\cos\theta\mathrm{p}}{2\tan\theta\mathrm{s}}(1-\tan^2\theta\mathrm{p})\right)\right]\frac{\Delta\mu}{\mu}\qquad(16)$$

This last equation (16) is an exact relationship combining Rpp(θ) with an angle and Vs/Vp scaled version of Rps(θ) expressed in terms of only the two fractional changes of moduli or reflectivity i.e. P-modulus Δ(λ+2μ)/(λ+2μ) and shear modulus Δμ/μ.

2) Replacing rigdity reflectivity Δμ/μ.

Rearranging the Rps equation (15);

$$\Rightarrow\frac{\Delta\mu}{\mu}=\frac{\mathrm{Vp}}{\mathrm{Vs}}\left[\frac{\left(\mathrm{Rps}(\theta)+\frac{\mathrm{Vp}}{\mathrm{Vs}}\frac{\tan\theta\mathrm{s}}{2}\frac{\Delta\rho}{\rho}\right)}{\left(\sin^2\theta\mathrm{p}\tan\theta\mathrm{s}-\sin\theta\mathrm{p}\cos\theta\mathrm{p}\right)}\right]\Rightarrow2\left(\frac{\mathrm{Vs}}{\mathrm{Vp}}\right)^2\sin^2\theta\mathrm{p}\frac{\Delta\mu}{\mu}=2\frac{\mathrm{Vs}}{\mathrm{Vp}}\left(\mathrm{Rps}(\theta)+\frac{\mathrm{Vp}}{\mathrm{Vs}}\frac{\tan\theta\mathrm{s}}{2}\frac{\Delta\rho}{\rho}\right)\frac{\sin^2\theta\mathrm{p}}{\left(\sin^2\theta\mathrm{p}\tan\theta\mathrm{s}-\sin\theta\mathrm{p}\cos\theta\mathrm{p}\right)}$$

$$\Rightarrow-2\left(\frac{\mathrm{Vs}}{\mathrm{Vp}}\right)^2\sin^2\theta\mathrm{p}\frac{\Delta\mu}{\mu}=2\frac{\sin\theta\mathrm{s}\cos\theta\mathrm{s}}{\cos(\theta\mathrm{p}+\theta\mathrm{s})}\mathrm{Rps}(\theta)+\frac{\sin\theta\mathrm{p}\sin\theta\mathrm{s}}{\cos(\theta\mathrm{p}+\theta\mathrm{s})}\frac{\Delta\rho}{\rho}\qquad(17)$$

next replacing $\frac{\Delta\mu}{\mu}$ in Rpp equation (14) using equation (17)

$$\Rightarrow\mathrm{Rpp}(\theta)=\frac{1}{4}(1+\tan^2\theta\mathrm{p})\frac{\Delta(\lambda+2\mu)}{(\lambda+2\mu)}+\left[\frac{1}{4}(1-\tan^2\theta\mathrm{p})+\frac{\sin\theta\mathrm{p}\sin\theta\mathrm{s}}{\cos(\theta\mathrm{p}+\theta\mathrm{s})}\right]\frac{\Delta\rho}{\rho}+2\frac{\sin\theta\mathrm{s}\cos\theta\mathrm{s}}{\cos(\theta\mathrm{p}+\theta\mathrm{s})}\mathrm{Rps}(\theta)$$

$$\Rightarrow\mathrm{Rpp}(\theta)-2\frac{\sin\theta\mathrm{s}\cos\theta\mathrm{s}}{\cos(\theta\mathrm{p}+\theta\mathrm{s})}\mathrm{Rps}(\theta)=\frac{1}{4}(1+\tan^2\theta\mathrm{p})\frac{\Delta(\lambda+2\mu)}{(\lambda+2\mu)}+\left[\frac{1}{4}(1-\tan^2\theta\mathrm{p})+\frac{\sin\theta\mathrm{p}\sin\theta\mathrm{s}}{\cos(\theta\mathrm{p}+\theta\mathrm{s})}\right]\frac{\Delta\rho}{\rho}\qquad(18)$$

In a similar, though simpler result, this last equation (18) is an exact relationship combining Rpp(θ) with an angle scaled version of Rps(θ) expressed in terms of P-modulus Δ(λ+2μ)/(λ+2μ) and density Δρ/ρ reflectivity. Both equations (16) and (18) can be expressed in linearly weighted forms;

$$\mathrm{Rpp}(\theta)+\mathrm{m}(\theta)\mathrm{Rps}(\theta)=\mathrm{A}(\theta)\frac{\Delta(\lambda+2\mu)}{(\lambda+2\mu)}+\mathrm{B}(\theta)\frac{\Delta\mu}{\mu}\quad(19)\quad\text{and}\quad\mathrm{Rpp}(\theta)+\mathrm{n}(\theta)\mathrm{Rps}(\theta)=\mathrm{C}(\theta)\frac{\Delta(\lambda+2\mu)}{(\lambda+2\mu)}+\mathrm{D}(\theta)\frac{\Delta\rho}{\rho}\quad(20)$$

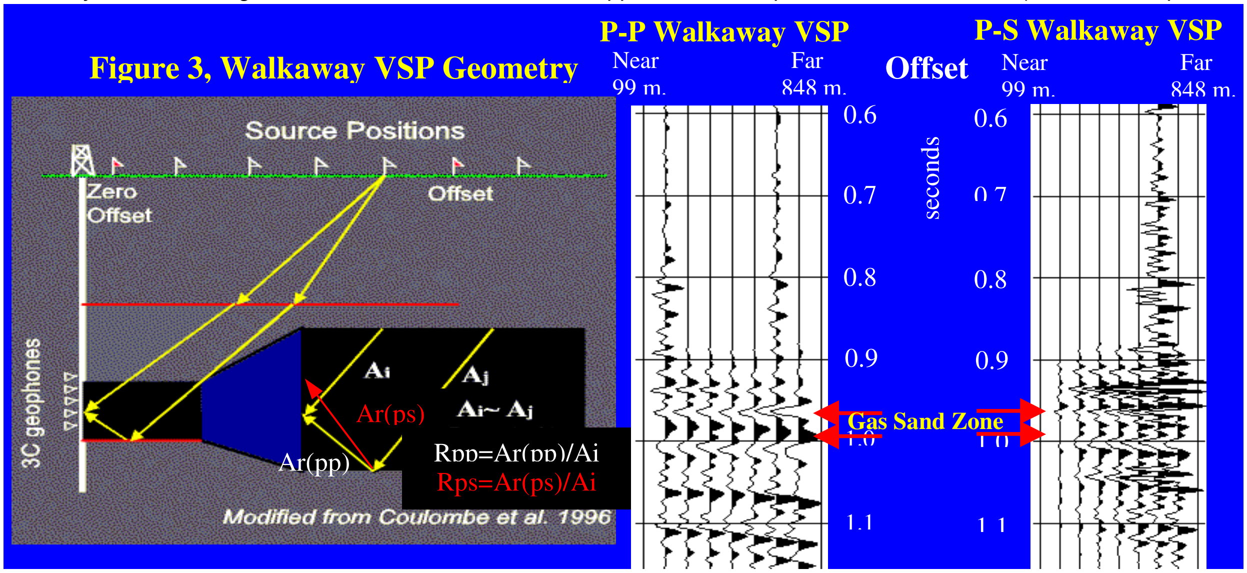
where m and n are angle dependent scalars and coefficients A, B, C and D are angle dependent weights.
Either equation (19) or (20) can be used in standard least-squares AVO model to data r(θ) error minimisation to give a pair of simultaneous equations that can be inverted for elastic parameter estimates Δ(λ+2μ)/(λ+2μ), Δμ/μ and Δρ/ρ as shown by the matrices below;

$$\begin{bmatrix}\frac{\Delta(\lambda+2\mu)}{(\lambda+2\mu)}\\ \frac{\Delta\mu}{\mu}\end{bmatrix}=\begin{bmatrix}\sum_{\vartheta=0}^{\vartheta\max}\mathrm{A}^2(\theta) & \sum_{\vartheta=0}^{\vartheta\max}\mathrm{A}(\theta)\mathrm{B}(\theta)\\ \sum_{\vartheta=0}^{\vartheta\max}\mathrm{A}(\theta)\mathrm{B}(\theta) & \sum_{\vartheta=0}^{\vartheta\max}\mathrm{B}^2(\theta)\end{bmatrix}^{-1}\begin{bmatrix}\sum_{\vartheta=0}^{\vartheta\max}\mathrm{A}(\theta)\mathrm{r}(\theta)\\ \sum_{\vartheta=0}^{\vartheta\max}\mathrm{B}(\theta)\mathrm{r}(\theta)\end{bmatrix}\text{ from equation (19)}$$

$$\begin{bmatrix}\frac{\Delta(\lambda+2\mu)}{(\lambda+2\mu)}\\ \frac{\Delta\rho}{\rho}\end{bmatrix}=\begin{bmatrix}\sum_{\vartheta=0}^{\vartheta\max}\mathrm{C}^2(\theta) & \sum_{\vartheta=0}^{\vartheta\max}\mathrm{C}(\theta)\mathrm{D}(\theta)\\ \sum_{\vartheta=0}^{\vartheta\max}\mathrm{C}(\theta)\mathrm{D}(\theta) & \sum_{\vartheta=0}^{\vartheta\max}\mathrm{D}^2(\theta)\end{bmatrix}^{-1}\begin{bmatrix}\sum_{\vartheta=0}^{\vartheta\max}\mathrm{C}(\theta)\mathrm{r}(\theta)\\ \sum_{\vartheta=0}^{\vartheta\max}\mathrm{D}(\theta)\mathrm{r}(\theta)\end{bmatrix}\text{ from equation (20)}$$

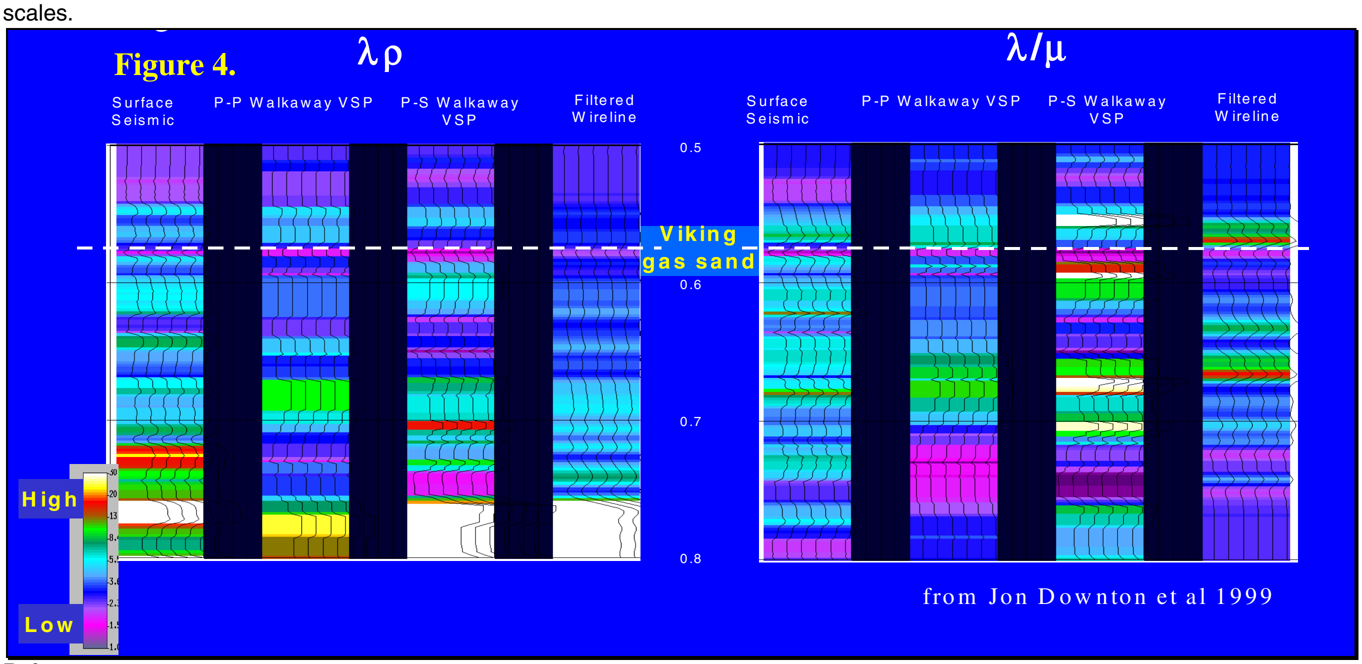
This combined PP/PS method is stable and robust with a further constraint in that the fractional P-modulus Δ(λ+2μ)/(λ+2μ) term estimated from both (19) and (20) matrix equations must converge. However the method does require a nearly perfect match between the recorded

PP and PS r(θ)'s, which may be problematic with surface data of high PP to PS bandwidth ratio, but is readily achievable with a walkaway VSP survey as shown in figure 3, where the PS data have an apparent 20Hz improvement in bandwidth (time axis compression) over PP.



Case study seismic and walkaway VSP AVO inversion for elastic parameters.

A walkaway VSP (figure 3) provides a real seismic data set for calibration of surface AVO, as well as the evidence of a quantifiable AVO response (Downton, Goodway & Chen 1999). This surface to borehole VSP seismic calibration has a number of advantages in that a VSP is a controlled experiment where the results are directly tied to well logs, so that it is possible to quantify the reliability of the different AVO methods described above. 3D surface seismic was also acquired and processed in a similar fashion to the VSP and an example of estimating elastic parameters from an AVO extraction on both the P-wave surface and VSP seismic data, as well as the VSP PS converted wave data is shown in figure 4. The Viking gas sand is clearly identified and remarkably well resolved at all measurement scales.



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