Seismic Scale Effects: Dispersion, Attenuation, and Anisotropy by Multiple Scattering of Waves
Yinbin Liu and Douglas R. Schmitt
Institute for Geophysical Research, Department of Physics, University of Alberta

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Summary

The multiple scattering of seismic waves in heterogeneous media causes velocity dispersion and waveform distortion. A propagator matrix approach is used to model the scale-dependent velocity dispersion and waveform distortion caused by the interference of intrabed multiple reflections in strong 1D heterogeneous media. The results indicate that the velocity transition from ray to effective media domains mainly takes place at \( R = \lambda/d = 10 \) (the ratio of wavelength to layer spacing), and that velocities are equal to ray velocity for small \( R \)-values and effective media velocity for large \( R \)-values. The waveform distortion results in scale-dependent low frequency behavior and coda wave. The low frequency wave occurs as either coherent scattering attenuation for small \( R \) or coherent scattering enhancement for large \( R \). Near the effective media region, the low-frequency energy transfers into an enhanced main wave-type and a high frequency coda wave. As \( R \) increases, the amplitude of coda wave decreases while the frequency increases. This work has implications to the problem of synthetic model mis-ties for reflection profiles.

Introduction

A seismic signal propagating through heterogeneous media undergoes velocity dispersion and amplitude attenuation because of intrinsic absorption from anelasticity and multiple scattering from heterogeneity. These effects result in delaying arrival time and waveform distortion. Intrinsic absorption, which converts seismic energy to heat, carries lithological information and is basically independent of seismic frequency; multiple scattering, which redistributes seismic energy, carries stratigraphic, structural and pore fluid information and is strongly dependent on seismic frequency. This work only studies the influence of multiple scattering on velocity dispersion and waveform distortion.

Seismic wave scattering is strongly dependent on the inhomogeneous scale of media. Ray theory best describes wave propagation when the scale of heterogeneity is larger than seismic wavelength (high frequency or short-wavelength approximation). Effective media theory (e.g., Backus, 1962) describes wave propagation when the scale of heterogeneity is much less than seismic wavelength (low frequency or long-wavelength approximation). For intermediate frequencies, where the scale of heterogeneity is comparable to seismic wavelength, statistical averages and probability densities are generally employed to describe scattering processes in weak inhomogeneous media. However, the accumulation zones of oil and gas are usually strong heterogeneity. To our knowledge, no effective method may be employed to study the multiple scattering for strong 3-D heterogeneity.

A universal feature of sedimentary sequences is that they tend to be layered (1-D heterogeneity). Reservoir thicknesses within these sequences are usually much less than seismic wavelengths. The multiple scattering of seismic wave within depositional sequences and reservoirs remains poorly understood. Therefore, the understanding for the multiple scattering of seismic wave in strong 1-D heterogeneous media has implications for doing more accurate stratigraphic interpretations and subtle reservoir evaluation. Many essential features for 3-D heterogeneity scattering can be captured with 1-D models. Insight gained from such simple models is an important aid in understanding more complicated 3-D case.

O'Doherty and Anstey (1971) discussed the influence of intrabed multiple scattering (or stratigraphic filtering) and intrinsic absorption on seismic amplitudes. They showed that the attenuation apparent on reflection seismic records can mainly be attributed to intrabed multiples in cyclic sedimentary system in which the impedances of layers tend to alternate rapidly between high and low values. Since this time, there are some theoretical and experimental discussions on velocity dispersion and apparent attenuation using well log data (e.g., Stewart et al., 1984; Banik et al., 1985; Kebaili and Schmitt, 1996). However, above discussions only considered the scattering in weakly heterogeneous media and did not study the multiple scattering in strongly heterogeneous media. This paper models the scale-dependent velocity dispersion and waveform distortion for strong 1-D heterogeneity following Marion et al's (1994) plastic/steel stack experiment.

Model and algorithm

Model

We consider plane wave propagation through a quasiperiodic plastic/steel stack embedded between two fluid half-spaces as shown in Figure 1 (Marion et al., 1994). For convenience, the velocity and density of fluid half spaces are assumed to be equal to the P-wave velocity and density of the first layer. The materials have the thicknesses \( d_1 \) and \( d_2 \) with the spatial period \( d = d_1 + d_2 \) and a periodic structure of 'M'. There are two reasons to choose a plastic/steel model: first, there is experimental result for this model and so we may compare the results between theory and experiment; second, the physical properties of the plastic and the steel differ substantially and the resulting medium may be considered to some degree an extreme case.

![Figure 1. Quasiperiodic structure with two constituent materials. Black boxes represent plastic while white boxes represent steel.](image)

The changing scale models (\( R = \lambda/d \), where \( \lambda \) is the seismic wavelength of transmission signal) are constructed by choosing a different periodic structure 'M' (or arrangement) and keeping the total thickness ('D') = M'd and the proportion of materials (\( P_i \) = 32.7%). Individual layers change thickness from near seismic wavelength (small M, ray theory) to much less than seismic wavelength (large M, effective media theory). The properties of
the constituent materials are given in Table 1. The corresponding anisotropic constants of transversely isotropic effective media (TIE media, low frequency limit) are shown in Table II. Thomson's anisotropic parameters (Thomson, 1986) are shown in Table III, where $\beta_i$ is the corresponding normal P wave effective velocity. The plastic/steel stack is strongly anisotropic.

### Table I. Parameters of constituent materials

<table>
<thead>
<tr>
<th>Medium</th>
<th>$v_1 (m/s)$</th>
<th>$v_2 (m/s)$</th>
<th>$\rho (g/cm^3)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plastic</td>
<td>2487</td>
<td>1048</td>
<td>1.21</td>
</tr>
<tr>
<td>Steel</td>
<td>5535</td>
<td>300</td>
<td>7.9</td>
</tr>
</tbody>
</table>

### Table II. Parameters of TIE medium

<table>
<thead>
<tr>
<th>Medium</th>
<th>$C_{11}$ (GPa)</th>
<th>$C_{12}$ (GPa)</th>
<th>$C_{13}$ (GPa)</th>
<th>$C_{44}$ (GPa)</th>
<th>$\rho$ (g/cm³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plastic/Steel</td>
<td>142</td>
<td>45</td>
<td>11</td>
<td>22</td>
<td>4</td>
</tr>
</tbody>
</table>

### Table III. Thomsen anisotropic parameters

<table>
<thead>
<tr>
<th>Medium</th>
<th>$\alpha_0$ (m/s)</th>
<th>$\beta_0$ (m/s)</th>
<th>$\epsilon$</th>
<th>$\delta$</th>
<th>$\delta^*$</th>
<th>$\gamma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plastic/Steel</td>
<td>1941</td>
<td>828</td>
<td>2.8</td>
<td>5.7</td>
<td>-0.13</td>
<td>-2.7</td>
</tr>
</tbody>
</table>

**Method** A propagation matrix method is employed to study the multiple scattering in 1-D heterogeneous media. The propagator method is an exact analytic method and so includes all multiple waves. In the following case, we consider only normal incidence and so the influence of shear wave can be ignored. The incident plane wave is a single cycle pulse with a 200 kHz dominant frequency.

### Scale-dependent velocity dispersion and waveform distortion

Figure 2 shows the calculated transmission pulse response for $M = 1$ to 128 when the total thickness is 52 mm with $d_1 = 17$ mm plastic and $d_2 = 35$ mm steel. The thickness of the individual layers change from $d_1 = 17$ mm and $d_2 = 35$ mm for $M = 1$ (ray theory) to $d_1 = 0.13$ mm and $d_2 = 0.27$ mm for $M = 128$ (effective media theory). The corresponding $R = \lambda_d$ varies from 0.4 to 31.7. The velocity transition and waveform distortion from ray (small R) to effective media (large R) can be clearly seen. In the transitional region (intermediate R), the multiple scattering results in strong scale-dependent velocity dispersion and waveform distortion.

### Velocity dispersion

The arrival times are early for the small $R$ case and late for large $R$ case. The intrabed multiple reflections are resolvable for a small $M$. The first pulse for $M = 1$ to 4 in Figure 2 is a direct transmission wave and the following pulses are the multiple reverberations. The amplitude of the direct wave drastically decreases with increasing $M$ because of transmission loss. Figure 3 reproduces the waveform at $M = 4$ with a factor 8 and shows that the amplitude of the direct wave is much less than that of the following multiple waves. The amplitude of the direct wave tends to zero for larger $M$ value and cannot be detected. Therefore, the first arrival at large $M$ values is not a direct wave but multiples. This results in the time delay or velocity dispersion, as seen in Figure 2. O’Doherty and Anstey (1971) and Banik et al. (1985) have pointed out this feature of multiple scattering. There are several approaches to picking the travel times in order to obtain the velocity of dispersive waves (e.g., Molyneux and Schmitt, 2000). In this paper, the travel times of the first arrivals are estimated by picking the time at 1% of the maximum amplitude. The average velocity was determined as the total thickness of the layered media divided by the picked arrival time, while the wavelength, $\lambda$, was taken as the average velocity multiplied by the observed dominant period — the time from the first arrival pick to the second zero crossing.

![Figure 2. Theoretical waveforms for the plastic/steel stack ($P_I = 32.7\%$) with $M = 1$ to 128 ($R = 0.4$ to 31.7) and a total thickness of 52.0 mm. The pulse is normal incidence with a dominant frequency $f_0 = 200$ khz. Dashed vertical lines RT and EMT denote the ray medium, small wavelength and effective medium, long wavelength traveltimes, respectively. The multiple scattering results in strong scale-dependent velocity dispersion, low frequency behavior, and coda wave. The low frequency wave occurs as either coherent scattering attenuation or coherent scattering enhancement.](image)

![Figure 3. Theoretical waveforms for the plastic/steel stack with $M = 4$ ($R = 1.0$) at normal incidence illustrating a weak arrival at the predicted ray theoretical time.](image)
Figure 4. Theoretical scattered P wave velocities calculated from first arrivals as a function of the ratio of wavelength to layer spacing, \( R = \frac{\lambda}{d} \) for normal incidence.

**Waveform distortion** Figure 2 shows that the multiple scattering results in low frequency behavior (stratigraphic filtering) and coda wave. The low frequency wave is due to the buildup of the interference of multiple reflections. Amplitude and frequency of the wave increase, whereas duration time decreases with decreasing inhomogeneous scale or increasing \( R \). The low frequency wave occurs as either coherent scattering attenuation for small \( R \), which is due to the destructive interference of the multiple scattering, or coherent scattering enhancement for large \( R \), which is due to the constructive interference of the multiple scattering. As \( R \) increases further, the low-frequency energy transfers into an enhanced main wave type and a high frequency coda wave. The amplitude and frequency of the coda wave decrease and increase, respectively, with increasing \( R \). In the effective media region, the energy of the coda wave decreases to zero and all of the energy transfers into a direct transmission wave in a transversely isotropic effective media (Liu and Schmitt, 2000).

**Conclusions**

The scale-dependent velocity dispersion and waveform distortion caused by the interference of the intrabed multiple reflections for strong 1D heterogeneity are numerically studied using a propagator matrix method. The velocities are equal to ray velocity for small \( R \) and effective media velocity for large \( R \). The transition from ray to effective media velocities mainly takes place at \( R = \frac{\lambda}{d} = 10 \). The buildup of multiple reflections results in scale-dependent low-frequency behavior, which occurs as either coherent scattering attenuation at large heterogeneity scales, or coherent scattering enhancement at small heterogeneity scales. Near the effective media region, the low-frequency energy transfers into an enhanced main wave-type and a high frequency coda wave. These results have implications towards the velocity dispersion seen between sonic log and seismic band frequencies.

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