Determination of the complete elastic stiffnesses from ultrasonic phase velocity measurements

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Summary

The complete set of 21 elastic stiffnesses of a well studied anisotropic composite material was found from ultrasonic measurements of the phase velocity. Using the τ-*p* method, both quasi-P and quasi-S wave phase speeds were determined at a variety of angles within a number of differing planes through the material. The 779 individual phase velocities were inverted to yield the elastic stiffnesses while assuming no *a priori* information about the symmetry of the material or orientation of the symmetry axes. The near orthorhombic nature of the material, observed in the elastic stiffnesses, is indicated by the texture. This is further confirmed by various bootstrap tests of the inversion assuming different levels of symmetry from triclinic to orthorhombic and using different subsets of the data.

Introduction

The intrinsic properties of the earth have only recently been taken into consideration when processing seismic data. This seismic data allows us to image deep beneath the earth's surface in the search for oil and gas. Such images are produced by the common midpoint method that almost exclusively assumes the earth is composed of isotropic layers. However, much of the rocks through which the seismic waves pass are anisotropic. In an anisotropic medium, there is sideslip of energy as it propagates leading to lateral positioning errors (Isaac & Lawton, 1999; Vestrum & Lawton, 1999). When conventional seismic processing does not take anisotropy into account the earth is poorly imaged (Vestrum et al., 1999) leading to erroneous interpretations of the structure of the earth and a lower likelihood of finding oil and gas.

In order to ameliorate this problem, one must first understand the intrinsic properties of the rocks through which the seismic waves pass. The velocity anisotropy present in the rock is directly related to the material's elastic properties and an important goal of experimental anisotropy determination is to determine the elastic constants of the material. Once this is known, these values can be used to model the complex wavefields as they propagate through the earth.

A method for determining the complete set of all 21 independent elastic constants from ultrasonic compressional and shear wave measurements is described. This builds on the earlier work using the τ-*p* transform as described in Kebaili & Schmitt (1997), Mah & Schmitt (2001a), and Mah & Schmitt (2001b).

Phase velocity determination

Specially constructed piezoelectric transducers are placed on the block in a colinear array in a manner similar to that described in Kebaili & Schmitt (1997) (Figure 1). Once the waveforms from the receivers are recorded, a τ-*p* transform is performed on the data. The τ-*p* transform, also known as the slant-stack (Radon) transform, is defined by Tatham (1984) as

$$F(t\,,p) = \int_{-\infty}^{\infty} f(x,t + px)dx$$

where τ is the intercept time, *p* is the horizontal slowness, *x* is the offset, and *f(x,t)* is the seismic data as a function of offset and time.

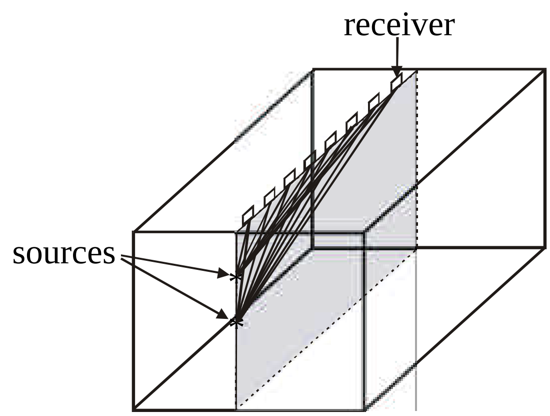

Figure 1. View of one plane of investigation showing the coplanar array containing the sources and receivers.

From the data in the τ-*p* domain, the phase velocities can be calculated as a function of the phase propagation direction. This allows us to accurately determine the phase velocities at off-axis angles that previous methods have had problems with. This method was applied to a block of phenolic in the laboratory (Kebaili & Schmitt, 1997). Phenolic is composed of layers of canvas held together by a phenol resin (Figure 2). The weaving of the canvas gives two directions of symmetry and the layering of the canvas gives a third direction of symmetry. This gives phenolic a predominantly orthorhombic symmetry. The phase velocities were determined with the direction of propagation within the phenolic. However, the elastic constants could not be determined by Kebaili & Schmitt (1997) since only P-wave transducers were used. In order to determine the elastic constants of any material both P-wave and S-wave velocities must be measured.

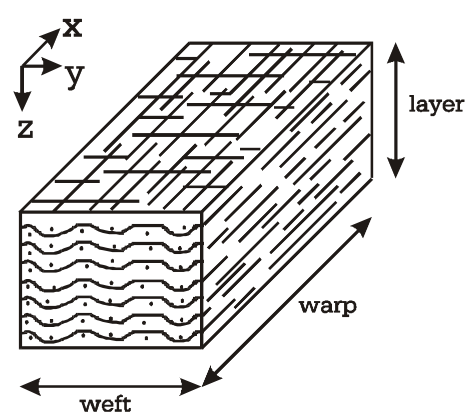

Figure 2. Diagram of weaving and layering of canvas in phenolic sample and orientation of the coordinate axes with respect to the observed fabric.

Using specially constructed piezoelectric transducers that are sensitive to the various particle motions, both the P-wave and S-wave arrivals were recorded by the coplanar arrays of sources and receivers. The phase velocities were determined in an isotropic glass block using the τ-*p* method (Mah & Schmitt, 2001a). The velocities that were determined using this method were compared with the velocities as determined using conventional pulse transmission methods and were found to match within 1%. This indicates that the methodology is accuate to within 1% of conventional methods in determining phase velocities. Using the calculated P-wave and S-wave phase velocities the two elastic constants needed to describe the glass block were determined.

This methodology was then extended to the anisotropic material phenolic (Mah & Schmitt, 2001b). These special coplanar arrays of sources and receivers were used to record the arriving waveforms from the three different polarizations in 4 different planes of investigation. The different planes of investigation were chosen on the assumption that the material was orthorhombic and the axes of symmetry were known based on the observed fabric. The phase velocities were determined for the block of phenolic in the different planes of investigation using the τ-p transform. The 9 elastic constants of the stress-strain tensor were then determined using an inversion of the calculated phase velocities.

The assumption that the axes of symmetry and the level of anisotropy of the material are known may not always be the case. If one miss-chooses the axes of symmetry of the material the symmetry observed becomes more complex. Also the anisotropy of the material may be more complex than expected or than what was assumed. According to Van Buskirk et al. (1986), only the velocities of the three modes of propagation need be measured in six well-chosen directions in order to determine all 21 elastic constants. This is valid if the associated particle motions are perfectly known. In our methodology, this indicates that if 3 perpendicular planes of investigation are used, all 21 elastic constants can be determined. For this talk, we have chosen 5 different planes of investigation in order to determine all 21 elastic constants (Figure 3).

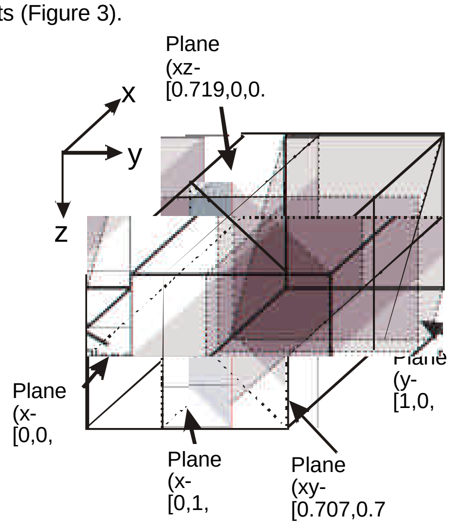

Figure 3. Diagram of the 5 different planes of investigation. Planes 1, 2, and 3 are parallel and perpendicular to the coordinate system chosen. Planes 4 and 5 are neither perpendicular nor parallel to one of the axes. The unit vectors normal to the planes are indicated in the square brackets.

An example of the observed waveforms for a co-linear array of transducers is shown in Figure 4. The corresponding τ-p transform is shown in Figure 5. Using the observed waveforms and the corresponding τ-p transforms from the 5 planes of investigation, 779 phase velocities were determined using the 3 different propagation modes in a block of phenolic. The phase velocities were then inverted to yield the 21 elastic constants of the stress-strain tensor. The material was found to be more or less orthorhombic with the "non-orthorhombic" elastic constants being quite small. To test if these non-zero "non-orthorhombic" elastic constants were caused by a poor choice of symmetry axes, a brute force search of possible rotations to reduce them was carried out. No rotation was found that would reduce these "non-orthorhombic" elastic constants. This indicates that the material is indeed predominantly orthorhombic and that the axes of symmetry chosen based on the observed fabric is valid.

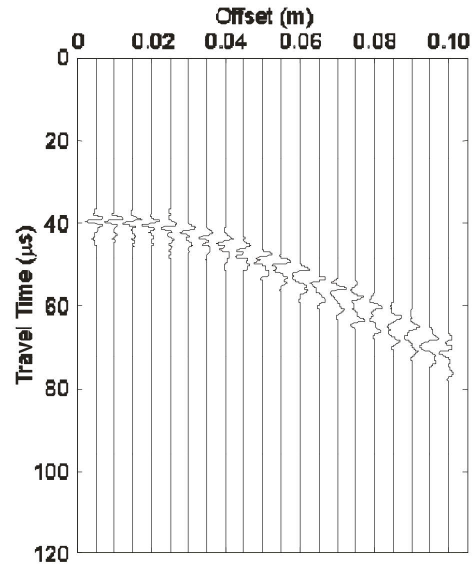

Figure 4. Observed waveforms in Plane 5 from a shear wave souce at 4.0 cm depth where the particle motions of the shear waves are predominantly aligned parallel to the sagittal plane.

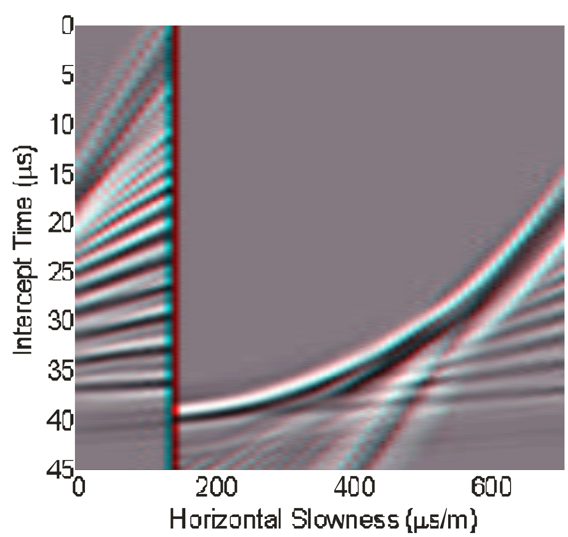

Figure 5. Corresponding τ-p transform of Figure 4.

The phase velocities were calculated in a forward manner from the elastic constants and were found to match the experimental phase velocities reasonably well (Figure 6). The errors in the elastic constants were determined and the corresponding error bars for the phase velocities were calculated using a Monte Carlo method (Figure 6). The phase velocities determined via the τ-p method were found to be coincident or near coincident were the different planes of investigation intersected at the z-axis and y-axis (Figure 6). This indicates this methodology is consistent in finding phase velocities. However, there are some discrepancies between the calculated phase velocities and observed phase velocities. This may be due to dispersion, such as that observed in Figure 4, not being accounted for in the analysis.

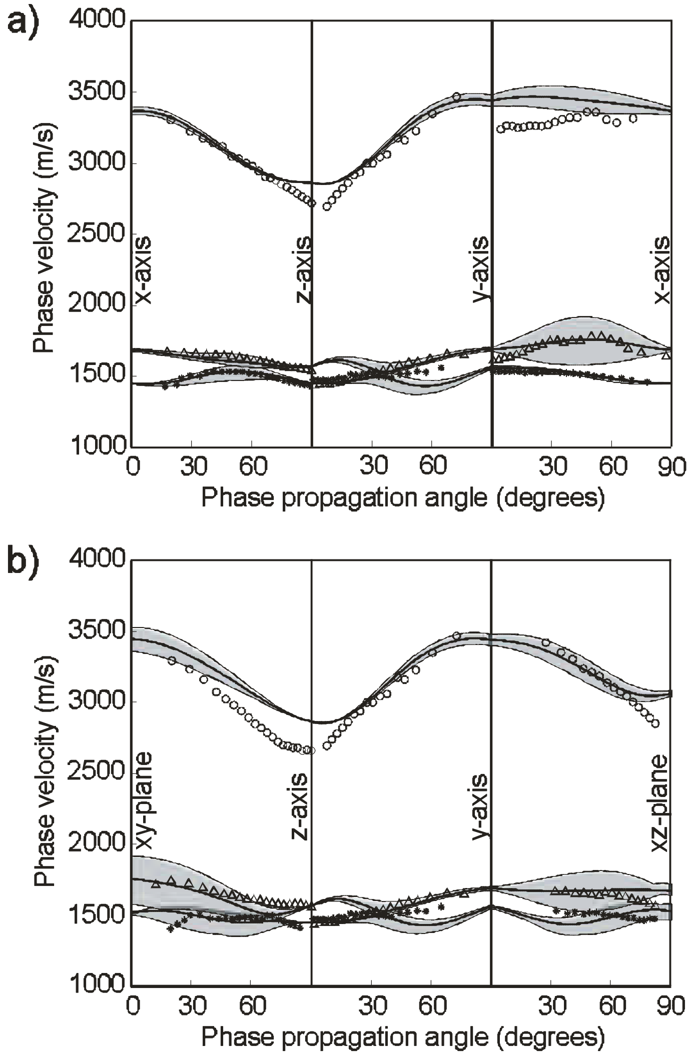

Figure 6. Summary of the phase velocities versus propagation angle q obtained from the τ-p analysis of the arrivals. Only every third data point has been plotted due to the high density of the data. a) A composite of the phase velocities in the x-z, y-z, and x-y planes. b) A composite of the phase velocities in the xy-z, y-z, and xz-y planes. Circle, triangle, and star symbols denote the velocities of the P-wave, S1-wave, and S2-wave phase velocities, respectively. The S1-wave is the fast shear wave and the S2-wave is the slow shear wave. The shaded bands are the calculated uncertainties in the phase velocities with the line in the centre of the shaded band being the theoretical phase velocity as calculated from the inversion results.

Bootstrap tests

Even though the 21 elastic constants were determined for the material phenolic, tests were performed to see how the differing assumptions of symmetry and the different planes of investigation affected our determination of the elastic constants of the material. So the inversion was done assuming differing levels of symmetry and using differing subsets of the data.

Three different levels of symmetry were assumed for the inversion, orthorhombic, monoclinic, and triclinic needing 9, 13, and 21 elastic constants to fully describe the anisotropy, respectively. As well, 4 different subsets of the data were used. One subset of the data examined used only the data gathered from the 3 perpendicular planes of investigation. Two subsets of the data used these 3 perpendicular planes of investigation and one of the diagonal planes. The last subset of data used all of the phase velocities available. In total, 12 different inversions were performed assuming 3 differing levels of symmetry and using 4 differing subsets of data. By comparing the elastic constants of these different inversions it was found the inversions essentially gave the same results so long as the symmetry assumed adequately describes the material and if the data coverage is adequate.

Conclusion

Phase velocities were determined directly as a function of phase propagation angle on an anisotropic composite material using special, near-point source transducers that were developed to impart and receive different elastic wave energies. Arrays of these transducers were constructed along 5 strategic planes of the composite material allowing 779 phase velocities to be obtained.

Making no assumption about the symmetry of the phenolic block, twenty-one independent elastic stiffnesses were obtained by a nonlinear inversion procedure. The inversion technique and experimental configuration are not symmetry dependent so long as the symmetry assumed and data collected adequately describe the material. Despite the number of constricting assumptions used in the inversion, the twenty-one independent elastic stiffnesses suggest the composite is predominantly orthorhombic. Phase velocities then calculated in a forward manner using the obtained elastic constants indicate they are in generally good agreement with those observed. However, some discrepancies remain and these may be due in part to the fact that there is very noticeable dispersion in all the waveform modes (i.e. pulse spreading with increasing propagation distance). This dispersion is not accounted for in the present τ-*p* velocity determination method and may contribute in part for the discrepancies.

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