A fast and accurate Q-inverse filter

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Abstract

Seismic waves propagating in an absorbing medium suffers from frequency dependant energy attenuation and phase distortion. These Q effects can removed by inverse Q filter. Usually, methods of inverse Q filtering are model based , i.e. a Q model that can simulate the Q effects is first adopted and then the inverse Q operator is found by mathematically inverting this modeling. However, the cost for calculating the inverse operator is computationaly expensive and some approximations are introduced. In this paper, we show that the most popular way of simply reversing the sign at the exponential term on the Q modeling operator will fail to give the expected results. We present a fast and accurate model based Q inverse algorithm together with numerical examples .

Introduction

The attenuation of seismic energy by the earth, and the resulting nonstationary of the recorded seismic traces, are fundamental problems in the processing of seismic data. Attenuation causes a loss of high frequency energy with increasing traveltime and also a time varying distortion of wavelet phase. Therefore, a number of different data processing techniques that provide a correction for seismic frequency loss (e.g. zero phase deconvolution, time variant spectral whitening) have been developed. The time varying phase distortion is often far more problematic. Wavelet phase, is of particular inportance if the processed data is to be input into inversion schemes that extract lithlogical information, e.g. Lame Parameters . The energy attenuation and phase distortion caused by the absorbing medium can be removed by inverse Q filtering . Futterman (1962) demonstrated that many researchers had attemped to derive an an inverse Q filter for showed that sound propagating through an absorbing medium. One technique that is used for approximate *Q*-compensation is to construct inverse filters within short windows of data. However, for accurate phase compensation, the number of windows need to be very large and the computational cost becomes expensive. Robinson (1979) derived an algorithm that corrected for the frequency dependant time shift caused by dispersion, by using frequency-shifting interpolated values of the Fourier transform of a data trace. Hargreaves *et. al.* (1991) further developed this method into one that is comparable to the Stolt's migration algorithm. With fast Fourier transforms this method is highly efficient to implement. However, when applying this method to variant Q model, recursion has to be applied. Therefore, the efficiency of this method is case dependent. Hale (1981) found that inverse Q filtering in time domain can be more efficient than that in frequency domain because the length of the operator used is much shorter.Therefore, he derived a faster inverse Q filter in time domain (Hale, 1981). In order to obtain more computational efficiency, Hale (1982) further extended his algorithm by a series expansions of the inverse Q-filter operator. However, this method is found to over compensate for the later events in seismogram. In order to obtain a resonable amplitude, the amplitude spectrum of the computed filter has to be clipped at some maximum gain to prevent undue amplication at later times. This gain limitation causes not only the ambiguity of amplitude but also influences the phase action of the filter since the minimum phase spectrum of his algorithm is determined by the clipped amplitude spectrum. Therefore, while the Hale's approach is efficient, the result could be one that is not desired. Moreover, a key problematic issue is that the inverse Q-filter operators in the methods , mentioned above , are based on the inverse operator which which was derived derived from the reversed sign approximation of the forward Q model. Therefore, some approximations are implicitly introduced. As we will show later ,such a inverse operator itself may introduce the error that could lead to misleading results.

Methods review

Futterman (1962) showed that a compressional wave propagating through an absorbing medium undergoes a frequency dependent attenuation :

$$W_Q(t,\tau,z) = IFT\{W(f,\tau,z)e^{-\pi\tau/Q[|f|+iH(|f|)]}\}\,, \qquad (1)$$

where Q is the quality factor, τ is the travelpath time, W(f,τ,z) is the amplitude spectrum of the propagating wave in pure elastic medium, f is frequency , and H(.) is the Hilbert transform to ensure the causal signal. With constant Q with frequency assuption, Futterman (1962) derived the *Q* model as

$$W_Q(t,\tau,z) = IFT\{W(f,\tau,z\}\exp\{-\pi fz/(v(f)Q)\}\,, \qquad (2)$$

where

$$V(f)/V(f_c) = \{1 + 1/\pi Q \ln|f/f_c|\}\,. \qquad (3)$$

and f_c is cut-off frequency. In equaton (2), the phase velocity is dependent on frequency, as a result of the requirement that a wave propagating in an absorbing medium must be causal (Aki and Richard, 1980). Actually, both equation (1) and (2) are equivalent when Q is assumed to be independant on frequency. Equation (1) has another form (Hale, 1981)

$$W_Q(f,\tau,z) = FT\{W(f,\tau,z)e^{-\pi\tau/Q[|f|+iH(|f|)]}\} \qquad (4)$$

Equation (4) can be considered as a nonstationary convolution while equation (1) is also a nonstationary convolution. The relations between both equations are discussed in detail by Margrave (e.g. Margrave, 1998)

In equation (3), the phase item in equation (1) is now replaced explicitly by a velocity dispersion relation. Equations (1) and (2) are equivalent when the frequency is equal to Nyquist frequency, which can be seen easily by a numerical test, because both of them are satisfied by the requirement of causality. The attenuation increases with both increasing frequency and increasing travel path length , and phase velocity increases with frequency up to an upper cut-off frequency .Now, let us discuss the problem in a ideal case scenario, i.e. no noise and no multiples, then both forms of convolution and combination can be formulated as a matrix equation (e.g. Margrave, 1998)

$$W_Q = A_{-Q} W(f,\tau,z) \qquad (5)$$

Where A_Q is the matrix created by applying a *Q* response to a time series via a generalized convolution. Each column of the Q matrix contains the impulse response for the input time of that column convolved with source waveform. Then, the inverse Q can be calculated by solving matrix equation (5). However, cost of solving equation (5) is thought to be too high and not practical (e.g. Hale, 1982) because of the huge dimensions of the matrix. Instead, the inverse of the matrix is approximatly calculated via inverse operator (e.g. Margraves et al. 1991, Hale, 1982)

$$W(t,\tau,z) = IFT\{W_Q(f,\tau,z)e^{\pi\tau/Q[|f|+iH(|f|)]}\} \qquad (6)$$

In equation (6), the inverse operator is simply the reverse sign at the exponential term on the Q modeling operator. It gives an approximate inverse of Q matrix because the operator is not orthogonal compared to that of the pure Fourier Transform kernel. Therefore, there are limitations in the use of equation 6.

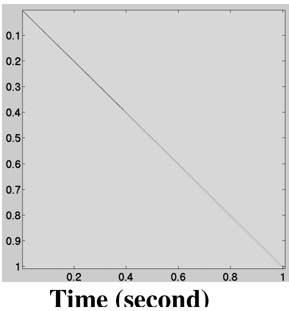

Figure1. Q matrix.

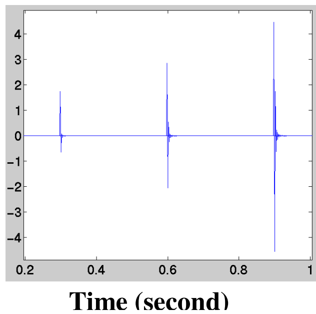

Figure2. Inverse response.

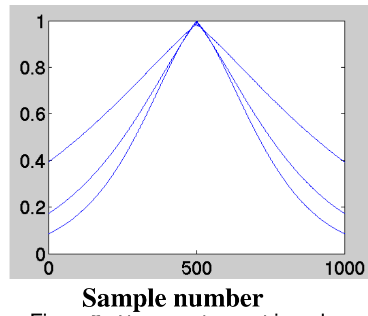

Figure3. The spectrum of impulse response of figure 2.

In order to show the problem clearly, we exame an example. Assuming Q=200, v=3000 m/s, figure 1 shows the matrix for forward Q modeling in time domain. Impulse responses for time at 0.3s, 0.6s and 0.9s are obtained from exact inverse matrix and their normalized spectrums are shown in figure 2 and figure 3, respectively. The impulse responses are at the same time as the approximation given by equation (5) but their normalized spectrums ,as shown by figure 5 are not. Comparing figure (2) and (3) with figure (4) and (5), the discrepancies of the responses are shown to be increasing with time delay. The result of applying an inverse operator derived from equation 5 on the forward Q model of a unit impulse is shown in figure 6. Instead of three unit impulses expected, the results in figure 5 show that the impulses at later time delays are amplified and the phase distortions are not eliminated. The over compensated phenomena was noted by Hale (Hale, 1982) ,and he used a clipping function set to some maximum gain to prevent undue amplification at later times. However, this gain limitation produces not only the ambiguity of amplitude but also influences the phase compensation, as the the minimum phase spectrum is determined from the clipped amplitude spectrum.

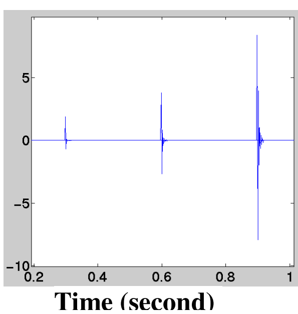

Figure 4. Inverse response of the approximate solution.

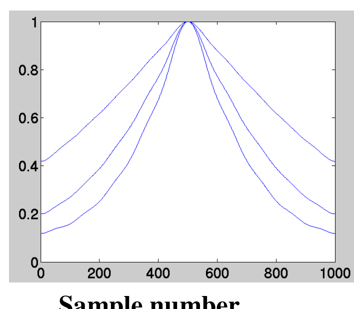

Figure 5. The spectrum of impulse response at figure 4.

Hale's approximation depends in part on the dominance of the diagonal elements within Q matrix. Unfortunately, this is not the actual case, especially at the later time delays. As figure 8 below shows, at 1.8 seconds, the maximum value of the impulse response obviously shifts from the onset position to that shown in figure 7. Therefore, while Hale approach is relatively efficient, and the result could be misleading.

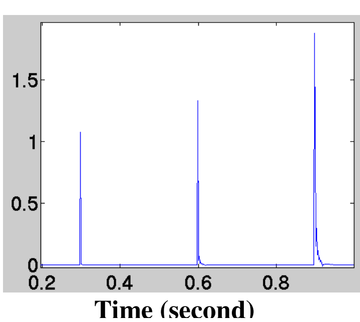

Figure 6. Inverse result of approximate solution.

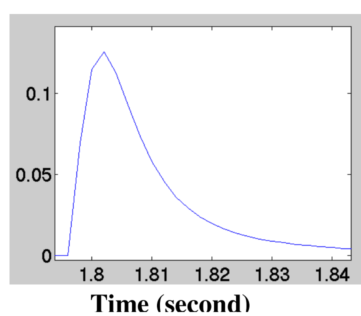

Figure 7. Impulse response at 1.8 seconds time delay.

Inverse Q filter

Based on the discussions above, the exact inverse provides an accurate result. However, the inverse of a *N* by *N* matrix needs at least N^3 Flops (floating point operations) which may prohibit it's use for the practical applications. In this section, we discuss how to solve the inverse *Q* Problem in the time domain.

The cost of computation is composed of two parts : the first is calculate the elements of the *Q* forward matrix ,and the other the other is to find a the solution of the inverse matrix. First, we will propose a method to calculate the matrix in time domain. With equation (5), we first have to first calculate each impulse response in frequency domain and then inverse Fourier transform it into time domain. Each step involves natural log and exponential operations. Taking a careful look at figure 1, it is shown that the matrix is most dominated by the near terms to the diagonal and the *Q* modeling responses is a practically δ like function. The spectrum of each response is wide. This property tells us that the operator in time domain is much shorter than that in the frequency domain. Another property is that the spectrum of each impulse response is very smooth. Taking these two properties into consideration ,it is seen that only a small number of samples in the frequency domain are required for the calculation of the time domain impulse response. In the example shown above, only 32 frequency samples are selected to be input into the calculation. If the trace sample is 1024, then we can gain 30 times improvement in computation time compared to considering all the samples. Figure 8 shows the comparison of the results of impulse responses at 1.2 second are nearly exactly the same as the input. This gain of efficiency cannot be obtained if we solve the equation in frequency domain, such as Hargreaves's method (Hargreave, 1991).

The compution the inverse of a Q matrix could indeed be expensive. However, the matrix is a narrowly banded low trianglar matrix. Instead of computing the inverse matrix, we propose to solve the equation directly. The solution for a banded lower trianglar matrix is very well developed and very fast. For example, given a chosen diagonal band width K, the the penalty for solving equation 4 is only 2*K*N flops ,where N is the dimension of the matrix (Golub and van Loan,1996). This cost is almost equivalent to the matrix multiplication. Therefore, with the band matrix solver, we should have nearly same efficiency as that of Hale's method (Hale, 1992). However, the accuracy is guarateed. Figure 9 shows the result from previous example and the three-unit impulse is exactly restored.

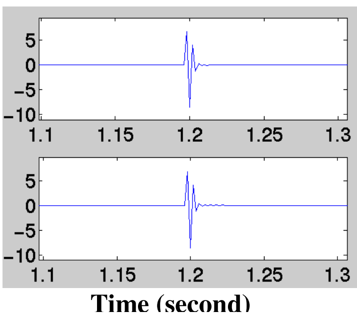

Figure 8. Comparison of impulse response.

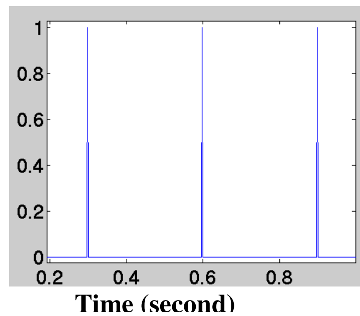

Figure 9. Result of inversion.

Conclusions

We have shown that inverse Q filter using Hale's approximate Inverse Q operator can only be valid for the early portion of seismogram. For the later portion, it could lead to a result that could be much different from the true result. The algorithm we presented solves the inverse Q problem directly and accurately from the Q modeling equation. Using the properties of the time domain impulse response, we can setup the Q time domain matrix equation and use the banded matrix solver to efficiently and rapidly solve the inverse Q filter . The computational speed of this method is comparable to the Hale's approximate inverse Q method now currently used. Numerical results presented have shown that our algorithm gives reliable results.

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