Estimating $R_{SS}(0)$ from $R_{SP}(\theta)$

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Introduction

The zero-offset shear-wave reflectivity, $R_{SS}(0)$, is a valuable quantity in exploration geophysics. From it we can obtain the shear impedance of earth layers using the same techniques as for P-impedance. One potential source of $R_{SS}(0)$ is from shear-wave seismic experiments. However, these are costly, and have not become standard in oil exploration. On the other hand, converted waves are becoming more commonly measured, and it is believed that information on R_{SS} can be extracted from the R_{PS} of converted wave data. This would provide explorationists with a valuable tool to help in delineating rock properties.

In some of the first work on this idea, Stewart and Bland (1997) showed that $R_{PS}(\theta) \approx 4\ (\beta/\alpha)\ \sin(\theta)\ R_{SS}(0)$. This expression is more accurate at small angles, and thus extrapolation to $\theta = 0$ could be used to estimate $R_{SS}(0)$. Goodway (2001) has recently developed a more accurate expression (see following section) which could be used with a simple stacking procedure to estimate $R_{SS}(0)$. In this study we develop additional expressions for $R_{SS}(0)$ and develop a more rigorous means of assessing their accuracy.

Theory

Goodway (2001) has presented a derivation of the following expression:

$$R_{SS}(0) \approx \frac{-R_{PS}(\theta)}{4\sin\varphi(\tan\varphi\sin\theta - \cos\theta)} \tag{1}$$

Here θ is the average of P-wave reflection and transmission angles, and φ is the analogous average for S-waves. The derivation of Eq. (1) assumes that the linear Aki-Richards approximation (Aki and Richards, 1980) is accurate, and also that the density contrast, $\Delta\rho/\rho = 2(\rho_2 - \rho_1) / (\rho_2 + \rho_1)$, is small.

To derive our first alternate expression, we also begin with the Aki-Richards approximation for R_{PS}:

$$R_{PS} = -\frac{\sin\theta}{\cos\varphi}\left[\left(1 - 2\sin^2\varphi + 2\frac{\beta}{\alpha}\cos\theta\cos\varphi\right)\frac{\Delta\rho}{\rho} - \left(4\sin^2\varphi - 4\frac{\beta}{\alpha}\cos\theta\cos\varphi\right)\frac{\Delta\beta}{\beta}\right] \tag{2}$$

This can be rearranged to

$$R_{PS} = -\frac{\sin\theta}{\cos\varphi}\left[\left(1 + 2\sin^2\varphi - 2\frac{\beta}{\alpha}\cos\theta\cos\varphi\right)\frac{\Delta\rho}{\rho} - \left(4\sin^2\varphi - 4\frac{\beta}{\alpha}\cos\theta\cos\varphi\right)\left(\frac{\Delta\rho}{\rho} + \frac{\Delta\beta}{\beta}\right)\right] \tag{3}$$

Noting that $\Delta\rho/\rho + \Delta\beta/\beta = -\,2\ R_{SS}(0)$, we could drop the remaining $\Delta\rho/\rho$ term to obtain Eq.(1). Instead we elect to approximate this term. To do this we employ results of studies that have shown that it is feasible to express Gardner's relation (Gardner et al., 1974) in terms of shear velocity instead of compressional velocity (Dey and Stewart, 1997; Potter and Stewart 1998; Potter 1999; Wang, 2000; Ursenbach, 2001). In one of these studies (Ursenbach, 2001) it was shown that for a lithology-independent expression, the exponent does not vary much from the 0.25 value of the original Gardner equation. This implies that $\Delta\rho/\rho \approx (1/4)\ \Delta\beta/\beta$ and allows us to replace $\Delta\rho/\rho$ as follows:

$$\frac{\Delta\rho}{\rho} = \frac{1}{5}\left(4\frac{\Delta\rho}{\rho} + \frac{\Delta\rho}{\rho}\right) \approx \frac{1}{5}\left(\frac{\Delta\beta}{\beta} + \frac{\Delta\rho}{\rho}\right) = -\frac{2}{5}R_{SS}(0) \tag{4}$$

Making this substitution yields

$$R_{PS} \approx -\frac{\sin\theta}{5\cos\varphi}\left(1 - 18\sin^2\varphi + 18\frac{\beta}{\alpha}\cos\theta\cos\varphi\right)R_{SS}(0) \tag{5}$$

This constitutes our first result. Note that a similar type of substitution has been employed (Stewart and Ursenbach, 2002) to improve the well-known AVO method of Fatti et al. (1994).

To derive our second result we again begin from Eq. (2), and note that it can be rearranged to the following result:

$$R_{PS} = -\frac{\sin\theta}{4\cos\varphi}\left[\left(1 + 2\sin^2\varphi - 2\frac{\beta}{\alpha}\cos\theta\cos\varphi\right)\left(\frac{\Delta\rho}{\rho} - \frac{\Delta\beta}{\beta}\right) + \left(1 - 6\sin^2\varphi + 6\frac{\beta}{\alpha}\cos\theta\cos\varphi\right)\left(\frac{\Delta\rho}{\rho} + \frac{\Delta\beta}{\beta}\right)\right] \tag{6}$$

The quantity $\Delta\rho/\rho - \Delta\beta/\beta = \Delta\ln(\rho/\beta)$ will be small if $\Delta\rho/\rho$ and $\Delta\beta/\beta$ are comparable. Furthermore, its coefficient is small for small angles and for β/α near ½. We thus neglect this term and obtain our final result:

$$R_{PS} \approx \frac{\sin\theta}{2\cos\varphi}\left(1-6\sin^2\varphi+6\frac{\beta}{\alpha}\cos\theta\cos\varphi\right)R_{SS}(0) \tag{7}$$

Finally, we include a description of one more method that is somewhat different from those above. It also begins with the Aki-Richards expression for R_{PS}, but expands R_{PS} in powers of sinθ and obtains coefficients for the linear and cubic terms. [See for instance Eqs (4a) and (4b) of Ramos and Castagna (2001). Their Eq. (4c) is incorrect but is not needed here.] These coefficients correspond to the intercept and gradient that would be obtained if one plotted $R_{PS}(\theta)/\sin\theta$ against $\sin^2\theta$. Both the intercept (Icpt) and gradient (Grdt) are linear combinations of $\Delta\rho/\rho$ and $\Delta\beta/\beta$ so that it is possible to combine them in such a way as to yield $R_{SS}(0)$. We have obtained this result as

$$R_{SS}(0) = \frac{1}{2}\left[\frac{\beta}{\alpha}\left(1+\frac{5}{2}\frac{\beta}{\alpha}\right)\times Icpt + \left(1-2\frac{\beta}{\alpha}\right)\times Grdt\right]\left[\frac{\beta}{\alpha}\left(1+\frac{\beta}{\alpha}\right)\right]^{-1} \tag{8}$$

This is a valuable result in its own right as it involves no approximations other than that of Aki-Richards. It involves converted-wave AVO however, rather than simple stacking, so it will not be compared here to the other methods above. Our preliminary studies though have indicated that its accuracy is similar to that of the Shuey method for P-wave AVO. We intend to explore its use more at a later time.

Calculations

To test the above stacking expressions we use the exact Zoeppritz equations to generate the ratio $R_{SS}(0)/[R_{PS}(\theta)/\sin\theta]$ for a large number of sets of earth parameters, taken from typical ranges. From these SS/PS ratio functions we calculate an average ratio. A number of different averages were tested (arithmetic, harmonic, geometric). In the end the only average which appeared to be centered in the main concentration of lines was a weighted average, in which each ratio was weighted by $R_{SS}(0)^2$. A set of calculated ratios, and their weighted average, are shown below in Figure 1.

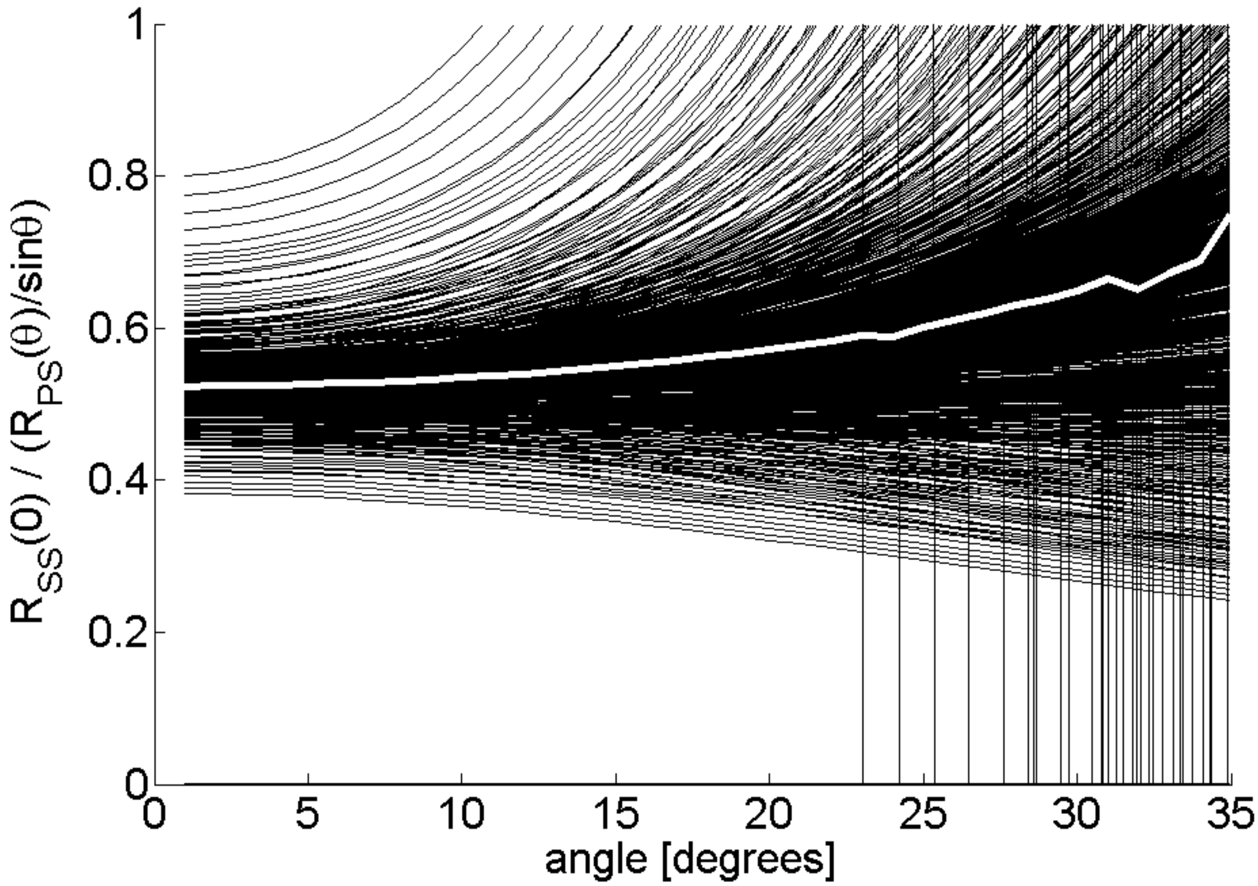

Figure 1. A display of the ratio of $R_{ss}(0)$ to $R_{PS}(\theta)/\sin\theta$. Each of the dark lines represents this SS/PS ratio calculated for a different set of earth parameters. The light line in the center is the weighted average, where $R_{ss}(0)^2$ has been used as the weighting factor. The velocity ratio β/α was constrained to a value of 0.47 for all lines in this figure, while $\Delta\alpha/\alpha$, $\Delta\beta/\beta$, and $\Delta\rho/\rho$ all varied between –0.1 and 0.1.

In Figure 1 the velocity ratio β/α was constrained to a value of 0.47, while $\Delta\alpha/\alpha$, $\Delta\beta/\beta$, and $\Delta\rho/\rho$ were sampled independently from a range of –0.1 to 0.1. Most of the ratio values are within about 0.05 of the weighted average line. Inspection of individual lines and their associated $R_{ss}(0)$ values and $R_{PS}(\theta)/\sin\theta$ functions (not shown) indicate that the outlying lines generally correspond to very small $R_{ss}(0)$ values. This is consistent with the weighted average being superior to the other averages. In practical terms, it is more important to estimate $R_{ss}(0)$ accurately when it is large than when it is very small.

Next we compare this weighted average to the three approximations given above. These results are displayed in Figure 2 below.

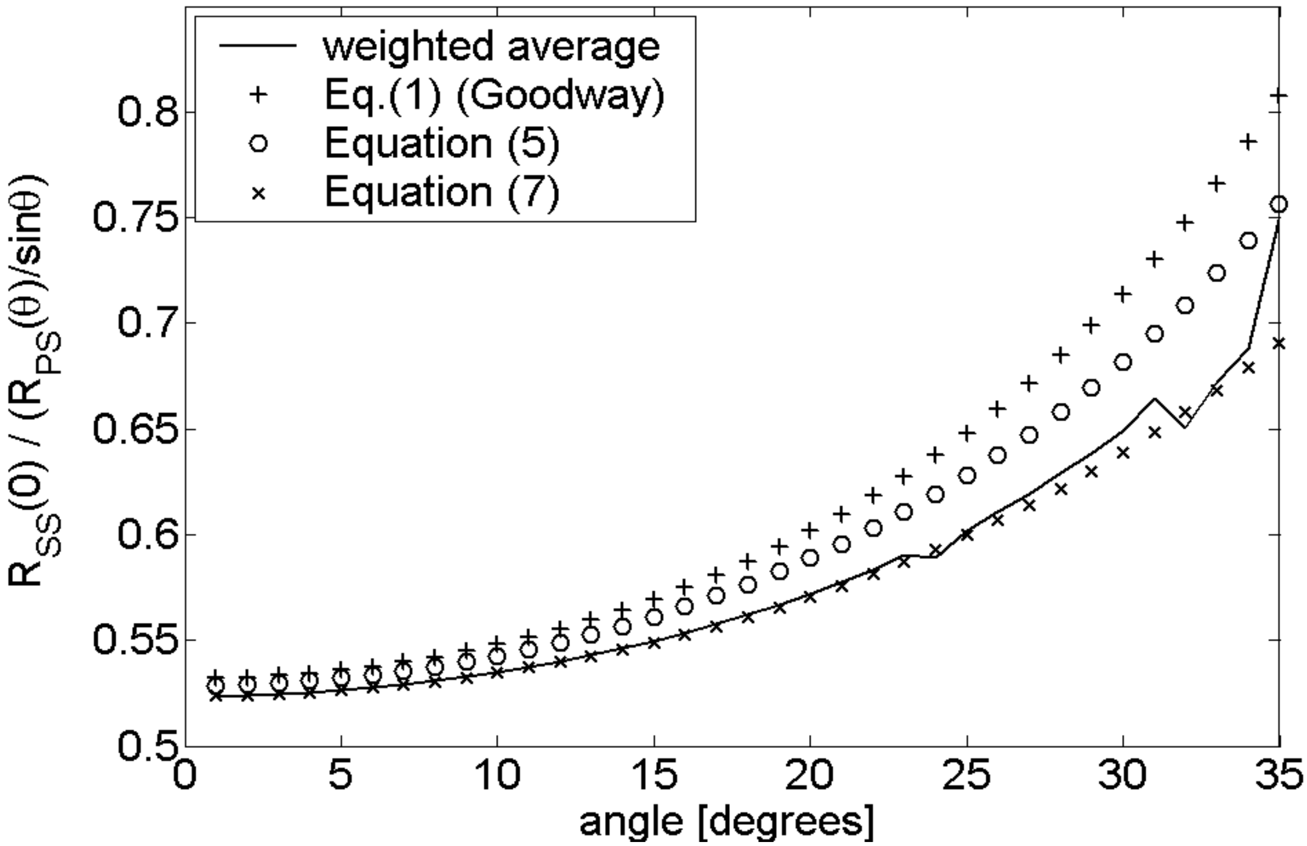

Figure 2. Comparison of the weighted average from Figure 1 with three approximate theoretical expressions defined in the text. Eq. (7) appears to represent the average quite accurately.

From Figure 2 we would conclude that Eq.(7) is the most accurate theoretical expression. However another issue concerns the choice of sample earth parameters. Suppose we felt that $\Delta\rho/\rho$ would on average be more representative of the real earth if it were related to velocity contrast through a Gardner-type relation. This possibility is explored in Figure 3 below. Figure 3a illustrates the case when $\Delta\rho/\rho = ¼\ \Delta\alpha/\alpha$, as per the original Gardner relation. $\Delta\alpha/\alpha$ and $\Delta\beta/\beta$ are still sampled independently from the interval [-.1,.1], and β/α is still fixed at 0.47. Figure 3b is similar but with $\Delta\rho/\rho = 0.235\ \Delta\beta/\beta$, as derived by Ursenbach (2001). In Figure 3a, Goodway's Eq.(1) is extremely accurate, while in Figure 3b Eq.(5) is most representative. This latter result in reasonable since it was derived using a shear-wave Gardner relation, similar to the one imposed on the averaged SS/PS ratios in Figure 3b. Calculations were also carried out using a generalized Gardner relation (Ursenbach, 2001) that sets $\Delta\rho/\rho = 0.080\ \Delta\alpha/\alpha + 0.164\ \Delta\beta/\beta$. These results are not shown, but display behavior intermediate between 3a and 3b, as one would expect. Thus, depending on what sort of understanding one has regarding the earth layers under consideration, one theory may be found to be more suitable than another.

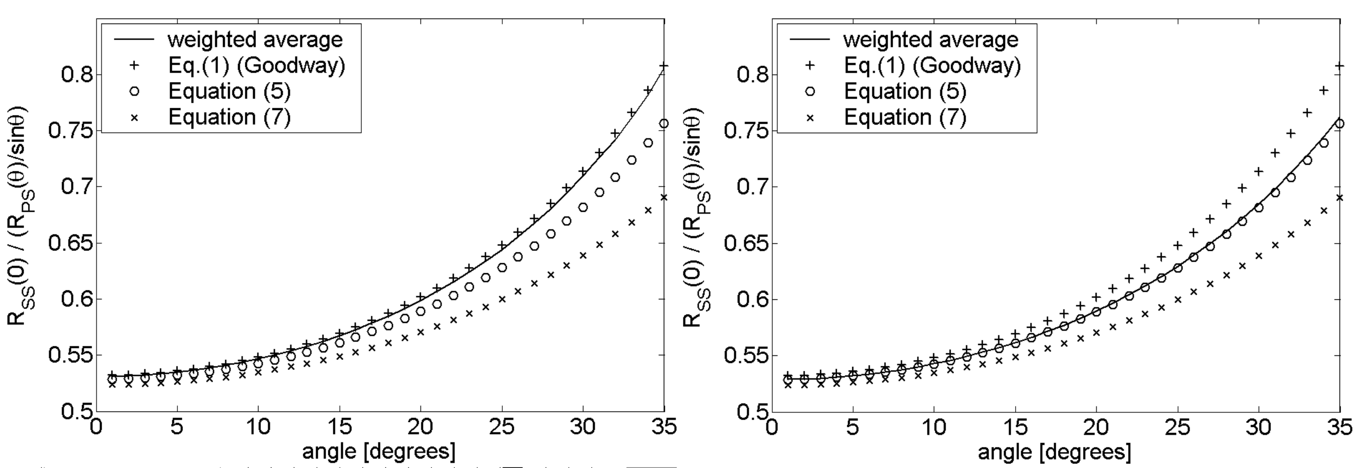

Figure 3. Weighted average SS/PS ratios with constrained Δρ /ρ values, compared with three approximate theoretical expressions defined in the text. In the left graph Δρ /ρ = ¼ Δα /α, while in the right graph Δρ /ρ = 0.235 Δβ /β. The former is best described by Equation (1) of Goodway (2001), while the latter is best described by Equation (5) of this paper.

All calculations to this point have used a typical but arbitrary velocity ratio of $\beta/\alpha = 0.47$. In Figure 4 we illustrate the effect on the weighted average ratios when β/α varies. $\Delta\rho/\rho$ is again allowed to vary independently of $\Delta\alpha/\alpha$ and $\Delta\beta/\beta$, as in Figure 2. The various theoretical expressions are not shown, but their behavior at other values of β/α is comparable to that shown in Figures 2 and 3. Also shown in Figure 4 is the fitting of an empirical function to the weighted averages. The fitting is only attempted within the range of 0 to 20 degrees. We note from Figure 1 that the ratios tend to vary more widely at higher angles, and this is reflected in unruly behavior above 20 degrees in Figures 2 and 4. Although the theoretical expressions are quite accurate, as shown above, the empirical expressions are simple in form and may be convenient for some applications. When a value of β/α is not available, for instance, a reasonable course is to assume that $\beta/\alpha = ½$, resulting in the empirical expression $R_{SS}(0) = [½ + ½ \sin^2(0.9\,\theta)]\, R_{SS}(\theta)/\sin\theta$, suitable for stacking of traces with $\theta < 20°$.

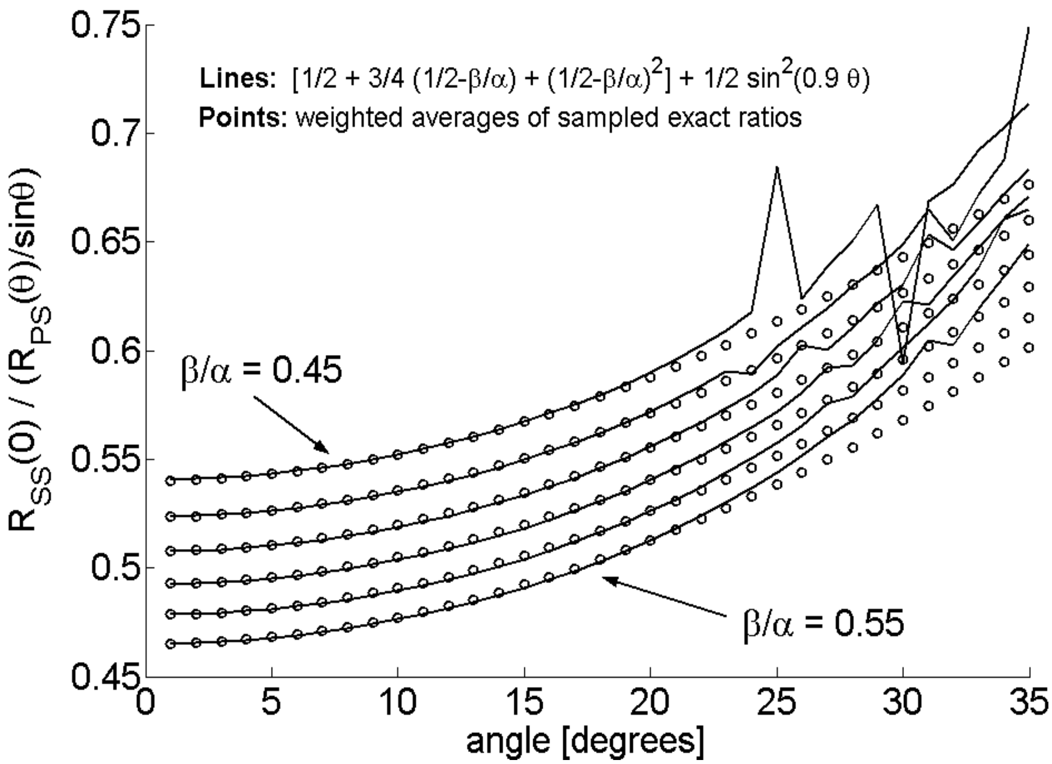

Figure 4. The weighted averages of SS/PS ratios for several values of β /α are shown as solid lines. This figure illustrates that the functional form of the SS/PS ratio average is largely independent of velocity ratio, but is shifted upwards as β /α decreases. Empirical fits are also shown as points on the graph. The simple form of these may be useful for some applications.

Conclusions

We have derived two new theoretical expressions relating the zero-offset shear reflectivity to offset-dependent converted wave reflectivities. These, along with an earlier expression by Goodway (2002), and also some simple empirical functions, have been shown to have encouraging potential for the stacking of converted wave traces to obtain shear reflectivity. We have also presented a new expression, subject only to the Aki-Richards approximation, which could be used in an AVO procedure to obtain the zero-offset shear reflectivity.

References

Aki, K., and Richards, P.G., 1980. *Quantitative Seismology: Theory and Methods*, W.H. Freeman and Co., San Francisco.

Dey, A.K., and Stewart, R.R., 1997. Predicting density using V_S and Gardner's relationship, in *CREWES 1997 Research Report*, Vol. 9.

Fatti, J.L., Smith, G.C., Vail, P.J., Strauss, P.J., and Levitt, P.R., 1994. Detection of gas in sandstone reservoirs using AVO analysis: A 3-D seismic case history using the Geostack technique, *Geophys.*, 59, 1362-1376.

Gardner, G.H.F., Gardner, L.W., and Gregory, A.R., 1974. Formation velocity and density – The diagnostic basics for stratigraphic traps. *Geophys.*, 39, 770-780.

Goodway, W., 2001. Elastic-wave AVO for lithology description and fluid detection, M.Sc.Thesis, University of Calgary.

Potter, C.C., 1999. Relating density and elastic velocities in clastics: an observation, in *CREWES 1999 Research Report*, Vol. 11.

Potter, C.C., and Stewart, R.R., 1998. Density predictions using V_P and V_S sonic logs, in *CREWES 1998 Research Report*, Vol. 10.

Stewart, R.R. and Ursenbach, C.P., 2002. The Full Offset AVO inversion approximation, EAGE Abstracts, submitted.

Ursenbach, C.P., 2001. A generalized Gardner relation, in *CREWES 2001 Research Report*, Vol. 13.

Wang, Z., 2000. Velocity-Density Relationships in Sedimentary Rocks, in *Seismic and acoustic velocities in reservoir rocks*, Vol. 3, Recent developments. *Soc. Expl. Geophys.*, Geophysics Reprint Series.