Estimation of Quality Factors – An Analytical Approach

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Introduction

Seismic waves travelling through the earth experience absorption, meaning attenuation and dispersion, due to the anelasticity and heterogeneity of the medium (Ricker, 1953; Futterman, 1962; White, 1983; Kneib and Shapiro, 1995). Understanding, estimating, and compensating for absorption of seismic waves are important in the quest to improve the resolution of seismic images, better interpret the effects of AVO, and invert for material properties.

Methods for estimating Q from surface seismic data are not well developed (Dasgupta and Clark, 1998). Some research has, however, been published concerning the estimation of Q from vertical seismic profile and cross well data (Tonn 1991). Almost all of these methods use the amplitude of received signals, but this amplitude information is often inaccurate due to noise, geometrical spreading, scattering, and other effects.

Quan and Harris (1997) presented a method for estimating seismic absorption based on the frequency shift observed in VSP data space. They developed a relationship between Q and the centroid of an amplitude spectrum which was represented by a Gaussian, boxcar, or triangular shape.

In this paper, we introduce some formulae to calculate quality factors from the peak frequency variation of a reflection at different arrival times. These formulae can be applied to estimate quality factors both from a CMP gather and a poststack trace. The formulae are derived from a prestack CMP gather based on the following two observations:

1. A CMP section represents multiple observations of an underground structure. It provides information in the time and offset domains, allowing for the extraction of information concerning structure, lithology and material properties such as velocity and Q.

2. Reflection arrival times are determined by interval velocities and the geometrical structure of the subsurface. Absorption of the received signals is only determined by the interval Q's and traveltimes in each layer. If the amplitude spectrum of a seismic wavelet is assumed to be Ricker-like, interval Q's can be computed solely from the variation of the peak frequency of a spectrum as a function of time.

We begin by developing an equation that relates absorption to spectral peak frequency variation. From this relationship, we estimate interval Q's using a layer stripping approach. Tests show that it determines interval Q values with reasonable accuracy.

Derivation of Analytical Equations

In seismic data processing, a recorded trace is commonly modeled as the convolution of a seismic source signature with a reflectivity series. The seismic source signature is generally unknown, although in some cases it can either be measured or assumed to be minimum phase. The effects of anelasticity may be incorporated into the model by convolving it with an earth filter. This filter is causal, minimum phase, and depends on Q (Aki and Richards, 1980).

Instead of studying the details of the absorption filter response, we will consider only the relationship between Q and peak frequency translation. Assuming that the amplitude spectrum $B(f)$ of the source wavelet can be well represented by that of a Ricker wavelet (Ricker, 1953), that is

$$B(f) = \frac{2}{\sqrt{\pi}} \frac{f^2}{f_m^2} e^{-f^2/f_m^2}, \qquad (1)$$

where f_m is the dominant frequency. In this paper, for convenience, we refer to the frequency of maximum amplitude as the peak frequency and denote it as f_p. For a wavelet at its initial state, the peak frequency is the dominant frequency. The evolution of the amplitude spectrum through time is now modelled as that of a Ricker wavelet travelling in a viscoelastic medium (geometrical spreading and other factors are not considered). After travelling for a time t, the amplitude spectrum is

$$B(f,t) = \frac{2}{\sqrt{\pi}} \frac{f^2}{f_m^2} e^{-f^2/f_m^2} H(f,t), \qquad (2)$$

where $H(f,t)$ is the absorption filter (Varela et al., 1993) which is determine by medium absorptive property and the ray path. Quality factors are often assumed to be independent of frequency (Ricker, 1953, Kjartasson, 1979, White, 1983). By considering the propagation of a wave in a half space with quality factor Q for t seconds, we determine the amplitude spectrum of the received signal as

$$B(f,t) = B(f) e^{-\frac{\pi f t}{Q}}. \qquad (3)$$

We can observe from this expression that, as time increases, absorption increases with frequency and results in the peak frequency translating towards lower frequency. This phenomenon is clearly illustrated in Figure 1. Due to absorption, the time width of the source wavelet increases and consequently the amplitude spectrum becomes narrower. If the travel time is known we can obtain Q from the spectral variation.

One layer case

To one layer case, there is only one quality factor. Including all Q unrelated functions into an amplitude term, we write the amplitude spectrum as

$$B(f,t) = A(t) B(f) e^{-\frac{\pi f t}{Q}}, \qquad (4)$$

where $A(t)$ is an amplitude factor independent of frequency and absorption. The relationship between Q and the shift of peak frequency is now

$$Q = \frac{\pi t f_p f_m^2}{2(f_m^2 - f_p^2)}. \qquad (5)$$

This equation shows that if the dominant frequency f_m is known, Q may be computed from the CMP gather using only one offset. In practice, of course, we do not know the initial f_m. It can be estimated, however, if we assume that the amplitude spectrum of the initial source wavelet is approximated by a Ricker wavelet.

Designating the peak frequencies at times t_1 and t_2 by f_1 and f_2, respectively, we can derive the dominant frequency of the source wavelet from the peak frequencies of a reflection at two different time points:

$$f_m = \sqrt{\frac{f_{p1} f_{p2} (t_2 f_{p1} - t_1 f_{p2})}{t_2 f_{p2} - t_1 f_{p1}}}. \qquad (6)$$

Equations (5) and (6) allow us to obtain an average Q, by using the peak frequency variation along all offsets, thereby allowing us to remove the effects of surface fluctuations and random noise, consequently improving the accuracy of Q.

For real seismic data, the amplitude spectrum of a seismic wavelet is not exactly that of a Ricker wavelet. In many situations, however, it can still be approximated by a Ricker spectrum (Ricker, 1963).

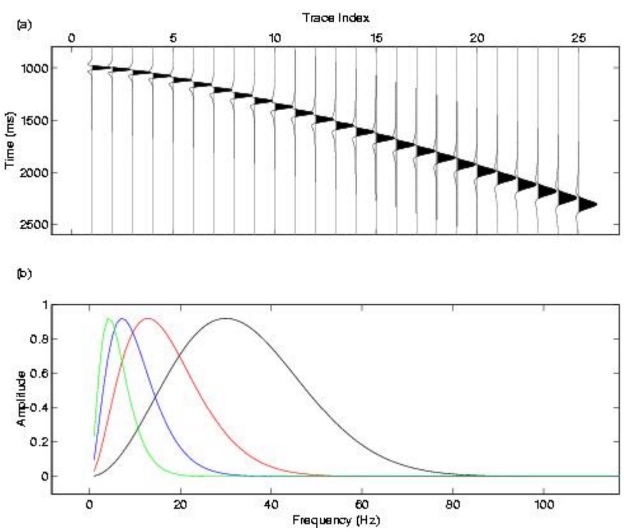

Figure 1: (a) An event in a CMP gather is generated by a reflector in an absorptive medium, where amplitude has been normalised with respect to the maximum amplitude. (b) Amplitude spectra of the events of the input source signature, trace 1, 11 and 21 respectively from the right to the left.

Multi-layer case

Assuming a multi-layer medium, we write the amplitude attenuation equation (4) as

$$B(f,t) = A(t)B(f)\exp\left(\sum_{i=1}^{N} \frac{\pi f \Delta t_i}{Q_i}\right) \qquad (7)$$

where Q_i and Δt_i are the quality factor and the traveltime in layer i, respectively. Taking an approach from velocity estimation, we assume straight ray paths and assume that quality factors and traveltimes in layers above layer N are known. Now we can calculate quality factor Q_N as

$$Q_N = \frac{\pi \Delta t_N}{\alpha - \beta}, \qquad (8)$$

where $\alpha = 2\frac{f_m^2 - f_p^2}{f_p f_m^2}$ and $\beta = \sum_{i=0}^{N-1} \frac{\pi \Delta t_i}{Q_i}$.

Q values can now be calculated layer by layer through layer stripping. Since a straight ray path approximation is used, the computed Q_N is not the actual interval quality factor. Analogous to RMS velocities, we refer to such Q values as RMS Q values.

Application of the Analytical Formula

To apply the foregoing derived equations (5), (6) and (8) to prestack data is very straight forward because they are derived from multi-offset surface observations. However, to apply them to poststack data, we should assume each trace in a stacked section is zero-offset.

Prestack

To estimate Q from CMP gathers, we assume that the arrival times for the main reflection events are known. Fourier transforms are computed in the window containing the reflection at each offset, each amplitude spectrum is fitted with a Ricker spectrum and the peak frequency of the spectrum is estimated. Using Equation (8), we can now calculate the Q's layer by layer from peak frequency variation. Figure 2 shows a simple test on a synthetic CMP gather with two events. Absorption is modeled by using low Q values to emphasize the effect: the Q values in the two layers are 10 and 20 respectively. The actual and estimated Q curves are shown in Figures 2b and 2c respectively.

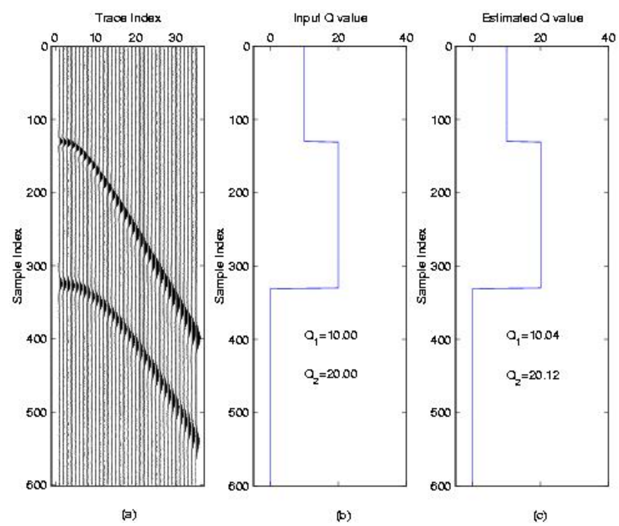

Figure 2: (a) A synthetic CMP gather of two reflections with absorption and 10% random noises, Traces have been normalised based on their maximum amplitude. (b) Values of input quality factors. (c) Computed quality factors.

Poststack

In geophysical exploration, observed signals are often time variant. Waves that reflect from the shallow subsurface are rich in high frequency components, whereas waves that return from the deep subsurface are dominated by low frequency components. To analyse such time variant signals, a windowed time-variant spectrum (WTVS) is calculated. If a single trace is assumed to be zero offset, from the peak frequency variation detected in a WTVS, quality factors can be calculated using equation (2) .

A WTVS spectrum can be displayed as a grey scale or contour plot as in Figure 3, with frequency as the horizontal coordinate and time as the vertical coordinate. Each row of the plot corresponds to the spectrum of the input data at a specific time. The relative detail in time and frequency in the WTVS is related to the window size and frequency intervals.

Figure 3 is a real data example, where Figure 3a is the windowed time variant spectrum, this WTVS gives a clear indication of the trend of the spectral variation. Picking the peak amplitude ridge from WTVS and fitting the ridge with a piece-wise straight line in the coordinates of f versus t , a quality factor can be calculated from each line segment using the formula (2).

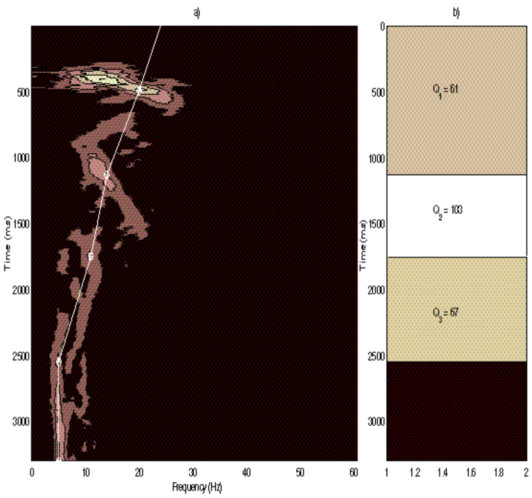

Figure 3: (a) A WTVS of a real seismic trace. The picked peak frequency points are marked by circles and connected by straight lines. (b) the estimated quality factors, the value of the last layer is infinite.

Conclusion

Underground lithology is characterised by its velocity, density and quality factor. While the quality factor does not affect the arrival times of reflections, it does affect amplitude and the signal's frequency content. Extracting velocity information from CMP gathers is a common practice. Analogously, here we have devised an analytical approach to estimate the absorption character from the variation of spectrum both with time and offset. The derivation is based on the reasonable assumption of a Ricker-like amplitude spectrum of the source signature. The derived formula can be applied to estimate quality factors from both prestack CMP gathers and poststack traces, for the later case, it must assumed that poststack traces are zero offset.

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