

# **Optimization approach to determine interval velocities in a medium with laterally inhomogeneous curvilinear layers**

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## **ABSTRACT**

Depth velocity models are an essential part of seismic data processing and interpretation. They are used in time-to-depth migration and transformation, and AVO analysis. The overburden velocity anomalies essentially influence stacking velocities and, therefore, may result in big errors in the velocity model determination.

Determination of interval velocities is often performed under the assumption that Dix's formula gives us reasonable values. This formula has been derived for a medium with horizontal homogeneous layers. If there are strong lateral changes in the interval velocities in the shallow part of the section or in the estimated layer, Dix's formula can cause large errors in the velocity model. There are several approaches for developing a Dix's type inversion (Chernjak and Gritsenko, 1979, Hubral and Krey, 1980, Goldin, 1986) for the medium with curvilinear boundaries. They realize layerwise determination of interval velocities and assume locally homogeneous layers. It means that the interval velocity does not change around the zero-offset ray. As analytically shown by one of the authors (Blias 1981, 1987, 1988, 2002), not taking into account lateral velocity changes may lead to significant errors in interval velocities.

There is another problem with layer-by-layer interval velocity calculation. A small error in the first layer causes a bigger error for the second layer. These two errors, in their turn, cause much bigger error in the third layer and so on. If we have more than three layers, this process may result in very large errors for the deep layers. The reason for these errors lies in the connection between stacking velocities for deep horizons and second derivatives of the shallow interval velocities (Blias, 1981, 1988). It implies that small errors in the second derivatives of interval velocities may lead to big estimation errors for deep layers.

These two problems show that for complicated geology (velocity anomalies) other methods should be developed.

## **Introduction**

There are different ways to solve this problem. One approach would be to use the normal incident raytracing and Dix-type formula (T. Krey and P. Hubral, S. Goldin, Chernjak and Gritsenko). This approach uses a layer-stripping inversion and often leads to big errors in the deep layers. The reason of these errors lies in

the connection between stacking velocities for deep horizons and second derivatives of the shallow interval velocities (Bias, 1981). These inversion methods consider local-homogeneous layers; that is, they assume that the interval velocity does not change along the ray. The problem is that, for deep reflectors, moveout function is generated by a wide interval in the shallow layers (Fig. 1).

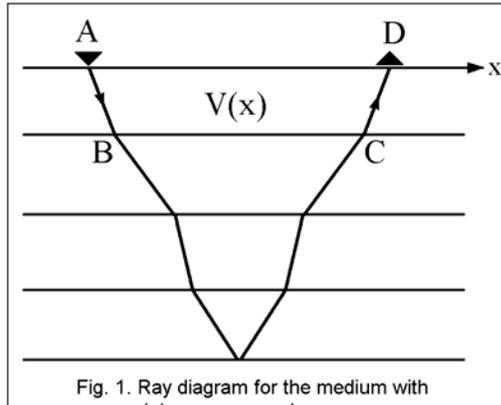


Fig. 1. Ray diagram for the medium with inhomogeneous layer

To obtain an accurate value of stacking velocities for deep reflectors, we have to use long-spread moveout functions. Interval velocity for the downgoing part of the moveout ray (AB) may differ from the one for upgoing ray (CD). As shown by one of the authors (Bias, 1987, 1988, 2002), the second derivatives of the shallow interval velocities cause big changes in stacking velocities.

Another reason for systematic errors in the interval velocity estimations is the change of stacking velocities with the spreadlength caused by non-hyperbolic moveout function.

To reduce these errors, iterative algorithms are used (Goldin, 1986). It decreases the errors, mostly caused by non-hyperbolic moveout, but does not allow minimizing the errors caused by shallow velocity anomalies. As shown by one of the authors (Bias, 1987, 1988), small errors in shallow velocities (and boundaries) estimations may cause big errors for deep interval velocities. The reason for this is that we need to determine not only the shallow interval velocities themselves, but also their second lateral derivatives. In the case of lateral shallow velocity variations, the second derivatives of interval velocities and curvilinear boundaries have a big influence on stacking velocities from deep reflectors. It implies that small errors in the second derivatives of interval velocities may lead to big estimation errors for deep layers.

To solve this problem, we developed an optimization approach. Optimization methods in travelt ime inversion for the layered medium have been considered by S. Goldin, A. Glebov and realized into 2D travelt ime inversion software (Goldin, 1986, Glebov, 1988). This approach allows determination of the interval velocities and reflection boundaries using the results of the velocity analysis. This approach needs an initial model, which can be obtained using a layer stripping inversion method. To improve interval velocities and boundaries, we describe them as a linear combination of basic functions with unknown coefficients. To find these coefficients, we minimize an objective function – squared deviation between model and real time arrivals. The Newton method is used to find this minimum with analytical derivative calculations. This approach allows us to obtain reliable interval velocities and to build a velocity-depth model. It also

allows us to find the shallow velocity model using some a priori information about deep interval velocities.

This approach can be used to map lateral velocity changes of the permafrost. Seismic velocities of the permafrost may vary between 1900 and 4200 m/sec (Calvert et al., 2001). These lateral changes cause large oscillations in stacking velocities from deep reflectors. Using additional velocity information, obtained from the log data, the travel-time inversion algorithm gives us tools to separate shallow and deep lateral velocity changes.

### Shallow velocity anomalies and their influence on stacking velocities

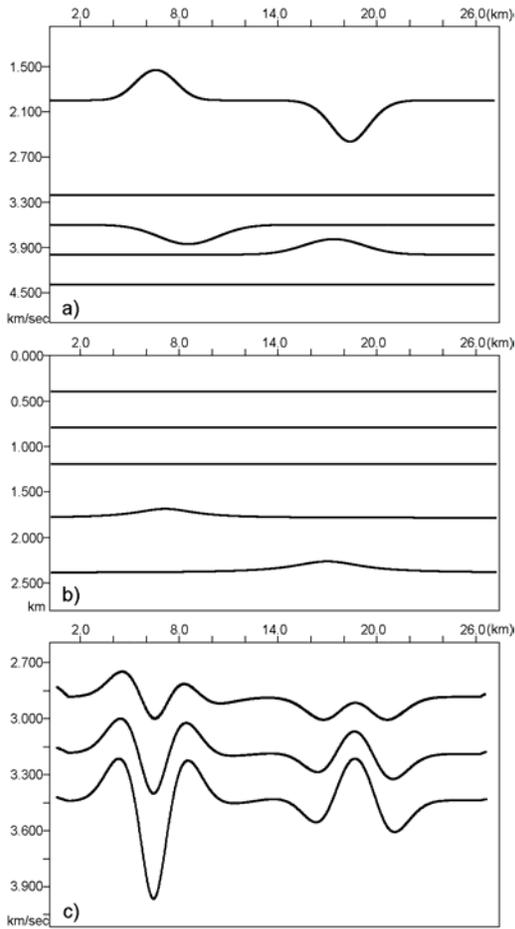


Fig. 2 Velocity model with shallow inhomogeneous layer  
a - Interval velocities  
b - boundaries  
c - Stacking velocities for the model with inhomogeneous layer

Before considering traveltimes inversion, let us put some remarks concerning velocity anomalies and their influence on the depth conversion. Let us consider the velocity model shown on *Fig. 3a* (interval velocities) and *3b* (boundaries). Geometry is the same for all considered models: group interval and shot distance are 40 m, each common shot gather contains 60 receivers and the smallest offset is 40m. As shown by the author (Blais, 1981, 1987, 1988), for deep reflector, stacking velocity repeats the behavior the second-order derivative of the shallow velocity  $v_1(x)$ . This is very well seen on the *Fig. 2c*, where the behavior of stacking velocities is very close to that (with some scalars depending on the reflector depth) of the second-order derivative of the interval velocity in the first layer (*Fig. 3a*). The same quality holds for the curvilinear boundary (Blais, 2002). From this point of view, we can completely understand the behavior of the stacking velocity when there is an overburden velocity anomaly (T. Armstrong et al. 2001, *Fig. 2*).

The stacking velocity behavior just reflects, with some scalar, the behavior of the second-order derivative of the curvilinear shallow boundary. This fact explains why we should not expect that stacking velocity repeats in any way average, or RMS, velocity behavior, when we have strong non-linear lateral velocity changes (interval velocities or curvilinear boundaries) in the shallow part of the section. We may mention here, that the anomaly behavior of the stacking velocity is not caused by non-hyperbolic moveout, as it was stated in the same paper, p. 82. To confirm this, we calculated standard (---) and maximum (---) deviations between

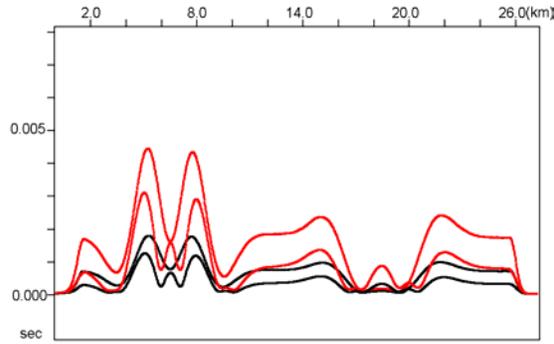


Fig. 3. Standard (—) and maximum (—) deviations between NMO function and its hyperbolic approximation

NMO function and its hyperbolic approximation for the NMO interval 2500m. Fig.4 shows these deviations for the two deepest boundaries. The standard deviation is less than 2 ms and maximum deviation for the most NMO function does not exceed 3 ms even for the close-to depth offset interval.

It confirms that stacking velocity anomaly is not caused by non-hyperbolic moveout, but by the second-order derivatives of shallow velocity

changes

### Velocity model description

Let's consider a layered velocity model with curvilinear boundaries and laterally changing interval velocities. We assume, that we obtained an approximate velocity model using seismic and log data. The problem of finding the approximate values for layered velocities and boundaries is considered by one of the authors (Blias, E., *Stacking and Interval Velocities in a Medium with Laterally Inhomogeneous Layers*, CSEG, 2003). The simplest way to obtain the reference model is to apply a strong smoothing of the stacking velocities, which makes them almost constant and then using Dix's formulas. This allows us to obtain approximate average interval velocities, though we miss the real lateral changes. The aim of the optimization approach is to improve the reference model, to recover lateral interval velocity changes and to map reflectors in a proper way.

We describe interval velocities and boundaries as the sum of some reference (known) functions and linear combinations of basic functions  $\varphi_k(x,y)$  with the coefficients  $\alpha$  and  $\beta$ :

$$s_m(x,y) = v_m^{-1}(x,y) = p_m(x,y) + \sum_k \alpha_{m,k} \varphi_k(x,y), \quad (1)$$

$$F(x,y) = H_k(x,y) + \sum_k \beta_{m,k} \varphi_k(x,y),$$

Here  $\varphi_k(x,y)$  are the basic functions,  $\alpha_{m,k}$ ,  $\beta_{m,k}$  are the unknown coefficients for the m-th layer;  $p_m(x,y)$  and  $H_k(x,y)$  are the slowness and boundaries for the zero approximation of depth velocity model (reference model). Let  $S(x_1, y_1, z_1)$  and  $R(x_2, y_2, z_2)$  be the source and receiver and  $Q_k(\xi_k, \eta_k, F_k(\xi_k, \eta_k))$  be the intersection point along the ray, Fig. 4.

$$T(R,S,M) = \sum_k t_k(Q_{k-1}, Q_k, M) \quad (2)$$

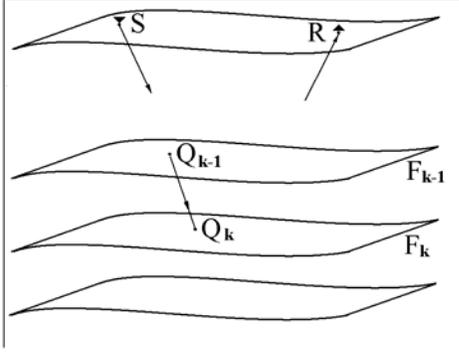


Fig. 4 Ray diagram for 3D medium

Here  $t_k(Q_{k-1}, Q_k)$  gives the time between the points  $(Q_{k-1}, Q_k)$ ,  $M(\alpha, \beta)$  – velocity model which includes boundaries and velocities (1).

Assuming that interval velocity does not change too much within the ray, we can write for the time  $t_k(Q_{k-1}, Q_k)$ :

$$t_k(Q_{k-1}, Q_k) = d_k \int_0^1 s_k(x_k(t), y_k(t)) dt \quad (3)$$

where  $d_k$  gives a distance between the points  $Q_{k-1}$  and  $Q_k$ :

$$d_k = [(\xi_k - \xi_{k-1})^2 + (\eta_k - \eta_{k-1})^2 + (F_k(\xi_k, \eta_k) - F_{k-1}(\xi_{k-1}, \eta_{k-1}))^2]^{1/2}$$

$x_k(t)$ ,  $y_k(t)$  give the parametric equation (with parameter  $p$ ,  $0 \leq p \leq 1$ ) of the ray between the points  $Q_{k-1}$  and  $Q_k$ :

$$x_k(t) = \xi_k + (\xi_k - \xi_{k-1}) p$$

$$y_k(t) = \eta_k + (\eta_k - \eta_{k-1}) p$$

### Traveltime inversion

To apply optimization approach we need a reference depth velocity model – zero approximation for our inversion. First we need a reference velocity model. To find interval velocities and boundaries, we minimize the objective function

$$F(\alpha, \beta) = \sum_{S,R,k} [T_k(R,S,M) - t_k(S,M)]^2 \quad (4)$$

Here  $k$  is the number of reflections that are used;  $t_k(S,M)$  is the observed time for the  $k$ -th wave,  $S$  and  $R$  stand for the source and receiver points. The observed times  $t_k(S,M)$  can be calculated through velocity analysis.

The velocity model  $M$  is described by the coefficients  $\alpha$  and  $\beta$  from representations (1) so we can think of the model  $\mathbf{M}$  as a vector:

$$\mathbf{M} = (\gamma_1, \gamma_1, \dots, \gamma_N)$$

where vector  $\mathbf{M} = (\gamma_1, \gamma_1, \dots, \gamma_N)$  includes all the coefficients  $\alpha$  and  $\beta$  from the presentations (1).

Instead of minimizing the objective function  $F(\mathbf{M})$ , we can find the solution of a non-linear system of equations with respect to  $\mathbf{M}$ :

$$\partial F / \partial \mathbf{M} = 0,$$

which we can write in the form:

$$\partial F / \partial \mathbf{M} = \sum_{S,R,k} [T_k(S,R,\mathbf{M}) - t_k(S,R)] \partial T_k / \partial \mathbf{M} = 0 \quad (5)$$

Here  $\partial T_k / \partial \mathbf{M}$  is a vector with the coordinates  $\partial T_k / \partial \gamma_i$ ,  $i=1, 2, \dots, N$ . To find the solution of the equation (4), we use the Newton method:

$$\mathbf{M}_{n+1} = \mathbf{M}_n - (\partial^2 F / \partial \mathbf{M}^2)^{-1} \mathbf{M}_n, \quad n=1,2,\dots$$

Here  $n$  is an iteration number,  $\partial^2 F / \partial \mathbf{M}^2$  is an  $N \times N$  matrix  $\mathbf{A} = ||a_{ij}||$  with the elements  $a_{ij}$ :

$$a_{ij} = \sum_{S,R,k} \frac{\partial T_k}{\partial \gamma_i} \frac{\partial T_k}{\partial \gamma_j} + \sum [T_k - t_k] \frac{\partial^2 T_k}{\partial \gamma_i \partial \gamma_j} \quad (6)$$

Matrix  $\mathbf{A}$  contains the first and the second order traveltime derivatives along the rays. To find these derivatives, we use an approach developed by one of the authors (Blas, 1985). This approach reduces first-order derivative calculation to the differentiation traveltime in one layer. To find the second-order derivatives, we have to solve a linear system. To use this method, we need to find an explicit formula for the travel time in one layer. If we don't take into account the curvilinear rays, we calculate the layer time through the integral along the straight ray, *formula (3)* (Appendix A). If we want to take into account a curvilinear ray within a layer, we can use more complicated formula derived by one of the authors (Blas, 1988). Thus, this approach allows us to take into account the curvilinear rays.

To stabilize the solution, we use penalty functions. Instead of function (4), we minimize the function

$$F_1(\alpha, \beta) = \sum_{S,R,k} [T_k(R,S,\mathbf{M}) - t_k(S,R)]^2 + \sum_{k,m} (u_m \alpha_{m,k}^2 + w_m \beta_{m,k}^2) \quad (4)$$

Here  $u_m$  and  $w_m$  are the positive weighting factors,  $m$  is the layer number. The weights factors depend on our knowledge about geology. We can use different weights for the velocities and boundaries for each layer. The bigger is the weight, the closer is parameter to its initial value. This allows us to make some interval velocities and boundaries more stable.

Raytracing is based on approach developed by E. Blas (1985), Blas and Lukovkin (1989). This approach reduces the raytracing problem to the solution of a non-linear algebraic system of equations with respect to the ray coordinates. To solve this system, we use the Newton method. Zero approximation for Newton method is obtained with the use of perturbation method.

## Model examples

Let us consider two close model examples for the traveltime inversion. The first model is composed of laterally inhomogeneous layers divided by curvilinear boundaries, *Fig. 5*. The main problem for this model is how to separate lateral velocity changes for curvilinear boundaries. Use of Dix's formula for this case gives us the wrong oscillations in interval velocities and, therefore, wrong boundary depths. These oscillations are caused by the strong stacking velocity oscillations (*Fig. 7*) as a result of shallow velocity anomalies.

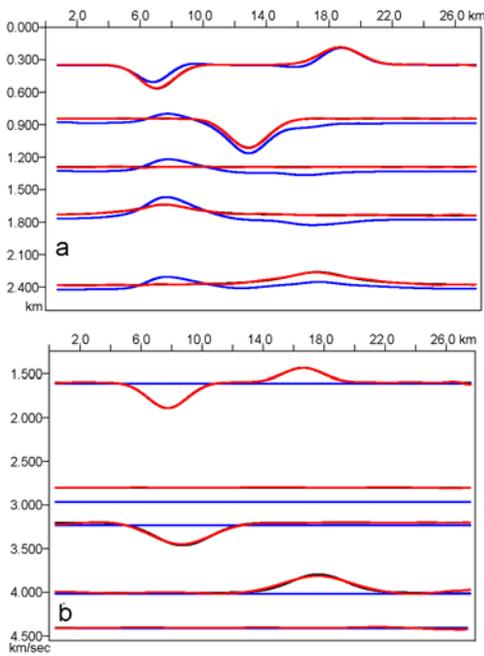


Fig. 6 Traveltime inversion

a. Boundaries: model (—), zero-approximation(—) and final (—)  
 b. Interval velocities: model(—), zero-approximation(—) and final(—)

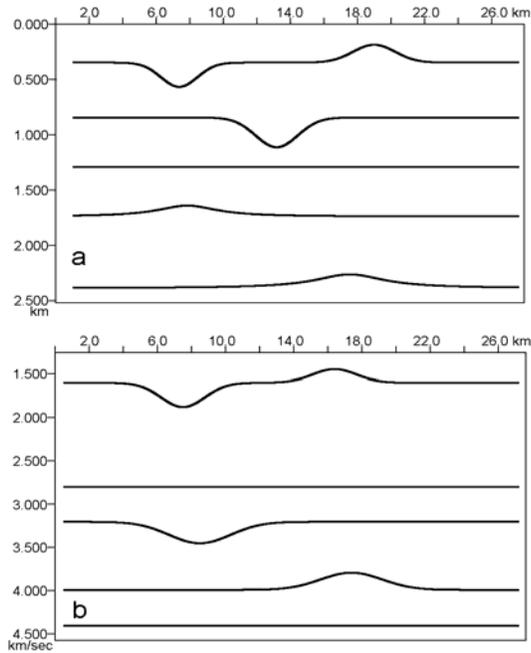


Fig. 5 Velocity model with variable interval velocities and curvilinear boundaries

a - boundaries    b - Interval velocities

To apply the optimization approach, we need a reference model. We can often obtain average values of interval velocities by just calculating them from Dix's formula and averaging them all along the line. We cannot restore velocity anomalies and proper depths, but this approach gives interval velocity estimations quite close to their average values. Knowing average values of the interval velocities, we can, using zero-offset times, find the boundaries and thus, obtain a reference depth velocity model (--- on Fig.6). This model is used as a zero-approximation model to be improved through the optimization process.

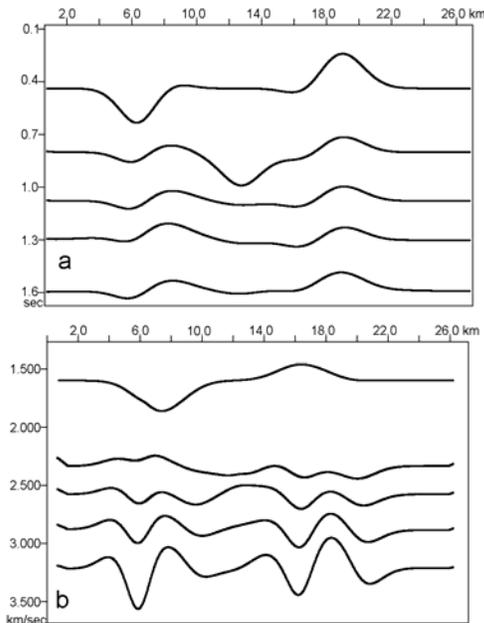


Fig. 7 NMO parameters

a - Zero-offset times    b - Stacking velocities

Optimization process allowed us to recover variable interval velocities and curvilinear boundaries for four iterations. Fig. 6 shows the optimization result: blue lines (---) show the initial approximation for the boundaries (a) and the interval velocities (b); red lines (---) show the resultant model after optimization and the black lines (almost hidden under the red) show model velocities and boundaries.

The next model contains a big lateral velocity variation (up to 1000m/s) in the first

layer, *Fig.8b*. This velocity anomaly can be considered as a permafrost model (Calvert et al., 2001). The model also contains lateral interval velocity changes and curvilinear boundaries in deep portion of the section. Here we assume that we don't have reflection from the bottom of the first layer, so we have to recover this layer (with the others) using deep traveltimes.

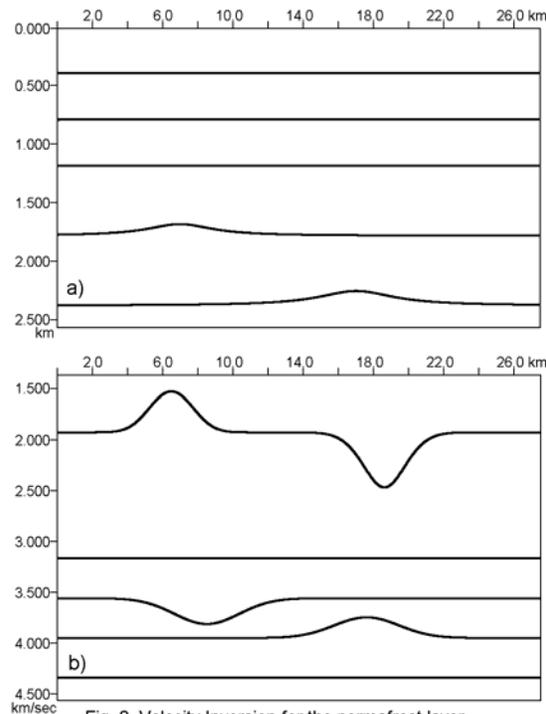


Fig. 8. Velocity Inversion for the permafrost layer  
a. Model boundaries  
b. Model interval velocities

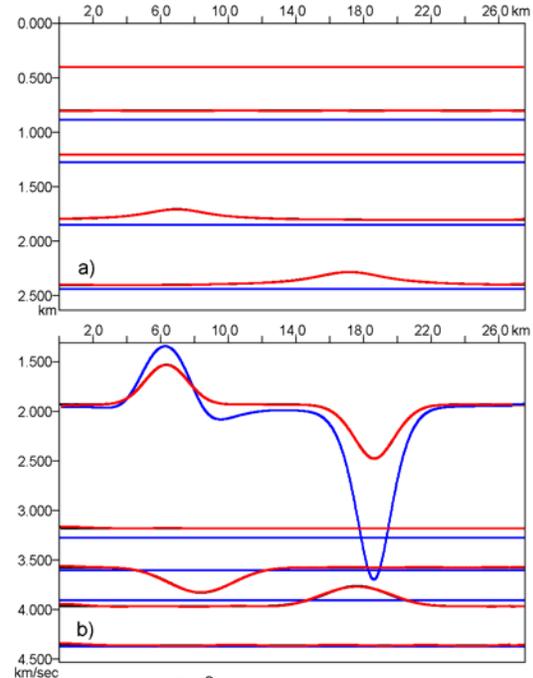


Fig. 9 Velocity Inversion  
a. Boundaries: model (—), zero-approximation(---) and final (---)  
b. Interval velocities: model(—), zero-approximation(---) and final(---)

To find the initial velocity model we assume that all the interval velocities, but the first one, are constant and all boundaries are horizontal. With these assumptions, we found a reference model showed on *Fig.9* with the blue lines (---). We see that all the structures and interval velocity variations are missing. The shallow velocity anomalies, we restored, have big errors. This model was used as a zero-approximation model for the optimization traveltimes inversion.

The result of optimization is shown on *Fig.9*. Blue lines (---) show initial approximation for the boundaries (a) and interval velocities (b); red lines (---) show the resultant model after optimization, and the black lines (almost hidden under the red) show model velocities and boundaries. We see that even in the case where we don't have a reflection from the bottom of the inhomogeneous shallow layer, there is a possibility to recover this velocity anomaly along with the depth velocity model.

## Conclusions

In this paper, we consider a combination of inverse and optimization approaches to build the depth velocity model from seismic data. The goal of the optimization

is to find depth layered velocity model, which brings the objective function to a minimum. To find this minimum, we run an iterative inversion process. In each iteration, we linearize the problem, so that it is reduced to a system of linear equations. To increase stability, different constraints have been applied. We use an optimization approach for the velocity model with laterally inhomogeneous layers, taking into account lateral velocity variation around the rays. This approach allows us to restore shallow velocity anomalies using traveltimes from deep horizons.

## Acknowledgments

The authors are very much grateful to Ludmila Chavina, who made most of the model calculations and helped a lot to obtain and arrange the results. They also would like to thank Michael Burianyk for his help.

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## Appendix A. Formula for the time in 3D inhomogeneous layer

Let's derive a formula for the time between two points  $Q_1(\xi_1, \eta_1, z_1)$  and  $Q_2(\xi_2, \eta_2, z_2)$  in an inhomogeneous layer with the slowness  $s(x, y, z)$ . We assume that lateral changes in interval velocities are relatively small (10% - 15%) with respect to their absolute values. It is well known (Lavrentiev et al., 1969) that in this medium, we can linearize traveltimes with respect to the slowness changes and calculate traveltimes along the straight ray connecting the source and the receiver.

The equation of the line  $Q_1Q_2$  is

$$\begin{aligned} x &= x(z) = \xi_1 + (z-z_1)(\xi_2 - \xi_1)/(z_2-z_1) \\ y &= y(z) = \eta_1 + (z-z_1)(\eta_2 - \eta_1)/(z_2-z_1) \end{aligned} \quad z_1 \leq z \leq z_2 \quad (\text{A-1})$$

The time  $t(Q_1, Q_2)$  between the points  $Q_1$  and  $Q_2$  can be found using the formula:

$$t(Q_1, Q_2) = \int_{z_1}^{z_2} \sqrt{1 + (x'(z))^2 + (y'(z))^2} n(x, y, z) dz \quad (\text{A-2})$$

where  $x = x(z)$ ,  $y = y(z)$  are equations for the ray between the points  $Q_1$  and  $Q_2$ . From (A-1) we obtain:

$$\begin{aligned} x'(z) &= (\xi_2 - \xi_1)/(z_2 - z_1) \\ y'(z) &= (\eta_2 - \eta_1)/(z_2 - z_1) \end{aligned} \quad (\text{A-3})$$

After substituting (A-1) and (A-3) into (A-2) and changing the variable of integration into  $p$ :  $p = (z-z_1)/(z_2-z_1)$  we obtain the formula

$$\begin{aligned} t(Q_1, Q_2) &= \sqrt{(\xi_2 - \xi_1)^2 + (\eta_2 - \eta_1)^2 + [F_2(\xi_2) - F_1(\xi_1)]^2} \times \\ &\times \int_0^1 s(\xi_1 + (\xi_2 - \xi_1)p, \eta_1 + (\eta_2 - \eta_1)p, z_1 + (z_2 - z_1)p) dp \end{aligned} \quad (\text{A-4})$$

Let's consider particular case when the ray is vertical and the points  $Q_1$  and  $Q_2$  are on the 2D curves  $z = F_1(x)$  and  $z = F_2(x)$  respectively. Then differentiating (A-4) twice and putting  $\xi_1 = \xi_2, \eta_1 = \eta_2$ , we obtain:

$$\begin{aligned} \partial^2 t / \partial \xi_1^2 &= n/d + 1/3 d \partial^2 n / \partial x^2 - n \partial^2 F_1 / \partial x^2 - \partial F_1 / \partial x \partial n / \partial x \\ \partial^2 t / \partial \xi_1 \partial \xi_2 &= -n/d + 1/6 d \partial^2 n / \partial x^2 + 1/2 (\partial F_2 / \partial x - \partial F_1 / \partial x) \partial n / \partial x \\ \partial^2 t / \partial \xi_2^2 &= n/d + 1/3 d \partial^2 n / \partial x^2 + n \partial^2 F_2 / \partial x^2 - \partial F_2 / \partial x \partial n / \partial x \end{aligned} \quad (\text{A-5})$$

These formulas coincide with the formulas, obtained by S. Gritsenko and V. Chernjak (Gritsenko and Chernjak, 2001).