Prestack F-xy Eigenimage Noise Suppression

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ABSTRACT

Summary

Last year it was shown how f-xy eigenimage filtering (Canadian and U.S. patents pending) can be used to remove random noise from stacked 3-D volumes. Here we adapt f-xy eigenimage filtering for removing noise from 2-D and 3-D prestack traces by having the x-y coordinates represent common shot and receiver. A new theoretical result states that this type of filtering is exact for noiseless that has a restricted number of dips in the CMP domain. Tests show that applying eigenimage filtering before stack can reduce noise to a degree not possible with poststack processes alone. They also show that, for very noisy data, preceding f-xy eigenimage filtering with a simple dip filter can be far more effective than applying either alone.

Properties of f-xy eigenimage filtering suggest that it can be applied before statics correction or even deconvolution. We find that not only can eigenimage filtering be applied before statics correction, it occasionally improves the estimated statics. Applying eigenimage filtering before deconvolution has so far not been a success, for reasons we don't understand.

Introduction

Andrews and Patterson (1976) demonstrated how the singular value decomposition, or SVD, can be used for noise suppression of digital images. The SVD is defined as (Golub and Van Loan, 1996)

$$\mathbf{A} = \mathbf{U}\Sigma\mathbf{V}^H .$$

We will consider A to be a square complex matrix of dimension *n,* but rectangular matrices are handled just as easily. U and V are *n*-by-*n* unitary matrices whose column vectors u_i and v_i are the left and right eigenvectors of A, respectively. Σ is a real diagonal matrix whose diagonal elements σ_i, known as the singular values, are ordered such that

$$\sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_n \geq 0.$$

It is easy to show

$$\mathbf{A} = \mathbf{I}_1 + \mathbf{I}_2 + \cdots + \mathbf{I}_n, \quad \mathbf{I}_i = \sigma_i \mathbf{u}_i \mathbf{v}_i^H .$$

The matrix I_i is referred to as the *i'th weighted eigenimage*. For *k* <= *n* define

$$F_k(\mathbf{A}) = \mathbf{I}_1 + \mathbf{I}_2 + \cdots + \mathbf{I}_k.$$

This is known as a truncated-SVD, or reduced-rank, approximation to A. We will refer to it as eigenimage filtering. Parameter *k* is called the *rank*. $F_k(\mathrm{A})$ becomes an increasingly better estimate to A as *k* increases, until finally $F_n(\mathrm{A}) = \mathrm{A}$. We generally, however, require only a few eigenimages to generate a reasonable image.

Many noise suppression schemes transform the data into a domain where signal and noise map onto separate regions. Eigenimage filtering is similar – it assumes that coherent energy maps onto the first few eigenimages, and that incoherent energy maps onto the remainder (Figure 1).

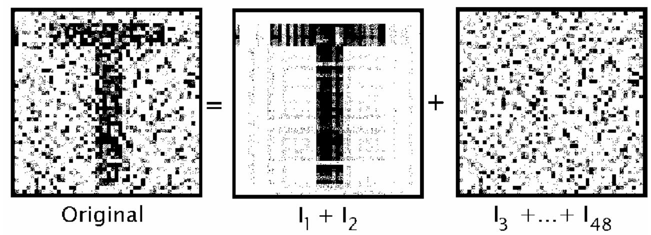

Fig. 1: A 48 x 48 pixel image of the letter T decomposed into the sum of its first two weighted eigenimage, containing most of the signal, and the sum of the remaining 46 eigenimage, containing most of the noise.

Ulrych, Freire, and Siston (1988) developed a number of seismic applications for eigenimage filtering. Trickett (2002) adapted eigenimage filtering for removing random noise from stacked 3-D volumes in the frequency domain. Starting with an *n*-by-*n* grid of stacked traces, the method, which we will refer to as *f-xy eigenimage filtering*, is as follows:

Take the DFT of each trace.

For each frequency…

> Form the n-by-n complex-valued matrix A from the DFT value of each trace.
>
> Calculate $F_k(\mathrm{A})$ for some small value of k.
>
> Replace the trace DFT values with the $F_k(\mathrm{A})$ values.

Take the inverse DFT of each trace.

The amount of attenuated noise can be increased by increasing the grid size *n* (from 15 to 30, say) and most importantly decreasing the rank *k*. By doing so, however, we also increase the chance of distorting signal. Typical values for *k* are 1 (harsh), 2 (strong), or 3 (mild).

The method was shown to have a number of properties:

Exactness Property: If a noiseless seismic section contains no more than *k* dips then f-xy eigenimage filtering does nothing.

*Statics Property**:*** F-xy eigenimage filtering is independent of x- and y-consistent statics.

*Filtering Property**:*** If a noiseless seismic section contains no more than *k* dips, and then has x- and y-consistent filters applied, then f-xy eigenimage filtering does nothing.

Methodology

How can we adapt this method for prestack noise suppression? Particularly, how do we group the traces together for filtering? The above properties suggest an answer; have one of the x-y coordinates represent common shot and the other common receiver. For 2-D lines this means arranging the unstacked traces like a surface stacking chart (*Fig. 2*) and then applying eigenimage filtering as one would a 3-D stacked volume. This scheme, together with the above properties, suggest the eigenimage filtering might be applied before surface-consistent statics correction, and perhaps even before deconvolution.

What do we do about non-uniform shooting geometries? Shots on land are rarely equally spaced – there are gaps, recovery shots, skids, and so on. This is answered by the following extension to the exactness property (proof not given):

Shooting Property: If a noiseless prestack seismic data set contains no more than *k* dips when viewed in the CMP domain then f-xy eigenimage filtering does nothing when the x-y coordinates represent common shot and receiver.

This is a fundamental and powerful result for prestack noise suppression. There is no restriction on shot and receiver positions – they can even be random or repeated. The assumption of a limited number of dips, however, suggests that traces should be NMO corrected beforehand, and that traces should be grouped so that the range of CMP's is as small as possible. The shooting property also highlights deficiencies in other methods; one cannot, for example, apply standard f-xy prediction filtering (Chase, 1992) to prestack 2-D data in this manner unless shots and receivers are uniformly spaced.

All of the above properties assume that there are no missing traces. Although not as bad as *Fig. 2* suggests, filtering prestack data invariably means that we must deal with incomplete rectangular grids of traces by, for instance, inserting zeroed traces. In practise this seems to have only a minor effect on the quality of the final result. One option, however, is the use of SVD's that can handle missing data (Hawkins, 2001).

Application to prestack 3-D land data also becomes straightforward. One scheme is to filter every combination of shot and receiver line as if they were prestack 2-D lines (*Fig. 3*). The fact that shot and receiver lines are typically orthogonal to, rather than inline with, each other is of no concern due to the shooting property.

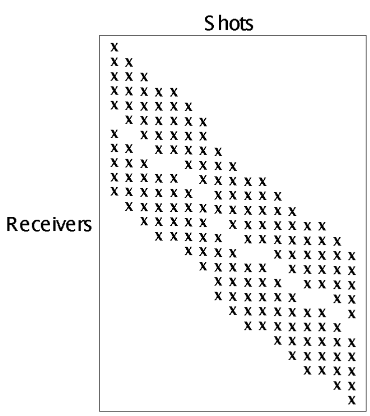

Fig. 2: A surface stacking chart where unstacked traces are positioned according to shot and receiver.

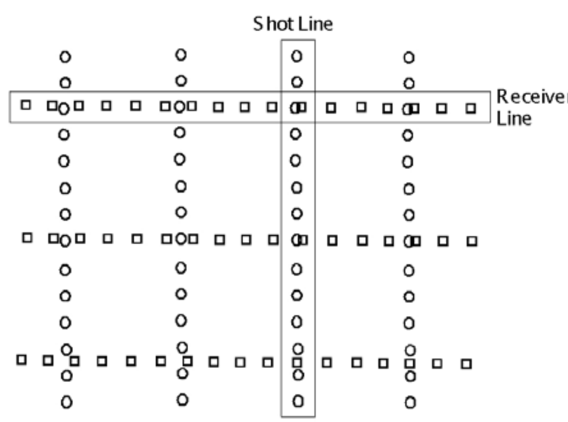

Fig. 3: For 3-D volumes, every combination of shot and receiver line can be filtered separately as if it were a 2-D line.

Data Results

We tried f-xy eigenimage filtering on a line having only 24 traces per shot, making prestack noise suppression particularly difficult. *Fig. 4* shows a single shot record before and after eigenimage filtering of the entire line. The difference indicates that little signal has been removed. Note also that offset-dependent effects seem to be well preserved.

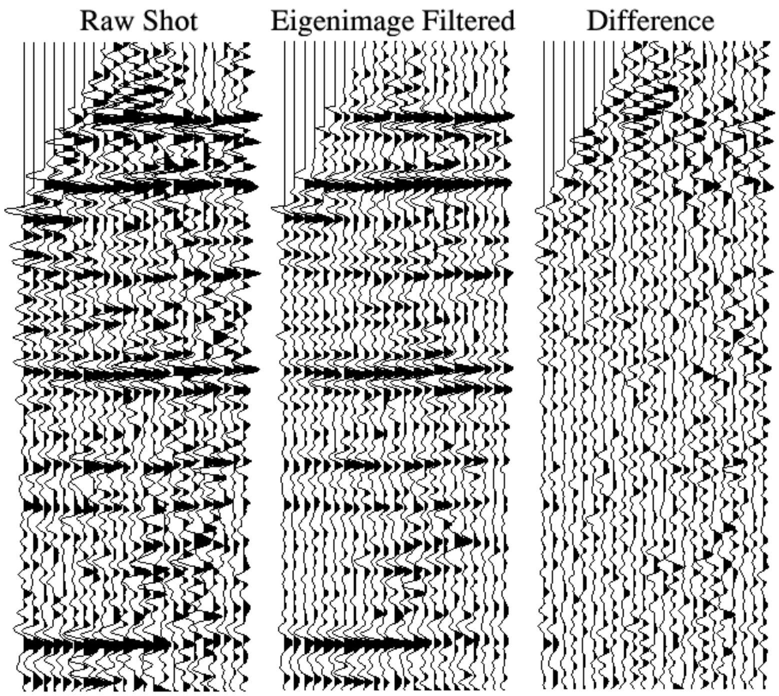

Fig. 4: A single 24-trace shot record before and after eigenimage filtering of the entire line.

When this data was stacked, migrated, and f-x projection filtered (*Fig. 5*), the stack with eigenimage noise suppression was considerably sharper, cleaner, and more continuous than the one without.

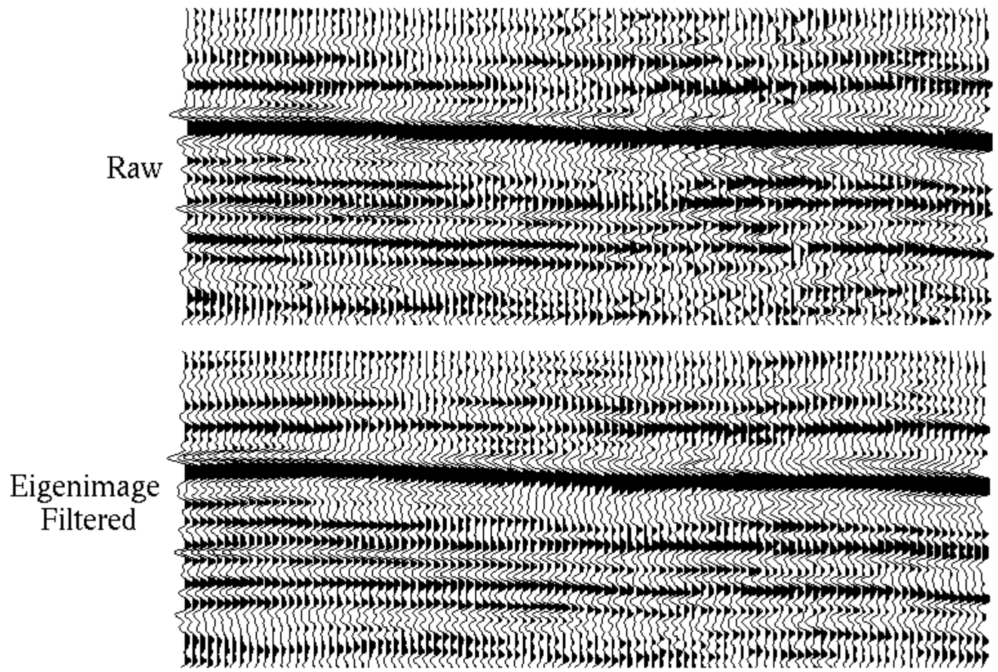

Fig. 5: Migrated stacks of shots in Fig. 4 without and with eigenimage filtering.

Fig. 6 shows an inline slice of a stacked 3-D volume, without and with prestack f-xy eigenimage filtering. All combinations of shot and receiver lines were filtered separately as if they were 2-D lines.

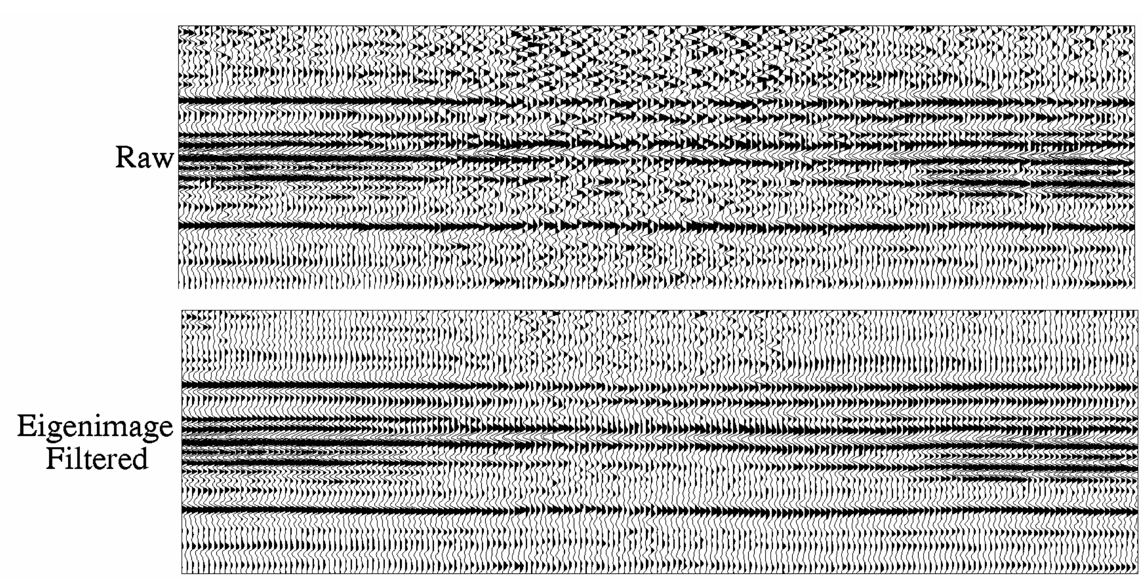

Fig. 6: An inline slice through a stacked 3-D volume, without and with prestack eigenimage filtering. A rank of 1 was used, which is harsh for all but very flat data.

The Statics Property suggests that f-xy eigenimage filtering can be performed before surface-consistent residual statics correction. We have found that not only is it safe to do so, but the resulting noise suppression occasionally improves the statics. Residual statics failed in one structured line, resulting in (according to the interpretor) a statics bust. We then ran harsh f-xy eigenimage filtering before residual statics estimation. The statics bust was repaired, resulting in a stack with which the interpretor was much more satisfied.

Discussion

We have demonstrated that f-xy eigenimage filtering (Canadian and U.S. patents pending) can be adapted for both 2-D and 3-D prestack random noise suppression by having the x-y coordinates represent common shot and receiver. The Shooting Property provides a strong theoretical basis for this scheme, whether we are working on flat or structured data. There are a number of reasons one might wish to apply prestack eigenimage filtering; better noise suppression in the final stack, better velocity analysis, better residual statics, and so on.

An intriguing idea is the cascading of noise filters of different types. Preceding eigenimage with dip or f-k filters seems particularly fruitful (assuming that statics have already been corrected for). In one very noisy area in particular, simple prestack dip filtering followed by eigenimage filtering gave dramatic results, whereas nothing else, including dip or eigenimage filtering alone, seemed to work. The reason may be that eigenimage filtering is effective only when the signal-to-noise level is above some threshold. Prediction-based methods, on the other hand, do not seem to behave well when immediately preceded by spatial filters, perhaps because they perform inversions which can become unstable.

Results of applying eigenimage filtering before surface-consistent deconvolution have so far looked poor, for reasons not fully understand. It may be, however, that the deconvolution operators derived from eigenimage filtered data may be superior. This requires further study.

Acknowledgements

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