# AVA Imaging of 3-D Common Azimuth Data

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**ABSTRACT**
This presentation summarizes our experience with pre-stack 2D/3D inversion of Amplitude versus Angle (AVA) gathers. As demonstrated by Kuehl and Sacchi (2003), AVA imaging can be posed within the linear inverse theory framework. This provides several advantages. First, we are able to incorporate model space weighting operators that improve amplitude fidelity in common angle gathers. In addition, the influence of improperly sampled data can be diminished. The latter leads to the attenuation of acquisition footprints.

In order to make our problem computationally tractable, we utilize 3D common azimuth data (Biondi and Palacharla, 1996). The inversion algorithm uses the method of conjugate gradients. We show that robust estimates of AVA attributes can be obtained by properly selecting the model and data space regularization operators. Finally, it is important to stress that the inversion of AVA gathers is the first step towards a robust and accurate estimation of physical rock properties and fluid indicators from seismic records. The latter is the ultimate goal of our research.

## Introduction

Common image gathers in angle domain (Stolt and Weglein, 1985; de Bruin et al., 1990) carry valuable angle dependent amplitude information. For this reason, AVA/AVP migration has gained increasing interest in recent years (Xu et al., 1998; Prucha et al., 1999; Wapenaar et al., 1999; Mosher and Foster, 2000; Sava et al., 2001). Kuehl and Sacchi (2002, 2003) showed that regularized least-squares wave equation migration could be used to mitigate imaging artifacts and acquisition-induced artifacts caused by missing observations.

In this article, we present an extension of the 2D AVA inversion algorithm introduced by Kuehl and Sacchi (2002) to the 3D case. We use the common azimuth operator proposed by Biondi and Palacharla (1996) in conjunction with a combination of a PSPI (phase shift plus interpolation) and split step correction (SSC) in order to account for lateral velocity variations in the 3D macro velocity field in both the forward (de-migration) and adjoint (migration) operators that are required by the inversion scheme. Common azimuth migration permits us for a considerable reduction of the data size and computational cost of the migration. This is crucial for any attempt to implement least-squares migration on real data.

### 3-D Common Azimuth Wave Equation Migration And Ava Imaging

Biondi and Palacharla (1996) proposed a phase-shift migration operator for 3-D common azimuth data. The algorithm downward continues the surface wave field using the following propagation scheme:

$$P(z+dz,\omega,k_{mx},k_{my},k_{hx}) = P(z,\omega,k_{mx},k_{my},k_{hx})\cdot e^{-ik_z dz} \qquad (1)$$

where the vertical wave number is calculated by a modified double square root equation:

$$k_z = \omega\left(\sqrt{\frac{1}{v(r,z)^2}-\frac{1}{4\omega^2}[(k_{mx}+k_{hx})^2+(k_{my}+\overline{k_{hy}})^2]}+\sqrt{\frac{1}{v(s,z)^2}-\frac{1}{4\omega^2}[(k_{mx}-k_{hx})^2+(k_{my}-\overline{k_{hy}})^2]}\right) \qquad (2)$$

$v(r,z)$ and $v(s,z)$ are the velocities evaluated at depth z and source and receiver lateral locations r and s. These velocities are replaced with the average velocity at a given depth, $v_{m(z)}$. Lateral velocity variation effects can be alleviated with velocity corrections terms like the pre-stack split-step correction (Popovici, 1996). For large velocity variations Gazdag's PSPI (1984) in conjunction split step is adopted (Kuehl and Sacchi, 2002). The spatial frequencies $k_{mx}$ and $k_{my}$ are the midpoint wavenumbers in in-line and cross-line directions respectively. In addition, $k_{hx}$ is in-line offset wavenumber. The resulting expression for $\overline{k_{hy}}$ is obtained by using the stationary phase approximation (Biondi and Palacharla, 1996):

$$\overline{k_{hy}}(z) = k_{my}\frac{\sqrt{\frac{1}{v_m{}^2}-\frac{1}{4\omega^2}(k_{mx}+k_{hx})^2}-\sqrt{\frac{1}{v_m{}^2}-\frac{1}{4\omega^2}(k_{mx}-k_{hx})^2}}{\sqrt{\frac{1}{v_m{}^2}-\frac{1}{4\omega^2}(k_{mx}+k_{hx})^2}+\sqrt{\frac{1}{v_m{}^2}-\frac{1}{4\omega^2}(k_{mx}-k_{hx})^2}} \qquad (3)$$

*Equations (2) and (3)* provide a routine to back propagate energy to different depths. At each depth, we can image the wave field at zero time by considering the following two steps. First, we use the radial-trace transform (Sava et al, 2001) to compute the contribution to the image of waves propagating with ray parameter $p_{hx}$. The relationship between offset ray parameter $p_{hx}$, frequency $\omega$ and offset wavenumber $k_{hx}$ is straightforward:

$$p_{hx} = \frac{k_{hx}}{\omega} \tag{4}$$

The second step is to sum all traces along radial lines in ($k_{hx,}$ $\omega$) domain with slope $p_{hx}$ (Mosher and Foster, 2000).

The above algorithm produces Common Image Gathers (CIG) in the ray parameter domain. These image gathers can be transformed to angle domain by the following expression:

$$\sin\theta = \frac{v(m,z)\,p_{hx}}{2\cos\varphi} \tag{5}$$

where $\theta$ is the incident angle, $v(m,z)$ is the velocity at the midpoint position $m$ and $\varphi$ is the dip angle of the interface.

**Least-Squares Wave Equation Ava Migration For 3-D Common Azimuth Data**

We consider seismic data as the result of a linear transformation on an earth model *m*

$$d = Lm + n \tag{6}$$

where $d$ denotes the observed data, $L$ is the forward operator, $m$ is common image gathers, and $n$ is the noise. Conventional migration entails applying $L'$, the adjoint of $L$, to the observed data. When the data are properly sampled, the amplitude in the CIG can be corrected by incorporating the Jacobian correction Sava (2001). This correction attempts to make the adjoint operator behave like the inverse operator. In general, this correction is not sufficient to achieve good amplitude fidelity. Sampling and migration artifacts are not suppressed by this correction. These artifacts can be attenuated, however, by constraining the solution to exhibit certain degree of smoothness along the ray parameter axis. In this case, we adopt the following cost function to retrieve a migrated image that "fits" the observations and, in addition, exhibits smoothness or continuity along the ray parameter axis:

$$F(m) = \| W(d - Lm) \|^2 + \lambda^2 \| D_{1hx} m \|^2 \tag{7}$$

where $W$ is a diagonal weighting matrix used to decrease the influence of “bad data” (missing observations) in the migrated image. The operator $D_{1hx}$ is a first order derivative operator along the in-line ray parameter-offset direction. Least-squares migration seeks a model $m$ by minimizing the sum of the two norms. The trade-off parameter $\lambda$ determines the amount of smoothing. We minimize the objective function using a conjugate gradients algorithm (Hestenes and Stiefel, 1952). In this case, the algorithm reduces to the sequential application of the following operators: migration ($L'$), de-migration ($L$), smoothing ($D_{1hx}$) and, sampling ($W$). It is important to stress that these operators are applied *in the flight*; in other words, there is no need of constructing equivalent operators in matrix form.

## Field Data Example

We tested our least-squares common azimuth migration algorithm using the Erskine data set provided by Veritas Geoservices. The data were first binned, and ensembles of common azimuth were created. The binned data consist of 157 in-lines and 40 cross-lines. The offset dimension ranges from zero to 3000 meters with a highly uneven distribution. The CDP gathers are quite sparse as the result of binning (*Fig. 1*). This is typical in orthogonal 3D surveys in land acquisition.

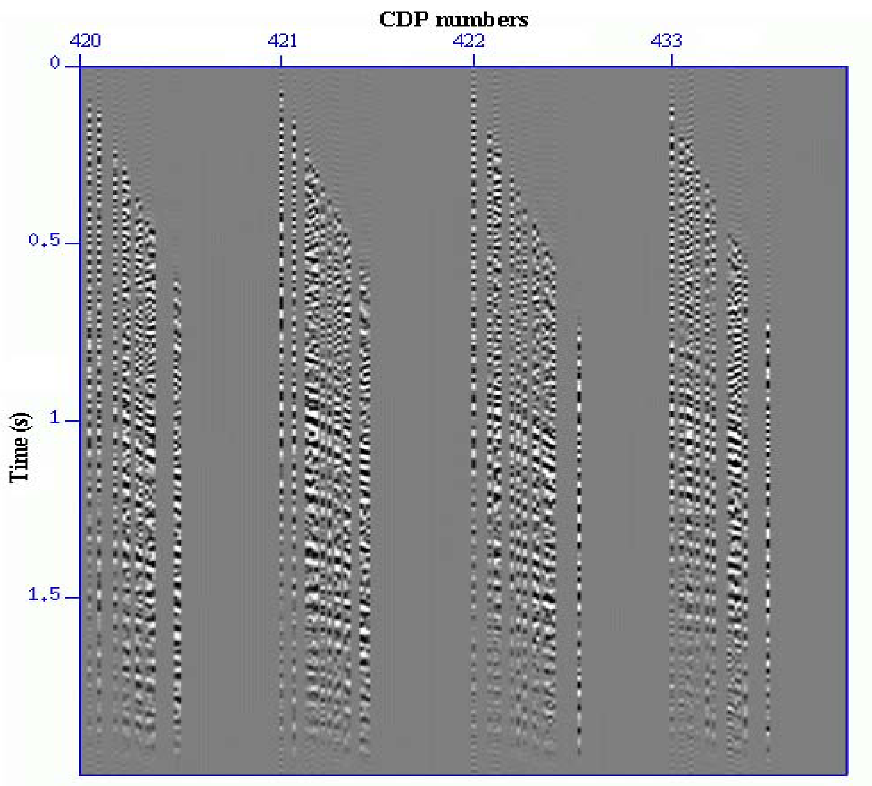


*Fig. 1. CDP gathers for in-line #10, each gather has 60 offsets with a spacing of 25 meters.*

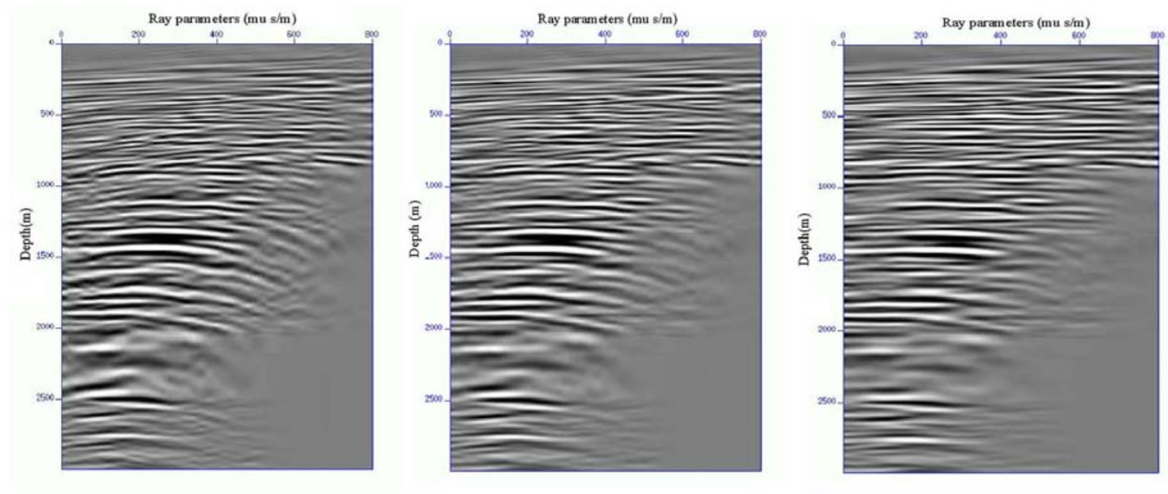


*Fig. 2. CIGs for in-line #10, cross-line #7 (left: iteration 1, center: iteration 2, right: iteration 4)*

CIGs were obtained with offset ray parameters in the range 0 to 800 μs/m, with a ray parameter interval of 20 μs/m. *Fig. 2* shows the calculated CIGs for the midpoint with position in-line #10, cross-line#7. *Fig. 2* (left) portrays the migrated imaged (equivalent to 1 iteration of the LS inversion). Artifacts along ray parameter, an effect caused by irregular data sampling, are clearly seen. *Figs 2* (center and right) portrays the least-squares inverted CIG after 2 iterations and 4 iterations, respectively. At iteration 4 we start to see an important attenuation of sampling artifacts.

**Conclusions And Discussion**

Least-squares AVA migration for common azimuth data has potential for deriving high resolution artifact-free CIG that can be subsequently used to extract rock and/or fluid properties. It provides high quality common image gathers in the angle domain and, in addition, a migrated image that can be used to reconstruct the seismic volume (de-migrate).

Our current implementation of LS migration uses the method of conjugate gradients in its simplest form. We are currently examining the possibility of using the total least-squares method (Arun, 1993) in an attempt to combat modeling operator errors (velocity mismatch) as well as sampling related artifacts.

**Acknowledgement**

The Signal Analysis and Imaging Group at the University of Alberta would like to acknowledge the financial support from the following companies: Encana, Geo-X Ltd., and Veritas Geoservices. Especially thank Veritas Geoservices for providing

Erskine common azimuth data. This research has been partially supported by Natural Sciences and Engineering Research Council of Canada, AOSTRA/Alberta Department of Energy and Schlumberger Foundation.

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