Approximation Errors in AVO-Analysis and Inversion

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ABSTRACT

The AVA-response of two-layer models for AVO-Classes 1 and 3 is computed with the Shuey-approximation, the Fatti-approximation as well as the Smith and Gidlow-approximation and is compared to exact Zoeppritz.
For both AVO-Classes, a weak reflector and a strong reflector are considered in order to test the small parameter change assumption.
Shuey's approximation is utilized in order to test the case of weak VTI-type anisotropy. The VTI-approximation error is found to be secondary to the isotropic approximation error for both Class 1 reflectors considered.
Smith and Gidlow style least-squares fitting is employed in order to fit all three Zoeppritz-approximations to exact Zoeppritz responses for the same range of two-layer models. Shuey's approximation gives the best fit for the Class 3 case, and for both Class 1 and Class 3, the Shuey-fit improves with reflector strength.
Exact Zoeppritz plane wave responses are least squares fitted to spherical wave AVO-responses of reflectors at 1000m depth. For both the weak reflector and the strong reflector, the fit is very good in the Class 3 situation. In the Class 1 case, the fit is poor in the vicinity of the critical angle and, in general, deteriorates with reflector strength.

Introduction

Amplitudes of reflected and transmitted plane waves at a planar boundary of two elastic media in welded contact are completely determined for all incidence angles by the Zoeppritz equations, and can be computed if the elastic parameters are known. A number of linearized approximations to the Zoeppritz equations have been developed that give more insight into the factors that control amplitude changes with offset/angle and simplify computations. The three most popular Zoeppritz equation approximations all start from an equation given by Aki and Richards (1980). The three-term version of Shuey's equation (1985) represents the Aki and Richards approximation except for rearranging terms and substituting Poisson's ratio in order to eliminate shear velocities. Smith and Gidlow (1987) make use of Gardner's relation between density and P-wave velocity in order to eliminate density. Fatti et al. (1994) rewrite the Aki and Richards equation in terms of acoustic impedances and make a small angle assumption by dropping some density terms to arrive at their final equation. In general, these approximations break down at incidence angles beyond 30 degrees and are valid only for small elastic parameter changes across the interface, limitations that do not exist for the exact Zoeppritz equations. However, even "exact Zoeppritz" is derived only for isotropic elastic materials.

Thomsen (1993), Tsvankin (1996) and Rueger (1996) describe approximations for the case of VTI-type anisotropy. Anisotropic P-wave reflection coefficients are computed by adding anisotropic terms to isotropic reflection coefficients. The additional anisotropic terms are controlled by the difference of Thomsen parameters, δ and ε, across the reflecting boundary.
What is the impact of approximation errors on AVO-inversion?
According to Drufuca and Mazzotti (1995), parameters are better constrained by greater contrasts across the interface and the inclusion of critical angles gives rise to unique solutions. Unfortunately, it is exactly under these conditions that Zoeppritz-approximations start to break down.
The purpose of this modelling study is to compare exact AVO-responses with various approximations for AVO-Classes 1 and 3.

Approximation errors in isotropic AVO-analysis

The starting point for Shuey's approximation (Shuey, 1985), which is taken from Aki and Richards (1980), is given by

$$R_{pp}(\theta) \approx \frac{1}{2}(1-4(\frac{W}{V})^2 \sin^2\theta)\frac{\Delta\rho}{\rho} + \frac{1}{2}(1+\tan^2\theta)\frac{\Delta V}{V} - 4(\frac{W}{V})^2 \sin^2\theta \frac{\Delta W}{W} \qquad (1)$$

where Rpp = P-wave reflection coefficient
V = average P-wave velocity across the interface
W = average S-wave velocity across the interface
ρ = average density across the interface
θ = average of $\theta 1$ and $\theta 2$
ΔV = V2-V1, etc.

This is the notation used by Smith and Gidlow (1987).
When applying Gardner's relation to equation (1) in order to eliminate density ρ, Smith and Gidlow (1987) obtain the following approximation

$$R_{pp}(\theta) \approx \frac{5}{8}\frac{\Delta V}{V} - (\frac{W}{V})^2 (4\frac{\Delta W}{W} + \frac{\Delta V}{2V})\sin^2\theta + \frac{\Delta V}{2V}\tan^2\theta \qquad (2)$$

Fatti et al. (1994) also start from equation (1) and, when neglecting a density term, arrive at

$$R_{pp}(\theta) \approx \frac{\Delta I}{2I}(1+\tan^2\theta) - 4(\frac{W}{V})^2 \frac{\Delta J}{J}\sin^2\theta \qquad (3)$$

where $\Delta I/I = \Delta V/V + \Delta\rho/\rho$ and
$\Delta J/J = \Delta W/W + \Delta\rho/\rho$

$\Delta I/I$ and $\Delta J/J$ are the zero offset P-wave reflection coefficient and zero offset S-wave reflection coefficient, respectively.
Fig. 1 shows a comparison of Class 1 AVO-responses computed with the above three approximations and the exact plane-wave response computed with the Zoeppritz equations. The two-layer model utilized for the computation of Fig. 1 is derived from Rutherford and Williams (1989) with Rpp(0)=0.1 .
There is a significant departure of all three approximations from exact Zoeppritz even below 20 degrees of angle.
Fig. 2 is computed for a Class 1 model adapted from Krail and Brysk (1983) with Rpp(0)=0.38 . Again, there is a significant departure from exact Zoeppritz even below 20 degrees of angle. Surprisingly, the relative departure appears to be less for the Rpp(0)=0.38 case when compared to the Rpp(0)=0.1 case of the weaker reflector.
Figs. 3 and 4 show a similar comparison of Class 3 AVO-responses. The two-layer model utilized for Fig. 3 is again derived from Rutherford and Williams (1989) for Rpp(0)=-0.1 . Fig. 4 is based on Ostrander (1984) with Rpp(0)=-0.365 .
The Fatti-approximation and even more so the Shuey-approximation are quite close to exact Zoeppritz in this Class 3 case, especially at incidence angles below 30 degrees, with the weaker reflector [Rpp(0)=-0.1] showing the best agreement. Note that there is no critical angle in the Class 3 case because of the P-wave velocity inversion. In all of the above Figures the Smith and Gidlow-approximation departs the most from exact Zoeppritz at small incidence angles. It is assumed that, for the models chosen, Gardner's relationship does not hold.

Approximation errors in VTI-type anisotropic AVO-analysis

VTI-type anisotropic reflection coefficients can be decomposed into an isotropic part and an anisotropic correction term (see for example Rueger, 1996)

$$R_{pp}^{VTI}(\theta) \approx R_{pp}^{iso}(\theta) + R_{pp}^{ani}(\theta) \tag{4}$$

where

$$R_{pp}^{ani}(\theta) \approx \frac{1}{2}(\delta_2 - \delta_1)\sin^2\theta + \frac{1}{2}(\varepsilon_2 - \varepsilon_1)\sin^2\theta\tan^2\theta \tag{5}$$

and ε,δ are Thomsen-parameters.

From Figs. 1 to 4 it is concluded that Shuey's approximation [equ.(1)] has the smallest overall approximation error. Therefore, for the approximate computations in this section, equ.(1) is substituted for the isotropic term in equ.(4). The exact VTI-type anisotropic AVO-responses are computed with equations derived by Graebner (1992) and Rueger (1996).
Figs. 5 to 8 utilize the same two-layer AVO-models as employed for Figs. 1 to 4 . Weak anisotropy (ε=0.15, δ=0.05) is assumed for the top layer in VTI computations.

Both Class 1 AVO-examples in Figs. 5 and 6 show the isotropic approximation error far exceeds the VTI-approximation error. This confirms the observation of some authors that, for their specific cases, anisotropy had little influence on AVO-responses (Hanitzsch et al.,1995).
For the two Class 3 AVO-examples displayed in Figs. 7 and 8, approximation errors are much smaller by comparison. The weaker reflector in Fig. 7 [Rpp(0)=-0.1] shows very good agreement between approximations and exact AVO-responses below 40 degrees. The small parameter change assumption appears to break down in Fig. 8 .

Least-squares fitting of approximations to exact Zoeppritz responses

Smith and Gidlow (1987) derive the equations for least-squares fitting of a set of AVO data points. With these equations, P-wave velocity reflectivity $\Delta V/V$ and S-wave velocity reflectivity $\Delta W/W$ can be computed. Instead of measured data points, AVO-responses determined via the Zoeppritz equations are utilized in this modelling study. The “optimum” velocity reflectivities arrived at by this least-squares fitting approach are then employed in order to compute “optimum” AVO-responses which can be compared to the original exact Zoeppritz responses.
The same least-squares fitting equations can be applied to Fatti’s approximation [equ.(3)] if $\Delta I/I$ replaces $\Delta V/V$ and $\Delta J/J$ replaces $\Delta W/W$. Similarly, if $\Delta J/J$ replaces $\Delta W/W$, Shuey’s approximation [equ.(1)] can also be adapted to Smith and Gidlow’s least-squares fitting.
Figs. 9 and 10 show Class 1 AVO-responses resulting from least-squares fitting of all three Zoeppritz-approximations. Figs. 11 and 12 show the equivalent Class 3 AVO-responses. Figs. 9 to 12 utilize the same two-layer AVO-models introduced in Figs. 1 to 4 . The mediocre fit for the weaker reflector in Fig. 9 [Rpp(0)=0.1] is not very convincing, but the improved fit in Fig. 10 appears to confirm Drufuca and Mazzotti’s observation (1995) that parameters are better constrained by greater contrasts across the interface. Figs. 11 and 12 show an improved fit for greater parameter contrasts also for the Class 3 case. The Shuey-approximation fit in both Figs. 11 and 12 is remarkably good. Because there is no critical angle in the Class 3 case, least-squares fitting for Figs. 11 and 12 includes the incidence angle range from zero to almost 90 degrees. Fig. 13 displays a variable angle range Smith and Gidlow fit to the Ostrander model [Rpp(0)=-0.365]. The smaller the angle range, the closer the fit to an exact Zoeppritz response. At zero to 40 degrees range, the two curves are practically indistinguishable (not shown).

Least-squares fitting of plane wave responses to spherical wave responses

The above derivations are based on the assumption of planar wave fronts. In reality however, wavefronts are curved. Krail and Brysk (1983) state that, for reflections from reasonable shallow interfaces, it is necessary to treat the incident wave as spherical rather than plane. In other words, the plane wave assumption is in itself an approximation.

For this modelling study, spherical wave AVO-responses are computed by employing the Weyl integral (Aki and Richards, 1980, page 217).
Figs. 14 and 15 show results for the same Class 1 models introduced in Figs. 1 and 2. Compared are a plane wave response, the spherical wave response for 1000m depth normalized to the plane wave zero offset reflection coefficient, the same spherical wave response with a spreading correction applied and the zero to 50 degree least-squares fit of an exact Zoeppritz plane wave to the spreading corrected spherical wave.
Only Fig. 14, for the weaker reflector, shows a good fit up to about 20 degrees. The achieved least-squares fit appears to be especially poor in the vicinity of the critical angle.
Figs. 16 and 17 show the result of the same procedure applied to the Class 3 models. The fit is excellent for both reflectors even beyond the included angle range of zero to 50 degrees.

Conclusions

The three Zoeppritz-approximations investigated do not always degrade with incidence angle but nearly always degrade with reflector strength in the isotropic situation.
Only the Shuey-approximation is investigated for the VTI-case, and the same observations apply. Compared to the isotropic approximation errors, VTI-approximation errors are of secondary importance only.
Least-squares fitting of approximations to the Zoeppritz equations improves with reflector strength for all Class 1 cases, but only for the Shuey-approximation in the Class 3 case. For both Class 3 reflectors investigated, the fit improves with incidence angle for both the Smith and Gidlow-approximation as well as the Fatti-approximation.
Least-squares fitting of exact Zoeppritz plane wave responses to spherical wave responses gives a good to excellent result for both Class 3 reflectors. In the Class 1 case, the fit deteriorates with reflector strength and appears to be especially poor in the vicinity of the critical angle of both reflectors.

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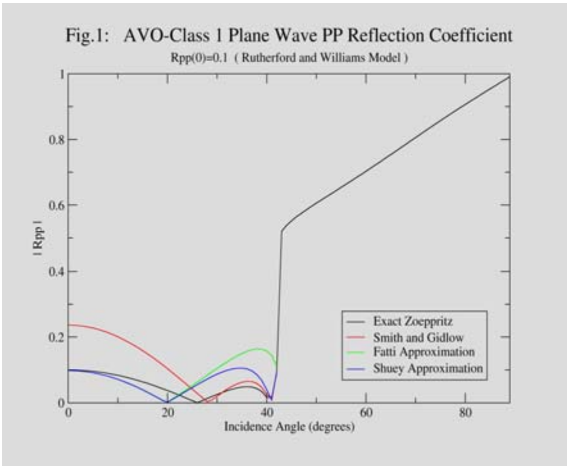


Fig.1: AVO-Class 1 Plane Wave PP Reflection Coefficient

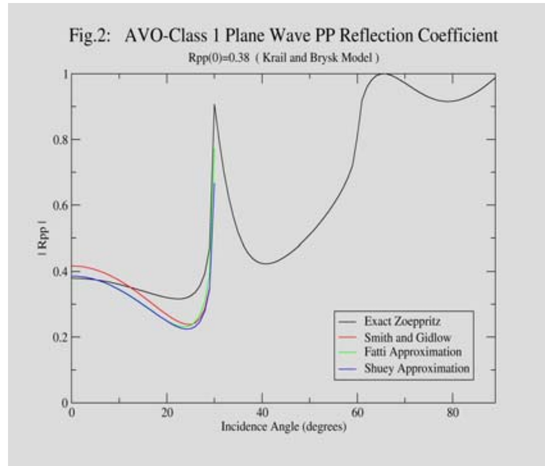


Fig.2: AVO-Class 1 Plane Wave PP Reflection Coefficient

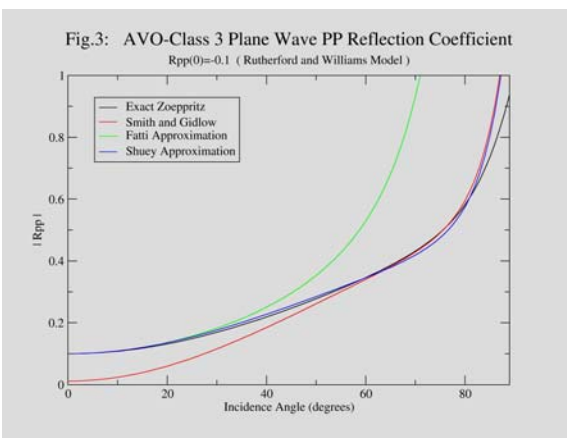


Fig.3: AVO-Class 3 Plane Wave PP Reflection Coefficient

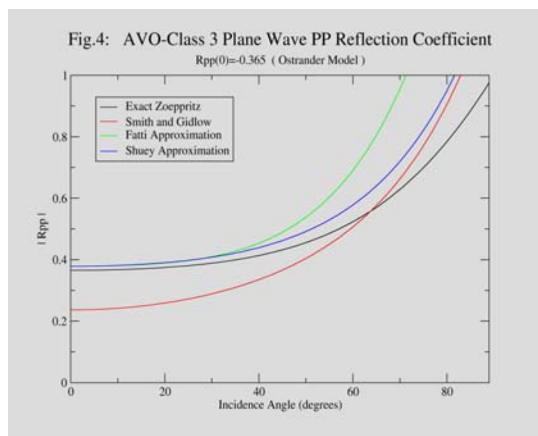


Fig.4: AVO-Class 3 Plane Wave PP Reflection Coefficient

Fig.5: AVO-Class 1 Plane Wave PP Reflection Coefficients

Rpp(0)=0.1 (Rutherford and Williams Model)

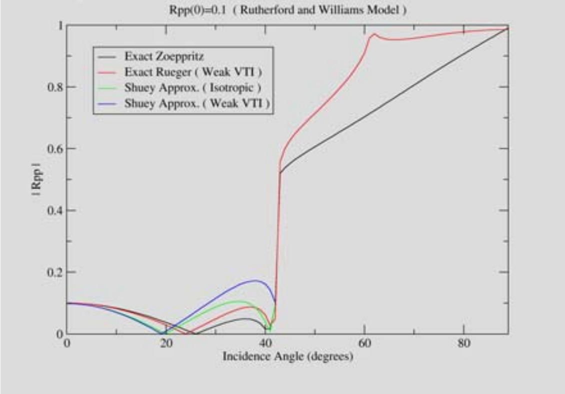


Fig.6: AVO-Class 1 Plane Wave PP Reflection Coefficients

Rpp(0)=0.38 (Krail and Brysk Model)

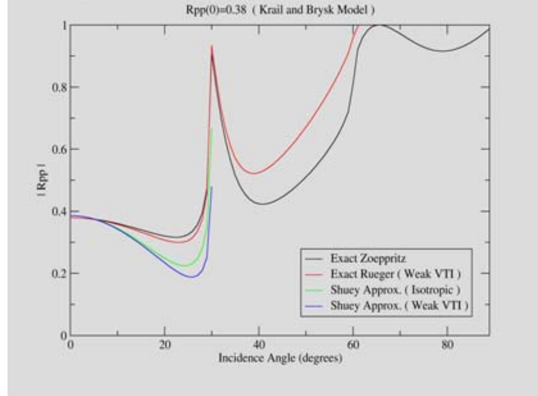


Fig.7: AVO-Class 3 Plane Wave PP Reflection Coefficients

Rpp(0)=-0.1 (Rutherford and Williams Model)

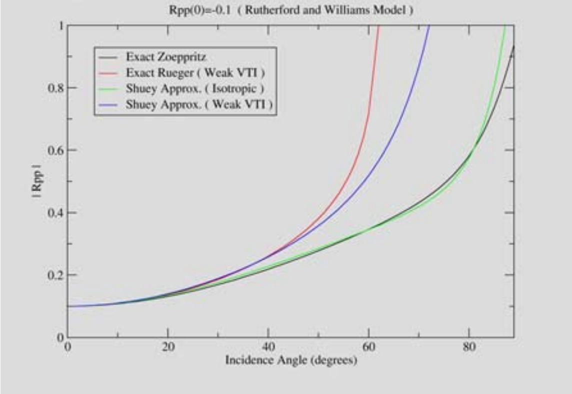


Fig.8: AVO-Class 3 Plane Wave PP Reflection Coefficients

Rpp(0)=-0.365 (Ostrander Model)

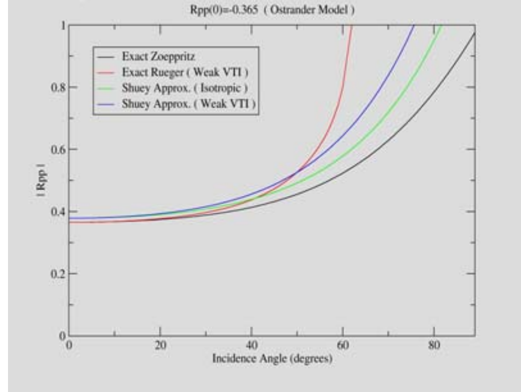


Fig.9: Least Squares Fit of Plane Wave Class 1 AVO Response

Rpp(0)=0.1 (Rutherford and Williams Model)

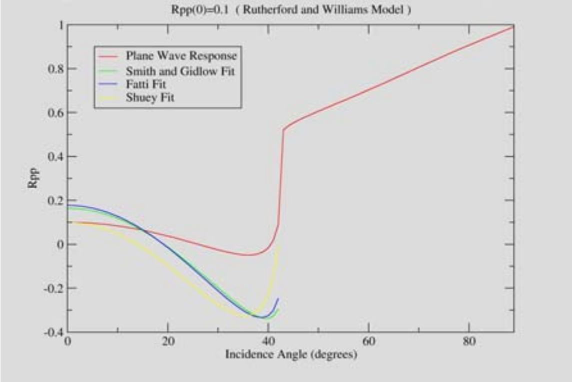


Fig.10: Least Squares Fit of Plane Wave Class 1 AVO Response

Rpp(0)=0.38 (Krail and Brysk Model)

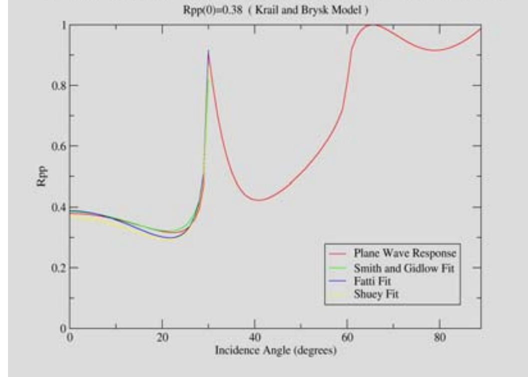


Fig.11: Least Squares Fit of Plane Wave Class 3 AVO Response

Rpp(0)=-0.1 (Rutherford and Williams Model)

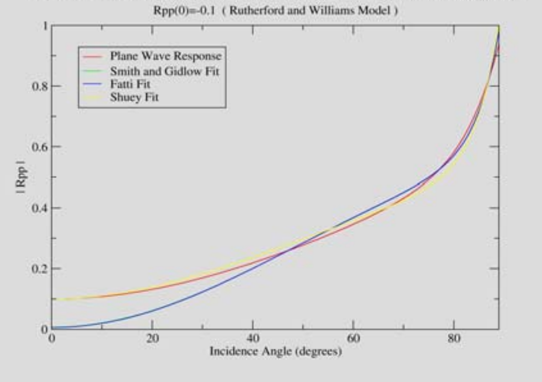


Fig.12: Least Squares Fit of Plane Wave Class 3 AVO Response

Rpp(0)=-0.365 (Ostrander Model)

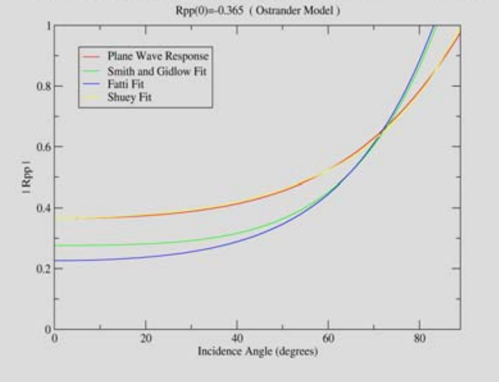


Fig.13: Least Squares Fit of Plane Wave Class 3 AVO Response

Rpp(0)=-0.365 (Ostrander Model, Smith and Gidlow Fit)

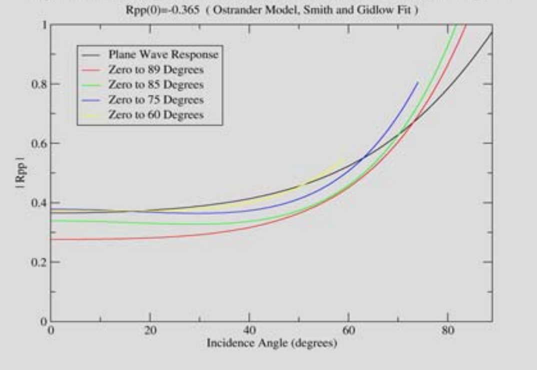


Fig.14: LSQ Fit of Class 1 Plane Wave to Spherical Wave

Rpp(0)=0.1 (Rutherford and Williams Model)

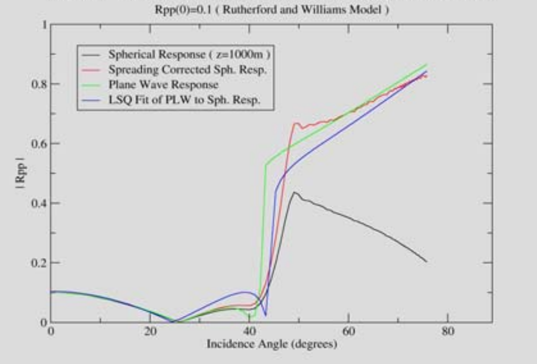
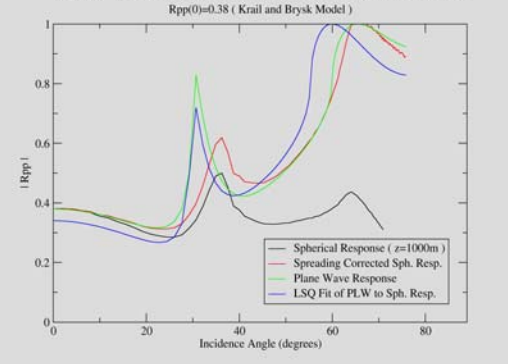


Fig.15: LSQ Fit of Class 1 Plane Wave to Spherical Wave

Rpp(0)=0.38 (Krail and Brysk Model)



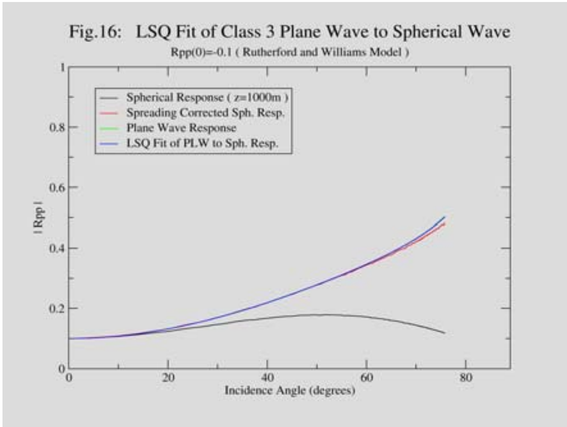


Fig.16: LSQ Fit of Class 3 Plane Wave to Spherical Wave
Rpp(0)=-0.1 (Rutherford and Williams Model)
Spherical Response (z=1000m)
Spreading Corrected Sph. Resp.
Plane Wave Response
LSQ Fit of PLW to Sph. Resp.
| Rpp |
Incidence Angle (degrees)

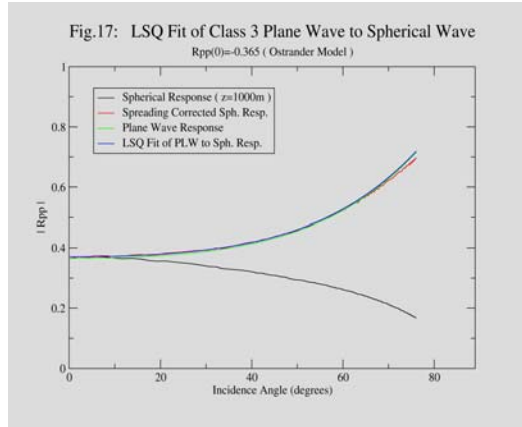


Fig.17: LSQ Fit of Class 3 Plane Wave to Spherical Wave
Rpp(0)=-0.365 (Ostrander Model)
Spherical Response (z=1000m)
Spreading Corrected Sph. Resp.
Plane Wave Response
LSQ Fit of PLW to Sph. Resp.
| Rpp |
Incidence Angle (degrees)