Imaging Using Multi-Arrivals: Gaussian Beams Or Multi-Arrival Kirchhoff?

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ABSTRACT

Summary

Single-arrival Kirchhoff migration is an accurate and reliable depth migration method except in cases of extreme geologic complexity, where it is not as accurate as most wavefield continuation methods. Also, where geology is extremely complex and multipathing occurs, its reliance on migrated image gathers indexed by offset (while more convenient than image gathers indexed by shot number) causes problems for both amplitude analysis and velocity analysis. On the other hand, most wavefield continuation methods are relatively expensive, and problems with amplitudes and migrated image gathers remain. Multi-arrival Kirchhoff migration can, to a large degree, overcome the problems associated with both single-arrival Kirchhoff and wavefield continuation methods, but the data flow problems can be serious in a production environment. Gaussian beams can be used to provide accurate Green's functions for multi-arrival Kirchhoff migration, and in the limiting case, kinematically accurate Gaussian beam migration can be modified to provide accurate amplitudes and migrated gathers indexed by angle.

Introduction

For the past decade, Kirchhoff migration has been the workhorse prestack seismic imaging method. Its versatility has allowed for time and depth migration methods to be written using the same basic program, for target-oriented migration, and for straightforward migration velocity analysis. While the imaging accuracy of single-arrival Kirchhoff prestack depth migration has been sufficient for all but the most challenging structural imaging problems, accuracy comparisons with many wavefield extrapolation methods have often brought out its shortcomings. Also, just in the last few years, algorithmic and hardware developments have allowed wavefield extrapolation methods to become viable imaging alternatives. Some problems remain with wavefield methods, though, such as completely rigorous amplitude treatment, and correctly-designed migrated gathers for amplitude studies and tomographic velocity analysis. Some types of multi-arrival Kirchhoff migration naturally solve these theoretical problems, and Gaussian beam migration efficiently solves many of the imaging accuracy problems of single-arrival Kirchhoff migration.

The present-day depth imaging hierarchy therefore ranges from single-arrival Kirchhoff migration to full shot-record migration, with multi-arrival Kirchhoff, Gaussian beam, and some wavefield methods (e.g., common-azimuth migration (Biondo and Palacharla, 1996)) in the middle. In this paper, we address multi-arrival Kirchhoff and Gaussian beam migrations. Our goal is to analyze these methods, with an eye to determining how large their niches are in the imaging hierarchy. If, for example, we can determine that Gaussian beam migration has accuracy comparable to an average single-arrival Kirchhoff migration and a cost comparable to shot-record migration by wavefield continuation, then we can state that the niche for this migration method is very small indeed. However, given impressive results on Gaussian beam migration reported by Hill (2001) and on multi-arrival Kirchhoff migration by Brandsberg-Dahl et al. (2001) and by Xu et al. (2001), we expect methods such as these to contribute significantly to future imaging projects. Eventually, these methods might supplant single-arrival Kirchhoff migration as a workhorse method in the years ahead, as greater accuracy becomes a standard imaging requirement.

The methods

- *Multi-arrival Kirchhoff migration*

Prestack Kirchhoff depth migration forms an image by summing an input (**x**, t) trace into an image (**x**, z) location using traveltimes that link both the source and detector locations with the image location. In single-valued migration, at most one traveltime links a source or detector location and the image location (*Fig. 1a*). This allows for a single traveltime table of a prescribed size to be constructed at each source or detector location, and for all the tables to be built, stored, and used conveniently. Typically, either the minimum time or the time associated with the maximum energy raypath is used. In multi-valued migration, many traveltimes might link the source or detector locations with the image location. If rays are shot from source and detector locations to image locations, a serious problem results: how many sheets (one sheet per traveltime value) will the traveltime tables include? If, on the other hand, rays are shot from the image locations to the source and detector locations with a prescribed increment in takeoff angle, these rays can accommodate any number of arrivals at a particular source or detector location very naturally (*Fig. 1b*). The prices to be paid for this flexibility are an added raytracing burden (rays shot from all subsurface locations instead of from all upper-surface locations) and, typically, a significant data flow overhead, forcing the migration to be output oriented rather than input oriented. In a multi-processor environment, this means that all the input traces must visit each of the processors.

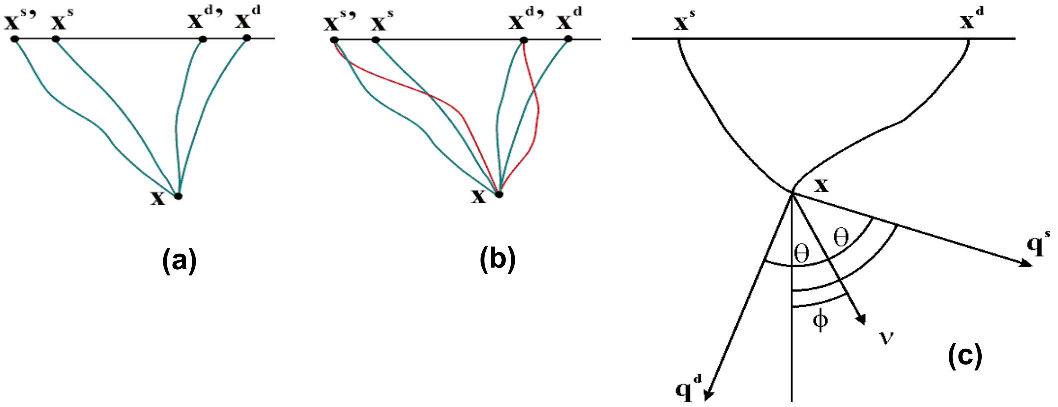

*Fig. 1. (a): A single arrival joins the image location **x** with each of the source-detector pairs $\mathbf{x}^s$, $\mathbf{x}^d$ and $\mathbf{x}^{s'}$, $\mathbf{x}^{d'}$. Single-arrival Kirchhoff migration handles this case easily and efficiently. (b): Multiple arrivals join the image location **x** with the source-detector pair $\mathbf{x}^{s'}$, $\mathbf{x}^{d'}$. In this case, the opening angle is the same for all the arrivals, and the dip angle increases for the three arrivals, the first joining **x** with $\mathbf{x}^{s'}$ and $\mathbf{x}^{d'}$, the second joining **x** with $\mathbf{x}^s$ and $\mathbf{x}^d$, and the third joining **x** with $\mathbf{x}^{s'}$ and $\mathbf{x}^{d'}$. A multi-arrival Kirchhoff migration that traces rays from image locations to source and detector locations handles these multiple arrivals naturally, while single-arrival Kirchhoff migration fails. (c): Geometry for the Beylkin determinant for an image location **x**. Raypaths from **x** to source and detector locations determine the dip angle ϕ and the opening angle 2θ for migrating an input trace to **x**. The Beylkin determinant uses this information in transforming the Kirchhoff integral from an integral over source and detector locations to an integral over migration dip angle.*

A major advantage of multi-arrival Kirchhoff migration is its ability to form migrated *incidence angle gathers*, i.e., gathers indexed by opening angle between source ray and detector ray at image locations. These gathers yield migrated amplitude vs. angle (AVA) information (Brandsberg-Dahl et al., 2001; Xu et al., 2001). These gathers are easier to use in reflection tomography than the more common amplitude vs. offset (AVO) gathers typically supplied by single-arrival Kirchhoff migration. If, in addition, correct weights are used in the migration, the AVA information can be used to invert for rock property contrasts along reflecting interfaces. Finally, if the raytracing is performed from the image locations to the source and detector locations, calculating and using correct migration weights (the Beylkin determinant – Bleistein, 1987) is straightforward to compute (*Fig. 1c*). The Beylkin determinant transforms the seismic experiment from the surface coordinates of the actual acquisition geometry to coordinates centered at each image location, essentially replacing a sum over surface coordinates with a sum over dip angles. In two dimensions, it is given by

$$|H| = \left[\frac{2\cos\theta}{v(x,z)}\right]^2 \left|\frac{d\nu}{d\alpha}\right|, \qquad (1)$$

where 2θ is the angle between the rays from the source and detector at the image location (x, z), ν is a unit vector in the direction of the gradient of total traveltime at (x, z), α is the variable of integration for the Kirchhoff integral, and v is velocity. When the raytracing is performed from the image locations, the raypaths carry information about the migrated dip at each image location, and the Beylkin determinant is very easy to compute. (For example, when α is chosen to be dip angle ϕ of *Fig. 1c*, the second factor in *Equation. (1)* is equal to one. For migrating into incidence angle gathers, the term containing the incidence angle θ can be taken outside of each migration integral.) In contrast, single-arrival Kirchhoff migration, with discontinuous traveltime and amplitude tables, fails to preserve amplitudes in the presence of multipathing.

Gaussian beam migration

Hill (2001) has formulated Gaussian beam migration as a wavefield continuation method that operates on common-offset, common-azimuth data volumes with the flexibility of Kirchhoff migration. The wavefield continuation formalism provides a kinematically correct imaging condition; otherwise the migration is performed as a Kirchhoff migration applied to local slant stacks of traces, using complex-valued traveltimes and amplitudes. The complex quantities come from expressing the wavefield as a sum of Gaussian beams, which are finite-frequency, ray-theoretic approximate solutions to the wave equation. In Hill's formulation, Gaussian beam migration is performed by imaging local slant stacks of traces from each common-offset data volume, and summing the contributions of all the local slant stacks. A particular local slant-stacked trace, centered at **x** and with slant-stack vector $\mathbf{p}^m = \mathbf{p}^d + \mathbf{p}^s$ (m for midpoint, d for detector, s for source), will be imaged using the complex Gaussian beams shot from $\mathbf{x}^m + \mathbf{h}$ and $\mathbf{x}^m - \mathbf{h}$ (**h** = half-offset vector) with takeoff angles determined by $\mathbf{p}^d$ and $\mathbf{p}^s$. The Gaussian beams provide complex values of time and amplitude for imaging. For example, the real part of time supplies a standard Kirchhoff imaging condition, and the imaginary part of time provides an exponential decay of wavefield strength away from a raypath (*Fig. 2*). In principle, this involves a double loop over $\mathbf{p}^m$ and $\mathbf{p}^h = \mathbf{p}^d - \mathbf{p}^s$, but Hill uses the locally planar geometry of the traveltimes to reduce the amount of work significantly from this, while still preserving multi-arrival imaging most of the time. The result is an efficient, kinematically accurate migration. Fig. 3 shows migrations of a pair of input spikes using single-arrival Kirchhoff migration and Gaussian beam migration, illustrating the multi-valued imaging capability of Gaussian beam migration.

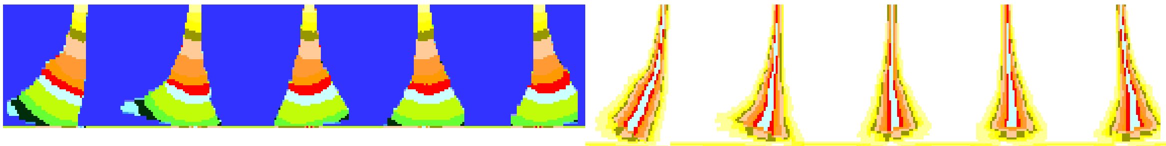

Fig. 2. Traveltime tables for five Gaussian beams all shot from the same surface location with different takeoff angles. The left panel shows the real part of the traveltime, and the right panel shows the exponential of the imaginary part of the traveltime at a reference frequency. The real part of the traveltime has moderate curvature, and the imaginary part of the traveltime causes exponential decay of energy away from the raypath.

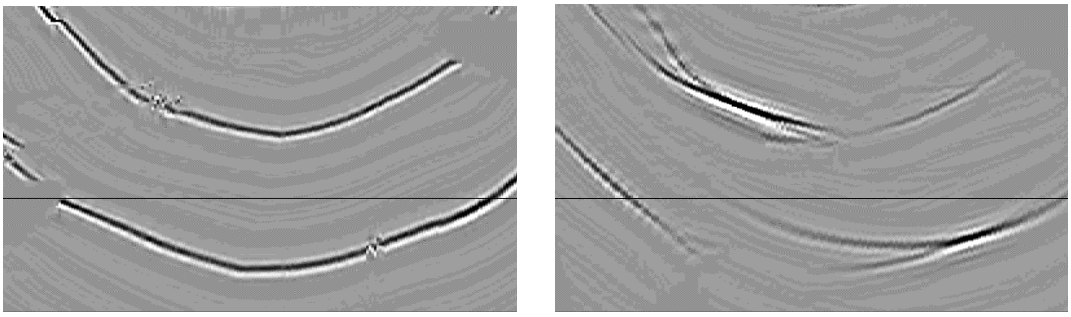

Fig. 3. Migrated impulses from single-arrival Kirchhoff migration (left) and Gaussian beam migration (right). The velocity model is the same for both migrations. The interpolation of traveltimes onto the migration grid (top) has produced glitches and sharply truncated holes in the migrated image. By contrast, Gaussian beam migration has preserved the multi-arrival nature of the actual wave propagation.

Gaussian beam migration replaces a single real-valued Kirchhoff traveltime table (plus, perhaps, an amplitude table) from each source or detector location with many complex-valued traveltime and amplitude tables from a restricted set of upper-surface locations (*Fig. 2*). The multiplicity of tables used by Gaussian beam migration provides its multi-valued imaging capability. Wavefield sampling theory provides rules for determining the total number of tables from each point, and for the total number of points that act as beam center locations. Typically, the total sizes of traveltime tables for Kirchhoff migration and traveltime and amplitude tables for Gaussian beam migration are within an order of magnitude of each other.

- *Multi-arrival Kirchhoff migration, revisited*

Multi-arrival Kirchhoff migration using raytracing from upper-surface locations is inconvenient because there is no simple way to control the number of arrivals at each image location. On the other hand, multi-arrival Kirchhoff migration using raytracing from the image locations forces an output-oriented structure for the migration program, with subsequent demands on data flow and parallelization. Gaussian beams can be put to use in multi-arrival Kirchhoff migration in a natural way to overcome both of these problems. In particular, Gaussian beams provide well-behaved traveltime tables that can be differentiated reliably to provide incidence angle information (*Fig. 2*). In two dimensions,

$$\frac{\partial t}{\partial x} = \frac{\sin \phi}{v(x,z)}, \qquad (2)$$

where t is traveltime at an image location with lateral position x, and ϕ is ray angle at x, measured from the horizontal. Gaussian beam traveltimes from a source location can be differentiated to give ϕ_s (angle of the source ray), and traveltimes from a detector location can be differentiated to give ϕ_d (angle of the detector ray). The difference between these angles is the opening angle when an input trace is imaged onto an image location. The opening angle determines which angle gather will be summed into when the input trace is imaged onto the image location. Also, the total traveltime gradient can be computed conveniently. From *Equation. (1),* both the opening angle and the total traveltime gradient are ingredients of the Beylkin determinant.

Conclusions

Single-arrival Kirchhoff depth migration is flexible, efficient, and generally very accurate. When the geology is moderately complex, its use of migrated gathers based on offset is useful for both velocity analysis and AVO analysis. In cases of extreme geologic complexity, however, its accuracy is limited by the assumption that a single raypath links each image location with any source or detector location. When the actual wavefield is multivalued, its migrated gathers become less useful. Multi-arrival Kirchhoff migration can be formulated in a number of ways, two of which we have discussed here. Using well-behaved Green's functions (e.g., Gaussian beams) to propagate energy between image locations and upper-surface locations can overcome problems associated with both of these approaches. When this is done, however, it should be a relatively small step to complete a full implementation of Gaussian beam migration that is flexible, efficient, and accurate, and that provides angle gathers for AVA and tomographic velocity analysis.

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