Enhancing Resolution via non-quadratic regularization - Next Generation of Imaging Algorithms

M. D. Sacchi*, C. Moldoveanu-Constantinescu and Jiang Feng
Department of Physics, University of Alberta, Edmonton, AB, T6G 2J1
sacchi@phys.ualberta.ca

ABSTRACT

The classical linear inversion approach in exploration seismology entails minimizing a cost function of the form **J=||Lm-d||** where **L** denotes the forward modeling operator, **d** the pre-processed seismic data and **m** the model of Earth property perturbations. Minimizing the cost function **J**, in the least squares sense, leads to the so-called least-squares migration methods. Migration algorithms, on the other hand, can estimate a blurred version of Earth material properties by using the adjoint operator **L'** (or a modified version of it). Migration methods, in general, do not attempt to fit the seismic data. Moreover, they have little control on the achievable resolution besides the one provided by the data. One way of improving resolution is by incorporating model space constraints. In this case, the cost function becomes **J = ||Lm-d|| + R(m),** where **R** is the regularization term utilized to force the solution to exhibit desirable characteristics. We discuss the implementation of non-quadratic constraints, similar to those used in image processing, to generate images with enhanced lateral and vertical resolution. In particular, we propose to use non-quadratic penalty terms and smoothing operators to adaptively smooth areas of lateral continuity and, at the same type, preserve edges and discontinuities along faults, and increase vertical resolution.

Introduction

Linearized inversion of seismic data entails the solution of the following problem:

$$\mathbf{L}\,\mathbf{m} = \mathbf{d} + \mathbf{n}, \qquad (1)$$

where **d** indicates the multi-source single/multi-receiver seismic experiment, **m** denotes an earth model that consist of physical model perturbations or an angle dependent reflectivity, the operator **L** is the Born forward single scattering operator computed on a known background model (macro-model), and **n** denotes coherent plus incoherent noise.

Rather than attempting to invert **L** via analytical methods, we propose to invert **m** using an approach that is closely related to least-squares inversion/migration methods (Nemeth et al., 1999). In this case, rather than seeking an analytical inverse we propose an optimization procedure to estimate model perturbations. What is the advantage of such a procedure? First, we can include covariance matrices in both model and data spaces, in other words the problem can be treated as a Bayesian inference problem where a priori correlations among parameters and observations can be included. Second, if we were to use the method to predict, for instance, velocities anomalies to detect over-pressure zones, one can also estimate a figure of confidence for the final estimate of the anomaly.

Inversion

In the numerical inversion we minimize the following Bayesian cost function (Youzwishen, 2001):

$$\mathbf{J = ||W\ (L\ m\ -d)\ ||\ +\ ||R\ m||.} \qquad (2)$$

The first term is the data misfit for a class of inference problems where we have considered Gaussian (and possible correlated) errors. The data covariance matrix can be replaced by an *empirical* expression that assign errors in terms of the distribution of sources and receivers; in *equation (2)* **W** is a matrix of weights proportional to the inverse data covariance matrix. The latter can be used to mitigate acquisition footprints (Kuehl and Sacchi, 2003). The interesting term in *equation (2)*, **R,** is often called the regularization term. This term, when obtained via the Bayesian framework, is associated to the a priori distribution of parameters. Model perturbations that are normally distributed and correlated lead to quadratic regularization terms.

Possible choices of **R**

When **L** is a focusing operator that attempts to collapse a certain class of events to points (i.e., Parabolic Radon Transform), **R** can be chosen to be a measure of sparseness, entropy or simplicity (Trad et al., 2003). Notice that in this case, **m** does not have a direct physical meaning. It is just a new domain, where it is possible to isolate, and filter undesired waveforms (this why we usually reserve the name *transform* when **L** is used in this context).

If **L** is an operator that maps angle dependent reflectivity to data space, then **R** should be an operator that attempts to enhance non-continuous events, and therefore, minimize the lack of continuity of events on Common Image Gathers. In this csae, **R** can be replaced by a first derivative (discrete) operator acting along ray-parameter or angle of incidence (Sacchi and Kuehl, 2003).

If **L** is an operator that maps 2-D/3-D *geological images* to data space, there is no general agreement (at least within our group) on how to design **R**. This is the most important (unsolved) problem at present time. There is an agreement, however, that if **R** is derived from a Bayesian prior, this prior should contain a covariance matrix that is capable of capturing the lateral correlation that exists between quasi-horizontal beddings (stratigraphic case) and to some degree introduce non-Gaussian features along the vertical direction to increase resolution (similar to 1-D sparse spike inversion). In the structural imaging case, on the other hand, one should probably adopt a simple strategy to smooth and unsmooth according to pre-defined partial knowledge of the spatial distribution of geological structures (i.e., avoid lateral smoothing in faulted areas.)

Example

In the following synthetic example, we demonstrate the benefits that could be obtained by adopting non-quadratic regularization methods to estimate Earth models. In particular, we adopt a non-Gaussian prior with a lateral smoothing term. This implies assuming a sparse reflectivity in depth/time and continuity of reflectors along the lateral spatial axis. Our numerical algorithm uses a Preconditioned Conjugate Gradients algorithm with a re-iterative scheme to update the non-quadratic term. The non-quadratic term is given by:

$$\|\mathbf{Rm}\| = \|\mathbf{F}\,(\mathbf{D}\,(\mathbf{m}))\|. \qquad (3)$$

Where **D** indicates the lateral derivative operator that penalizes non-continuous reflectors and **F** is a non-linear functional derived under the non-Gaussian reflectivity assumption. In our numerical implementation, we have assumed a Cauchy-distributed reflectivity. The latter leads to an algorithm that can attain an important enhancement of vertical resolution by properly de-blurring the migration operator. A similar strategy was adopted by Sacchi and Ulrych (1995) to compute high resolution Radon gathers

We have constructed a data set that is composed of common offset gathers synthetically computed over a simple geological model using a Born forward modeling code. The data inverted using least squares with a quadratic constraint (damping) is shown in *Fig. 1* (Left). The same data were inverted with the non-quadratic regularization term discussed in *equation (3)*. The resulting reflectivity model is portrayed in *Fig. 2* (Right). We notice an important increment in the vertical resolution and a considerable attenuation of imaging artifacts.

Discussion

Imaging/inversion with the introduction of non-quadratic constraints could lead to a new class of imaging algorithms where the resolution of the inverted image can be enhanced beyond the limits imposed by the data (band-width and aperture). This is not a completely new idea. Exploration geophysicists have been using similar concepts to invert post-stack data (sparse spike inversion) in an attempt to construct highly resolved impedance profiles.

Several problems remain to be solved. In particular, what constitutes a flexible regularization term to model both continue and discontinue reflectors is an active area of research in our group.

References

Kuehl, H. and Sacchi, M.D., 2003, Least-squares wave-equation migration for AVP/AVA inversion: Geophysics, 68, 262-273.

Nemeth, T., Wu, C. and Schuster, G.T., 1999, Least-squares migration of incomplete data: Geophysics, 64, 208-221.

Sacchi, M.D., and Ulrych, T.J., 1995, High-resolution velocity gathers and offset space reconstruction: Geophysics, 60, 1169-1177.

Trad, D., Ulrych, T.J., and Sacchi, M.D., 2003, Latest views of the sparse Radon transform: Geophysics, 68, 386-399

Youzwishen, C., 2001, Non-linear sparse and blocky constraints for seismic inverse problems, Msc Thesis, Department of Physics, Signal Analysis and Imaging Group. UofA, [http://cm-gw.phys.ualberta.ca/~sacchi/saig/theses.php]

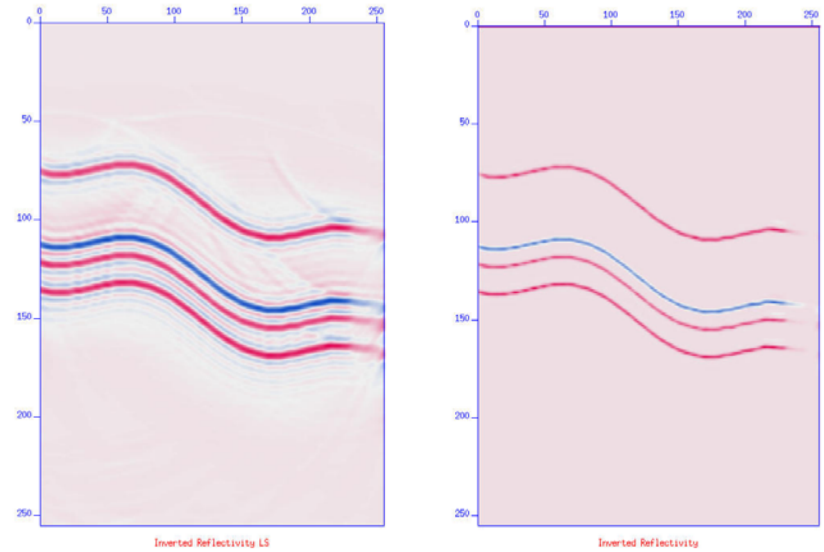

Fig. 1. Left: Reflectivity model derived using least-squares and quadratic smoothing (damping). Right: Reflectivity model derived using non-quadratic regularization to enhance vertical resolution and lateral continuity.

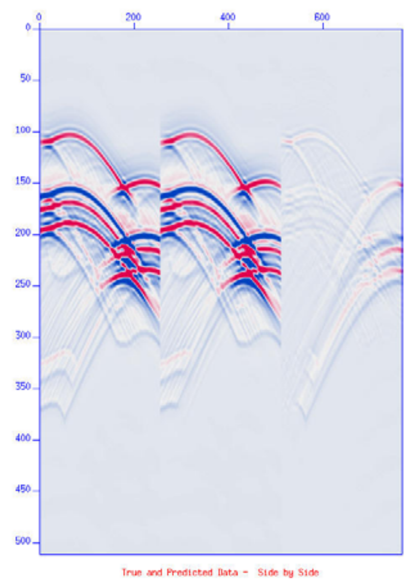

Fig. 2. Left: True data (Common Offset Gather). Center: Data predicted with the reflectivity model shown in Figure 1(Right). Right: Error panel.