Deconvolution with Multigrid

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Abstract

Multigrid methods are a familiar way to solve linear and non-linear systems in applied mathematics. They provide solutions to large and difficult equation sets at a fraction of the computational cost of more traditional methods. Most of the applications of the algorithms have been applied in the field of fluid dynamics, few applications in exploration seismology have been realised. The efficient solution to linear systems is an important part of many routines in geophysical data processing and inversion. In this paper a review of the basic concept of multigrid methods is provided, and one possible application is briefly explored.

Introduction

Multigrid methods work on the concept of decomposition of scale. Instead of solving the entire system, an approximate solution is found on a coarser grid. This solution is then interpolated to a higher sampling interval. This interpolated result becomes the starting point for an iterative correction scheme. Once the iterative method reduces the error to a specified tolerance, this updated solution is interpolated again, and the process is repeated until a solution at the desired sampling interval is achieved.

Possible applications of multigrid are numerous. In this paper we develop a multigrid framework for use in deconvolving a seismic trace. Deconvolution is a familiar 1-D process, and so is therefore useful for demonstrating the application of the algorithm.

Iterative methods

For iterative methods, an initial estimate of the solution is refined until satisfactory convergence is achieved. There are many types of iterative methods, including conjugate gradients, incomplete LU factorization, Jacobi method and Gauss-Seidel.
Consider Laplaces equation in 1-D,

$$Lu = 0, \tag{1}$$

where *L* is the matrix representation of the discrete ∇^2 operator, and *u* is a vector containing the solution. Substituting the second order approximation to the second derivative (in index notation) yields

$$\frac{u(i+1) - 2u(i) + u(i-1)}{2\Delta x^2} = 0. \tag{2}$$

The Gauss-Seidel correction is derived by solving for the i^{th} entry in *u* ,

$$u(i) = \frac{u(i+1) + u(i-1)}{2}. \tag{3}$$

For each iteration, the values in *u* are updated successively, until the desired convergence is achieved.
Assigning homogeneous boundary conditions to equation (1), the solution is *u=0* everywhere, and therefore the error is equal to the current estimate of the solution. In figure 1, an initial estimate is provided by assigning a random number to the interior points. This initial estimate is shown in figure 1a). Figures 1b), 1c) and 1d) show the remaining error in the trial solution after 3, 10 and 20 iterations of the Gauss-Seidel correction are applied.
In Figure 1, it is shown that the high frequency component of the error is reduced by very few passes of the correction, while the longer wavelength component of the error is much more difficult to reduce.

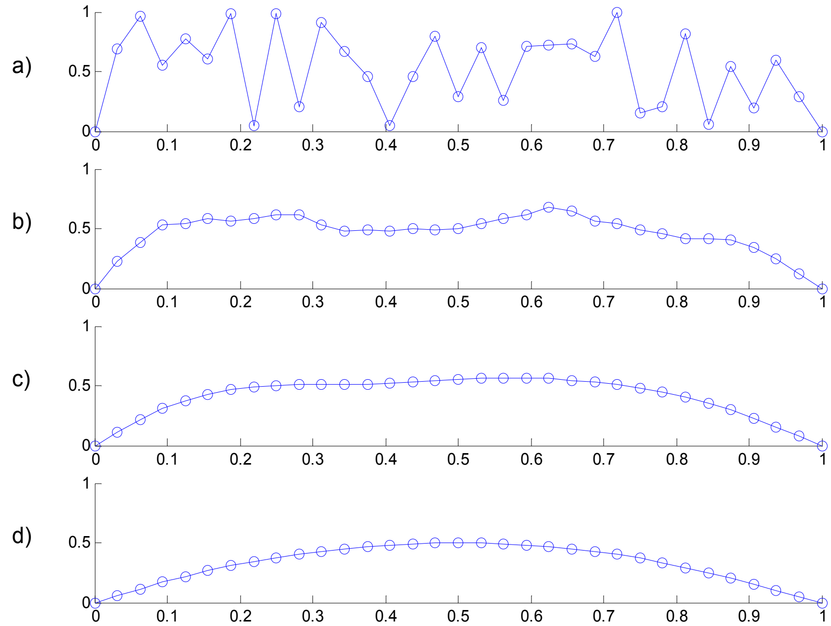

FIG. 1 a) Initial error of estimate of equation (1). b) Error after 3 iterations of the Gauss-Seidel correction. c) Error after 10 iterations. d) Error after 30 iterations. Note the difficulty in reducing the long wavelength error.

Multigrid Correction

One strategy for reducing the long wavelength error is to use an iterative method on a coarser grid. In figure 2a), the same estimate to equation (1) is provided. Instead of reducing this error at this grid spacing, the solution is *restricted* (opposite of interpolation) to a sampling interval 8 times that of the initial problem, displayed in figure 2b). Figure 2c) shows the remaining error after 3 passes of the Gauss-Seidel correction. This result is then interpolated back to the original sampling interval in figure 2d).

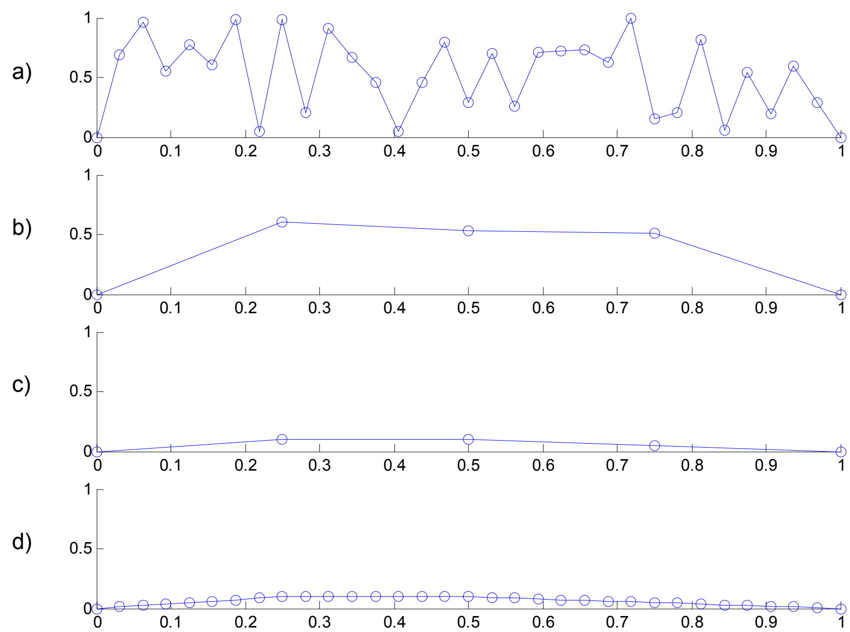

FIG. 2. a) The initial error. b) Error restricted to 8 times the sampling interval. c) Error after 3 iterations of the Gauss-Seidel correction. d) The solution re-interpolated to the original grid.

Application

One possible application of multigrid is in seismic deconvolution. Consider the convolutional model,

$$Wr = s, \tag{4}$$

where, *r* is the unknown reflectivity series, *W* is a wavelet matrix, which when multiplied with the reflectivity is the equivalent of convolution, yielding the seismic trace *s*. In order to use multigrid on this linear system, the wavelet matrix must have certain properties, one being diagonal dominance. In order to assure this in the case of a minimum phase wavelet, we use

$$W^T W r = W^T s. \tag{5}$$

To derive the Gauss-Seidel correction for this system of *N* equations and unknowns, we inspect the i^{th} equation,

$$\sum_j [W^T W](i,j) r(j) = [W^T s](i), j = 1, N \tag{6}$$

Solving (6) for $r(i)$,

$$\frac{\sum_j W^T W(i,j) r(j) - W^T s(i)}{W^T W(i,i)} = r(i), j = 1, N, j \neq i \tag{7}$$

As is seen in figure 3, the Gauss-Seidel algorithm will in time reproduce the reflectivity accurately. As is predicted, the high frequency components of the reflectivity are resolved quite quickly, however a small long wavelength component is still present, even after several passes of the correction. After a large number of iterations, the long wavelength error may become tolerably low, but the computational cost is high.

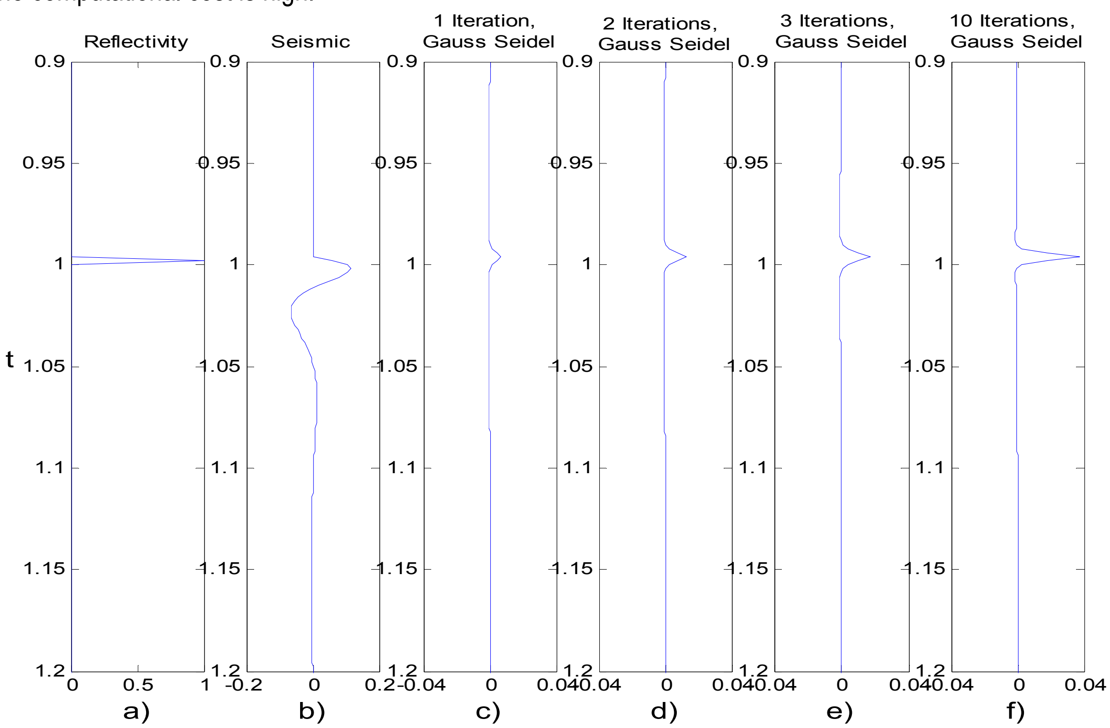

FIG. 3. a) Reflectivity b) Seismic trace c), d), e), and f), Seismic trace after 1,2, 3, and 10 Gauss Seidel corrections respectively. These inversions are using the exact wavelet, demonstrating the optimum scenario.

Figure 4 shows a comparison of how a multigrid correction performs against the Gauss-Seidel correction. A wavelet is estimated on the restricted trace, where the sampling interval is reduced to 0.008ms from 0.002ms. The correction is applied to the coarse trace, then interpolated up to the original interval, where the correction is applied again. The decrease in quality of the deconvolution in figure 4 is due to the fact that we are now using an estimate of the wavelet from the input trace, rather than the exact wavelet which was used in figure 3.

The main advantage of the coarse grid correction is that the initial estimate for the correction at the higher grid levels has less long wavelength error, and is computationally much less expensive, due to the decrease in both the number of trace samples and the width of the correction operator. For a direct comparison, 4b) and 4c) could each be though of as initial estimates for the Gauss-Seidel operator.

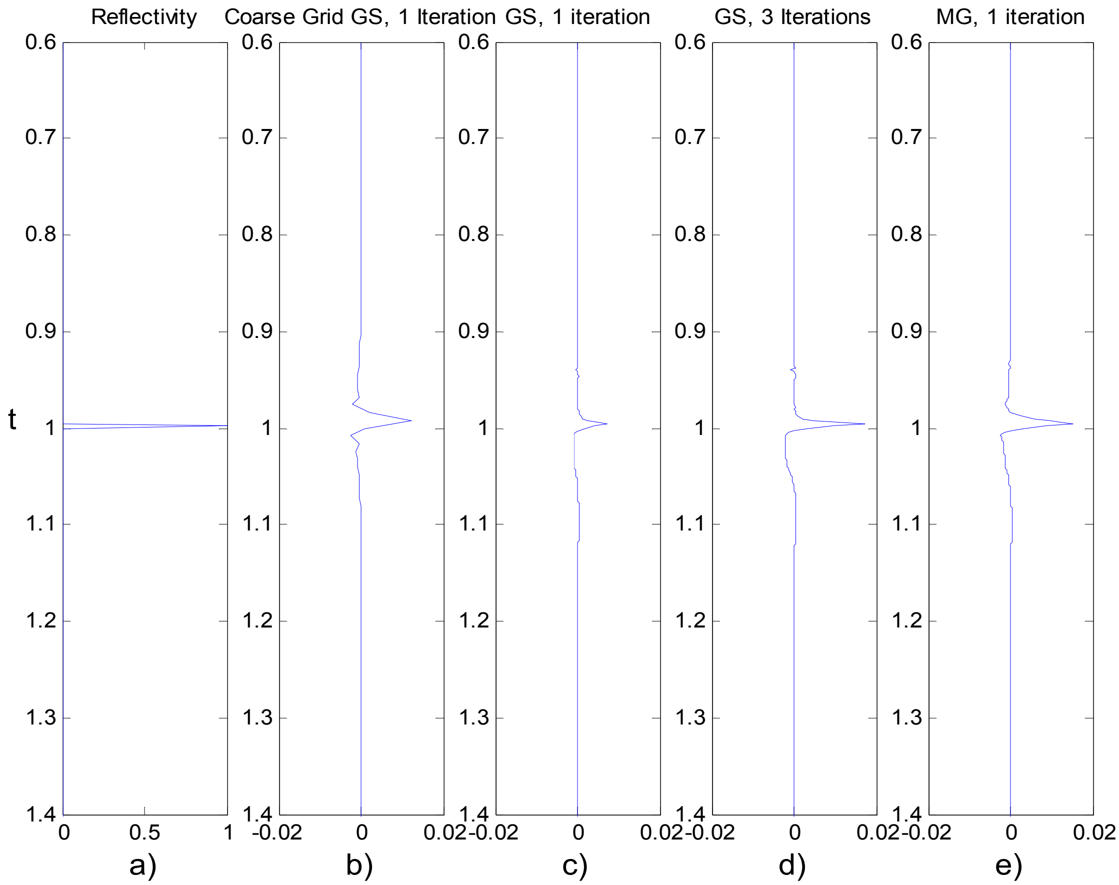

FIG. 4. a) Reflectivity b) Trace after coarse grid correction c) Trace after Gauss-Seidel correction. d) 3 Gauss-Seidel corrections e) 1 multigrid correction. A wavelet that was estimated from the seismic trace was used for the inversion.

Conclusions

Initial results show that the multigrid method is a practical way to solve the deconvolution system of equations. Tests on data that include noise, multiples and interference are pending. Before multigrid deconvolution is used on production data, certain questions, such as how best to restrict and interpolate the data, and how wavelet estimation on reduced grid spacing affects the frequency content and resolution of the data need to be answered.

Further applications of the multigrid concept to be researched include how it may be used for the migration of data, elastic modeling, tomography and inversion.