Abstract
A simple crosscorrelation method to estimate the Quality Factor Q is described. Q-waves are used to crosscorrelate with the seismic trace to estimate Q. The technique is applied on a synthetic seismic trace that has been Q-filtered with Q value of 70. The Q-wavelet (with a Q value of 70) among all the tested wavelets gave highest crosscorrelation coefficients.

Introduction
The decrease in the strength of seismic signal in the course of its propagation thru the medium is defined as the seismic wave attenuation (Sheriff, 1991). Absorption of Seismic energy which is quantized by the quality factor (Q) is one of the mechanisms by which seismic waves are attenuated in an inelastic earth.

The estimation of attenuation and correcting for it is one of the challenges in the seismic data processing. Dasgupta and Clark (1998) state that there are not many techniques to estimate Q from the surface seismic data.

VSP data is frequently used to estimate Q (Tonn, 1991).

In this paper we neglect the near-surface effects, therefore equation (3) can be written as:

\[ S(t) = e(t) \ast W_s(t). \]  

Theory

The Quality Factor ‘Q’

The quality factor, Q, is defined as the loss of energy per unit cycle (Aki and Richards, 1980).

Let \(-\Delta E\) and \(-\Delta A\) be the energy and amplitude respectively, lost in each energy cycle due to inelasticity, then the Q can be written as:

\[ \frac{1}{Q(\omega)} = \frac{-\Delta E}{2\pi E} \quad \text{and} \quad \frac{1}{Q(\omega)} = \frac{-\Delta A}{2\pi A}. \]  

In a perfectly elastic medium \( Q = \infty \) and the loss factor \( 1/Q = 0 \) (Aki and Richards, 1980). Q is dependent on frequency, but it has been shown that it varies a little in the range of seismic frequencies (Aki and Richards, 1980).

Convolution Model

There are various simplifying assumptions involved in the deconvolution theory and they are encapsulated and referred as convolution model (Sheriff and Geldart, 1995).

This model can be expressed mathematically as

\[ S(t) = l_r(t) \ast W_s(t). \]  

where,

\(l_r(t)\) is the earth impulse response,
\(W_s\) is the source waveform, and
\(S(t)\) is the earth response to the source waveform.

\(l_r(t)\), the impulse response can be written as follows, shown by Sheriff and Geldart (1995) as

\[ l_r(t) = n_s(t) \ast p(t) \ast e(t), \]  

where,

\(n_s(t)\) represents the near-surface effects beneath the source and receiver,
\(p(t)\) represents the effects such as multiples, absorption, mode conversions etc which can’t be included in the earth model and
\(e(t)\) is the reflectivity of the earth, or the impulse response of the target reflectors.

In this paper we neglect the near-surface effects, therefore equation (3) can be written as:

\[ l_r(t) = p(t) \ast e(t). \]  

In an attenuating medium, equation (4) can be written using (3) as

\[ S(t) = e(t) \ast p(t) \ast W_s(t). \]  

In this paper, we assume that the factor \( p(t) \) represents only the absorption of the medium ignoring all the other effects like multiples and mode conversions etc.

Now equation can be written as:

\[ S(t) = e(t) \ast W_{sa}(t), \]  

where \(W_{sa}(t)\) is the attenuated wavelet.

The amplitude spectrum of \(W_{sa}(t)\) can be written as follows,

\[ W_{sa}(f, \tau) = W_{sa}(f) e^{-\frac{\pi f \tau}{2}}, \]  

(Zhang and Ulrych, 2002), where,

\(Q\) is the quality factor,
\(f\) is the frequency,
\(\tau\) is the median time of the time window,

It is apparent from the above expression that, absorption increases with time.
Crosscorrelation
The crosscorrelation of two digital signals $s$ and $r$ can be written as:

$$c_j = \sum_k s_k r_{k+j}$$

(8)

Normalized Crosscorrelation
Normalised crosscorrelation relation can be written as equation (9)

$$c_{norm}^j = \frac{1}{\sqrt{\sum_k s_k^2 \sum_k r_k^2}} \sum_k s_k r_{k+j}$$

(9)

Q Estimation
A seismic trace is crosscorrelated with Q-filtered versions of the source wavelets which are formed at different times and with different values of Q. The Q-filtered wavelets are generated using the equation (7)

Methodology
The technique discussed above will be used to estimate Q over a synthetic seismic trace.

The first step in this methodology is to form Q-filtered wavelets. The source wavelet in this paper is a Ricker wavelet with a dominant frequency of 50 Hz. The Ricker wavelet is shown in Figure 1.

These wavelets are shown in Figures 2.

$Q$ Estimation

This Q filtered seismic trace is crosscorrelated with the 9 Q-filtered wavelets which were discussed earlier. The plots of the crosscorrelation coefficients are shown in Figure 6.

Test Cases
We tested this method on synthetic traces formed over two kinds of reflectivity series.

Case 1
It has three spikes with a very little noise added to it. The three spikes are at 1 s, 3 s and 5 s. This reflectivity is shown in Figure 3.

Fig. 3. The reflectivity series with spikes at 1s, 3s, and 5s.

Fig. 4. The seismic trace formed by convolving a 50Hz Ricker wavelet with the spiked reflectivity.

Fig. 5. The Q-filtered seismic trace with Q=70.
FIG. 9. The Q-filtered seismic trace with Q=70.

This Q filtered seismic trace is crosscorrelated with the 9 Q-filtered wavelets which were discussed earlier. The plots of the crosscorrelation coefficients are shown in Figure 10.

The seismic trace formed by convolving with source wavelet is shown in Figure 8.

**Case 2**

We then tested this method over a pseudo-reflectivity, this reflectivity is shown in Figure 7.

From Figure 10 it is obvious that the crosscorrelation with the wavelet (with Q=70) gives the maximum cross correlation coefficient at respective times. The crosscorrelation coefficient is 0.9 at 1 s when the trace is correlated the wavelet (Q=70, 1s), the crosscorrelation coefficient is 0.98 at 3 s when the trace is correlated the wavelet (Q=70, 3s) and the crosscorrelation coefficient is 0.8 at 5 s when the trace is correlated the wavelet (Q=70, 5s).

**Conclusion**

A very intuitive and simple method to estimate Q is proposed in this paper. The measure of crosscorrelation of a Q-wavelet over a seismic trace is used here to estimate Q.

This method is used to estimate Q on two cases. In the first case, the reflectivity series is made up of three spikes at 1, 3, and 5 s. This series is convolved with a 50 Hz Ricker wavelet and Q-filtered with Q=70. This is then crosscorrelated with the 9 wavelets. The wavelet with Q value of 70 gives the maximum crosscorrelation coefficients. The same procedure is repeated for a random reflectivity series and it is shown that wavelet with Q value of 70.

It has been observed that estimating Q at shallow depths is not very accurate with this method as the shape of the wavelet remains constant with various values of Q.
The value of $Q$ estimated thus may be used as an initial estimate in more sophisticated algorithms which need an initial guess of $Q$.

One of the main assumptions of this paper is the knowledge of the source wavelet and this may be a major disadvantage of this technique.

**Acknowledgements**
We acknowledge the CREWES sponsors for their generous support.

**References**


