De-mystification of “errors” in stacking velocities. Classification, analysis and improvement of understanding

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Abstract
This paper is a continuation of the paper by Blias (2005). Here we analyze so-called “errors” in stacking velocities in geologically complex cases. Using a simplified analytical connection between stacking and RMS velocities and velocity modeling, we show that the real reason for the anomalous behaviour is non-linear lateral velocity changes in shallow layers (the second-order derivative). We analyze some of statements about the cause of the stacking velocity anomalies and confirm our conclusions with velocity modeling on several models.

Introduction
In geologically complex areas interpretation of seismic reflections and their conversion to depth is an important problem. Several authors investigated this problem using velocity modeling: Miller (1974), Honeyman (1983), Blackburn (1980), Armstrong et al. (2001), Pickard (1992), Armstrong (2002). They created some depth velocity models, which were important for solving some geological problem. For each specific geological situation, one can create its’ own model, calculate rays and time arrivals and see how stacking velocities correspond to real (model) velocities but this does not give us much information and general knowledge about the connection between stacking and velocities. It’s understandable that the number of specific depth velocity models tends to infinity so it’s very hard to come to right conclusions using only empirical knowledge.

For example (Blackburn, 1980), we can read conclusions like this "Ray tracing through a number of different models has highlighted some of the difficulties in velocity determination in geologically complex areas. Errors in conversion of stacking velocities to true average velocities are due to timing errors as a result of migration and also raypath distortions due to the complex overburden. The study highlights the need for migration of all traces in a CDP gather prior to stacking velocity determination". These conclusions were made 25 years ago but they seem to be acknowledged by many geophysicists. However, it’s interesting to mention that each part of this statement is not quite correct except the first one about the difficulties.

(i) We cannot call “Errors” what we would not want to see but what is caused by the nature. We call these “errors” because “one wants the stacking velocities to approximate real velocities”, Blackburn (1980, p. 1466). Maybe this reason is not enough to interpret the difference between the stacking velocities (what we see on real data) and average velocities (what we would like to see) as “errors”.

(ii) We don’t have to convert stacking velocities to true average velocities but to interval velocities!

(iii) Migration and “raypath distortions” influence are not the reasons that causes the “errors” in stacking velocities. A medium with horizontal homogeneous layers also has “raypath distortions” if we mean that the rays are not straight.

(iv) There is no need for prestack migration prior to stacking velocity determination in the presence of a shallow velocity anomaly and relatively small dips of deep reflectors. Migration may be needed only if we have reflectors with big dips.

In a more recent paper (Armstrong et al. 2002), we can read: “Derivation of conventional stacking velocities is based on the assumption that of hyperbolic moveout. This is only true for horizontal layers of constant interval velocities. In the presence of velocity anomalies, stacking velocities show systematic errors”. “If an anomalous stacking velocity response at a target horizon is observed to correlate with the position of an overburden velocity anomaly, then the time delays must have survived the process of wavefront “healing”. This follows because the origin of the anomalous response is non-hyperbolic moveout within CMP gathers introduced by time delays caused by the velocity anomalies.”

In these two sentences we can also find some not correct statements:

(i) Derivation of conventional stacking velocities is not based on the assumption of hyperbolic moveout. It is based on the assumption that moveout is close to hyperbolic.

(ii) This is not correct for horizontal layers of constant interval velocities because for a layered medium, even with horizontal layers of constant interval velocities, moveout is not hyperbolic.
In the presence of velocity anomalies, stacking velocities don’t show systematic errors. It depends on where these velocity anomalies are and what we mean by “systematic errors”.

The origin of the anomalous response is not non-hyperbolic moveout within CMP gathers.

These kinds of sentences lead to statements like this: “Stacking velocities, as derived from NMO correction of CDP gathers, need have no physical significance to the true velocity distribution below the gather location” Blackburn (1980, p.1466).

We will show the real reason for “error” behaviour of the stacking velocity. We will see that there are no “errors” in “strange” stacking velocities and these velocities can be used to obtain accurate interval velocities. In my paper (Blias, 1981, 2003) I described an analytical explanation of so called “errors" in stacking velocities. Here I will show model examples that confirm statements (i) – (vi) and (i) – (iv) above. We will also come to some general conclusions, based on a simple analytical description of stacking velocities and modeling results.

The analytical connection between stacking and RMS velocities shows that the difference between them is not an error. To make it easier to obtain more clear understanding of this connection, let us simplify the formula for stacking velocities (Blias 2005). With some assumptions, we can rewrite this formula as:

\[
\frac{1}{V_{\text{NMO}}} = \frac{1}{V_{\text{RMS}}} \left[ 1 + \frac{1}{V_1 - V_2} \right] \left( 1 - \frac{V}{H} \right)^2 \]  

where \( F \) is the depth of the curvilinear boundary, \( H \) is a reflector depth and \( V_{\text{AVE}} \) is an average velocity for the reflector depth (Fig. 1).

This formula gives us understanding of what can happen with stacking velocity in different situations. We should mention that if we have several local velocity anomalies, we just add the similar terms for each anomaly.

(i) First of all, the difference between NMO and RMS velocities depends on the values of the product

\[
P = \frac{1}{V_1 - V_2} \left( 1 - \frac{V}{H} \right)^2 H
\]

If the value of \( P \) is small then the expression in the square brackets is close to 1 and the stacking velocity is close to the RMS velocity and we don’t have a problem to find intervalvelocities using Dix’s formula. If the value of \( P \) is relatively big then the expression in the square brackets differs from 1 and the difference between the stacking and RMS velocities becomes significant.

(ii) Second, the NMO velocity can be bigger or less than the RMS velocity. It depends on the sign of \( F''(1/V_1 - 1/V_2) \):

\[
V_{\text{NMO}} > V_{\text{RMS}} \quad \text{if} \quad F''(1/V_1 - 1/V_2) < 0 \quad \text{and} \quad V_{\text{NMO}} < V_{\text{RMS}} \quad \text{if} \quad F''(1/V_1 - 1/V_2) > 0
\]

(iii) From (2) IT follows that if \( V_1 < V_2 \) (we have a low velocity anomaly) then at the points \( X_1 \) and \( X_3 \) \( V_{\text{RMS}} < V_{\text{NMO}} \) because at these points \( F'' > 0 \). At the center of anomaly \( X_2, F'' < 0 \) and therefore \( V_{\text{RMS}} \) is bigger than \( V_{\text{NMO}} \). If we have a high velocity anomaly \( V_1 > V_2 \), the correlation between the stacking and RMS velocities is the opposite.

(iv) Formula (1) shows that there are two factors, which influence stacking velocity values. First is RMS velocity (what we like) and the second is non-linear lateral changes in the interval velocity – the anomaly itself. If the reflector depth \( H \) is not far from the anomaly depth \( F \) then \( (1-F/H)^2 \) is small and the anomaly influence is small. With the reflector depth increasing, the value of \( (1-F/H)^2 \) is also increasing and from some depth it becomes significant. The deeper we go, the less the lateral changes of RMS velocity (as any average velocity) and the bigger lateral changes of the value in the square brackets. If we have a strong shallow anomaly then, for close reflectors, the NMO velocity is close to RMS. For deep reflectors, the difference between the stacking and RMS velocities can be very big.

(v) From (1) it follows that the expression in the square brackets can be negative. This means that we will observe reverse time arrivals – minimum time will be at non-zero offset. In this case \( 1/V_{\text{NMO}}^2 < 0 \) so we cannot use stacking velocity at all to describe NMO function. We have to use coefficient \( b = 1/V_{\text{NMO}}^2 \), which is negative for this case.

This sounds strange but this is what the formula (1) implies and modeling confirms this (model example will be in fig. 6-7). Moreover, the proper statement is even stronger: If \( F''(1/V_1 - 1/V_2) < 0 \) then there exists such a critical depth, that all horizontal
reflectors, deeper than this depth, will create a reverse NMO function. From (1), we can derive an approximate formula for this critical depth:

$$ H_{CR} \approx \frac{1}{(1/V_1^2 - 1/V_2^2)V_{AVE}^2} F^* $$

(3)

For most cases this critical depth is too big to see this effects but it is nonetheless possible. Let us consider three model examples to confirm stated above.

Model 1. This model shows that the derivation of conventional stacking velocities is not based on the assumption of hyperbolic moveout. That is, the origin of the anomalous response is not non-hyperbolic moveout within CMP gathers. Fig 2a shows the model with a shallow velocity anomaly created by a quite strong curvilinear boundary ($V_1=1.6$ and $V_2=2.8$ km/sec).

Fig 2a. Depth velocity model

Fig 2b. Zero-offset times

Fig 2c. Stacking velocities

Fig 2d. Average difference between NMO function and hyperbolic approximation

Fig 2e. Maximum difference between NMO function and hyperbolic approximation

Fig 2c - 2e and 3 show that in spite of small difference between the time arrivals and their hyperbolic approximation (standard deviation is less than 1 msec and maximum difference is less than 4 msec), stacking velocities (fig. 2c) show big “errors”: that is big oscillations. Fig 4 shows the curvilinear boundary and its second-order derivative. For deep reflectors, as stated, stacking velocity repeats the behavior of the second-order derivatives (compare the lowest curve on fig. 2c and the second-order derivative, fig. 4), while for shallow reflector it’s close to RMS velocity.

3. Model 3 confirms that if we have deep velocity anomalies (horizontal changes), they don’t cause anomalous behaviour. This also follows from (1). Fig. 5 shows a depth velocity model with complex boundaries.

Fig 4. Boundary and its second-order derivative

Fig 5a. Depth velocity model

Fig 5b. Stacking velocities

Fig 5c. RMS velocities
Comparing fig. 5b and fig. 5c, we see that in spite of complex deep boundaries (complex geology), stacking velocities are quite similar to the RMS velocities. This confirms that “bad” stacking velocities are not caused by migration or by the raypath distortions.

Let us consider another velocity model with a strong velocity anomaly, which causes reverse a NMO function for deep reflectors. The depth velocity model is plotted in Fig. 6a. Fig. 6b and 6c show zero-offset times and stacking velocities. In the center of anomaly (around the point \( X_{\text{CDP}} = 5 \) km), the NMO functions are reversed. It means that the zero-offset time \( T(x=0) \) is a maximum point but not the minimum as it should be in the “normal” case. This is clearly seen in fig. 7. Fig. 7a shows CDP gathers for no-anomaly interval. Time arrivals have their minimum at zero offset. Fig. 7b shows CDP gathers around the center of the anomaly. We can see that, from deep horizons, the zero-offset trace time is bigger than the time for the traces with non-zero offset. This corresponds to the fact, described above (see formula (3)), that in the presence of strong shallow anomaly, NMO function may have maximum time at the zero-offset trace.

Conclusions
Analytical connections between stacking and RMS velocities and velocity modeling showed that the non-linear change of the velocity in the shallow part is the main reason for anomalous behaviour. So-called “Errors” in stacking velocities are not real errors and they are not caused by non-hyperbolic moveout or by migration or by raypath distortions as one can read sometime. Complex geology itself (dipping reflectors) does not cause a big difference between the RMS and stacking velocities. Model calculations confirm these statements. For reflectors close to the local anomaly, stacking velocity is close to RMS velocity. With the reflector depth increasing, the second term in (1) becomes more significant and causes “errors” in stacking velocities.

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