Nonlinear estimation of R_J from AVO intercept and gradient

Chuck Ursenbach, CREWES, University of Calgary, Calgary, Canada

Abstract

Two methods are derived for incorporating the quadratic shear reflectivity term into estimation of shear impedance reflectivity from AVO intercept and gradient data. These both give improved estimates over current linear methods, as illustrated with calculations on synthetic data.

Introduction

A variety of AVO methods exist, but the most common industrial methods involve extracting the AVO intercept and gradient. This approach is generally attributed to Shuey (1985), who rearranged the Aki-Richards linearization of the Zoeppritz equations into three terms. These terms depend on various functions of $\sin^2\theta$, and the coefficients are expressed in terms of a Poisson ratio contrast. Today a two-term truncation is commonly used, $A + B\sin^2\theta$, and A and B are generally expressed in terms of velocity and density contrasts. Such forms are still referred to as the two-term Shuey equation.

In some cases it does not matter what quantities are used to express A and B, as the intercept and gradient are often interpreted directly. Other times though they are used to estimate quantities of interest, in particular the P-wave and S-wave impedance reflectivities, R_I and R_J ($I=\rho\alpha$, $J=\rho\beta$). The first of these is a straightforward identification, $R_I = A$. The second is generated by theoretical approximations, such as $R_J \approx (A - B) / 2$. The purpose of this paper is to employ knowledge of the non-linear nature of reflection coefficients in order to make more accurate estimations of R_J from A and B.

In a previous study we have shown how to add key quadratic corrections to the Fatti approximation (Fatti et al., 1994), which is a stacking-type AVO method, without having to resort to iterative solution techniques (Ursenbach, 2004). Now we extend this notion to intercept-gradient AVO methods.

Theory

In the Aki-Richards linearization of the Zoeppritz equations (Aki & Richards, 1980), the P-wave reflectivity is given by

$$R_{PP}(\theta) \approx \frac{1}{\cos^2\theta} R_\alpha - 8\gamma^2 \sin^2\theta R_\beta + \left(1 - 4\gamma^2 \sin^2\theta\right) R_\rho , \tag{1}$$

where θ is the average of the angle of incidence and the angle of transmission of P-waves at the interface, $\gamma \equiv \beta/\alpha = (\beta_1 + \beta_2) / (\alpha_1 + \alpha_2)$, $R_\alpha = (\alpha_2 - \alpha_1) / (\alpha_1 + \alpha_2)$, and R_β and R_ρ are defined similarly. α_i, β_i and ρ_i are the P-wave velocity, S-wave velocity and density for the i^{th} interface.

We have previously shown that, in many practical cases of interest, the most important correction to this linear expression is the quadratic shear-wave term (Ursenbach, 2004). Usually it is even more important than the linear density term. This quadratic term can readily be derived, and the expanded expression is given as

$$R_{PP}(\theta) \approx \frac{1}{\cos^2\theta} R_\alpha - 8\gamma^2 \sin^2\theta R_\beta + 16\gamma^3 \sin^2\theta \frac{\cos^2\varphi - \sin^2\theta}{\cos\theta\cos\varphi} \left(R_J\right)^2 + \left(1 - 4\gamma^2 \sin^2\theta\right) R_\rho , \tag{2}$$

where φ is the average of the reflection and transmission angles for converted waves, and $\cos\varphi \approx \sqrt{(1 - \gamma^2\sin^2\theta)}$.

There are two ways we can use Eq. (2) to obtain expressions for A and B. In the first, which we will call the **expansion method**, one can expand Eq. (2) in powers of $\sin^2\theta$. The first term again yields $A = R_I$, while the second term gives

$$\begin{aligned} B &= R_\alpha - 8\gamma^2 R_\beta + 16\gamma^3 R_J^2 - 4\gamma^2 R_\rho \\ &= R_\alpha + R_\rho - 8\gamma^2\left(R_\beta + R_\rho\right) + 16\gamma^3 R_J^2 + \left(-1 + 8\gamma^2 - 4\gamma^2\right) R_\rho \\ &= A - 8\gamma^2 R_J + 16\gamma^3 R_J^2 - \left(1 - 4\gamma^2\right) R_\rho . \end{aligned}$$

The expression for B is a quadratic polynomial in R_J, but for it to be useful we must estimate the R_ρ term. We will do this by three different methods: (1) Set $R_\rho = 0$; (2) Set $R_\rho = R_I / 5 = A / 5$ (Gardner's relation, [Gardner et al., 1974]); (3) Set $\gamma = ½$. Solving the quadratic equation then results in the three following estimates of R_J:

$$R_J^{\exp}(R_\rho = 0) = \left[1 - \sqrt{1-(A-B)/\gamma}\right]/(4\gamma), \tag{3}$$

$$R_J^{\exp}(R_\rho = A/5) = \left[1 - \sqrt{1-[4A(1+\gamma^2)/5 - B]/\gamma}\right]/(4\gamma), \tag{4}$$

$$R_J^{\exp}(\gamma = 1/2) = \left[1 - \sqrt{1-2(A-B)}\right]/2. \tag{5}$$

If we had started from Eq. (1) instead of Eq. (2), the same procedures would have yielded, respectively,

$$R_J^{\exp}(R_\rho = 0;\ \text{linear}) = (A-B)/(8\gamma^2), \tag{6}$$

$$R_J^{\exp}(R_\rho = A/5;\ \text{linear}) = [4A(1+\gamma^2)/5 - B]/(8\gamma^2), \tag{7}$$

$$R_J^{\exp}(\gamma = 1/2;\ \text{linear}) = (A-B)/2. \tag{8}$$

Thus by expanding Eq. (2) [or Eq. (1)] in powers of $\sin^2\theta$ we can obtain various estimates of R_J. [Eqs (6)-(8) can also be obtained from appropriate linear Taylor expansion of Eqs (3)-(5).]

A second approach for obtaining estimates is simply to set Eq. (2) equal to the form implied by Shuey's two term approximation, $A + B\sin^2\theta$, for specific choices of θ. We will do this for two different choices of angle, $\theta = 0$ and $\theta = \theta_{max}$, where θ_{max} corresponds to the maximum offset employed in a given AVO calculation. We will call this the **two-point method**. It yields the following two expressions:

$$A = R_\alpha + R_\rho,$$

$$A + B\sin^2\theta_{\max} = \frac{1}{\cos^2\theta_{\max}} R_\alpha + R_\rho - 8\gamma^2 \sin^2\theta_{\max} R_\beta - 4\gamma^2 \sin^2\theta_{\max} R_\rho + 16\gamma^3 \sin^2\theta_{\max} \frac{\cos^2\varphi_{\max} - \sin^2\theta_{\max}}{\cos\theta_{\max}\cos\varphi_{\max}} (R_J)^2.$$

These two equations are linear in the two variables A and B. Solving for these two quantities yields $A = R_I$ and

$$B = \frac{R_I}{\cos^2\theta_{\max}} - \left(\frac{1}{\cos^2\theta_{\max}} - 4\gamma^2\right) R_\rho - 8\gamma^2 R_J + 16\gamma^3 \frac{\cos^2\varphi_{\max} - \sin^2\theta_{\max}}{\cos\theta_{\max}\cos\varphi_{\max}} R_J^2$$

$$\equiv \frac{R_I}{\cos^2\theta_{\max}} - \left(\frac{1}{\cos^2\theta_{\max}} - 4\gamma^2\right) R_\rho - 8\gamma^2 R_J - 16\gamma^3 G_2^{\max} R_J^2,$$

which implicitly defines G_2^{max}. This equation can be rewritten as

$$16\gamma^3 G_2^{\max} R_J^2 - 8\gamma^2 R_J + \left[\frac{A}{\cos^2\theta_{\max}} - B\right] - \left[\frac{1}{\cos^2\theta_{\max}} - 4\gamma^2\right] R_\rho = 0.$$

Again we obtain a quadratic equation in R_J, with coefficients depending on A and B, and again we must make some approximation to the R_ρ term. We can make two of the same choices as before: (1) Set $R_\rho = 0$; (2) Set $R_\rho = A/5$. The third choice is slightly different: (3) Set $\gamma = 1/(2\cos\theta_{max})$. Solving the quadratic equation then yields

$$R_J^{\text{2-pt}}(R_\rho = 0) = \frac{1 - \sqrt{1 - \frac{G_2^{\max}}{\gamma}\left[\frac{A}{\cos^2\theta_{\max}} - B\right]}}{4\gamma G_2^{\max}}, \tag{9}$$

$$R_J^{\text{2-pt}}(R_\rho = A/5) = \frac{1 - \sqrt{1 - \frac{G_2^{\max}}{\gamma}\left[\frac{4}{5}A\left(\gamma^2 + \frac{1}{\cos^2\theta_{\max}}\right) - B\right]}}{4\gamma G_2^{\max}}, \tag{10}$$

$$R_J^{\text{2-pt}}\left(\gamma = \frac{1}{2\cos\theta_{\max}}\right) = \frac{1 - \sqrt{1 - 2G_2^{\max}\left[A - B\cos^2\theta_{\max}\right]/\cos\theta_{\max}}}{2G_2^{\max}/\cos\theta_{\max}}. \tag{11}$$

The analogous results that would be obtained using Eq. (1) instead of Eq. (2) are as follows:

$$R_J^{\text{2-pt}}(R_\rho = 0;\ \text{linear}) = \frac{A/\cos^2\theta_{\max} - B}{8\gamma^2}\,, \tag{12}$$

$$R_J^{\text{2-pt}}(R_\rho = A/5;\ \text{linear}) = \left[\frac{4}{5}A\left(\frac{1}{\cos^2\theta_{\max}} + \gamma^2\right) - B\right]/(8\gamma^2)\,, \tag{13}$$

$$R_J^{\text{2-pt}}\left(\gamma = \frac{1}{2\cos\theta_{\max}};\ \text{linear}\right) = \frac{A - B\cos^2\theta_{\max}}{2}\,. \tag{14}$$

Again, Eqs (12)-(14) can also be obtained by appropriate linear Taylor expansion of Eqs (9)-(11).

Application

We have applied the twelve methods of Eqs (3)-(14) to 110 different AVO calculations. Each of the 110 calculations is based on synthetic reflection data generated from elastic parameter data obtained from well-logs and laboratory studies (Castagna & Smith, 1994). For each of the 110 calculations, an estimate of R_J is made, and the error, ΔR_J, is calculated, relative to the exact value. To give a rough indication of the efficacy of each method, the average of the absolute values of all 110 ΔR_J is calculated for each of the twelve methods, and the results are tabulated below:

Table I: The average absolute value of the errors for the twelve methods of estimating R_J represented by Eqs (3)-(14).

	Linear		Quadratic	
	Expansion method Eqs (6)-(8)	Two-point method Eqs (12)-(14)	Expansion method Eqs (3)-(5)	Two-point method Eqs (9)-(11)
$R_\rho = 0$	.0265	.0245	.0170	.0173
$R_\rho = R_I/5$	.0257	.0220	.0161	.0121
γ = ½ or $1/(2\cos^2\theta)$	.0334	.0345	.0290	.0274

We note some obvious trends from Table I. The error of the quadratic methods is significantly less than that of the linear methods. It is also clearly better to approximate R_ρ than γ (assuming some reasonable estimate of γ exists). It also appears that the two-point method is slightly better than the expansion method, and that the Gardner relation estimate of R_ρ is slightly preferable to setting R_ρ to zero. Consistent with these trends, the best method overall according to this data is Eq. (10), with an average absolute error of .0121 over the 110 calculations.

Below we present a more detailed picture of the individual errors for Eq. (10) (always shown as blue crosses) and compare these with individual error values for some of the other methods. In Figure 1 we compare the errors of Eq. (10) with those of its linear analogue, Eq. (13). We display the same data twice, but with different ordering. In Figure 1a it is plotted against the exact R_J. This shows that Eq. (10) removes a quadratic trend that is present in the error of Eq. (13). In Figure 1b the same data is plotted against the exact R_ρ, and this shows that large errors of Eq. (10) in Figure 1a are more commonly associated with large density reflectivities.

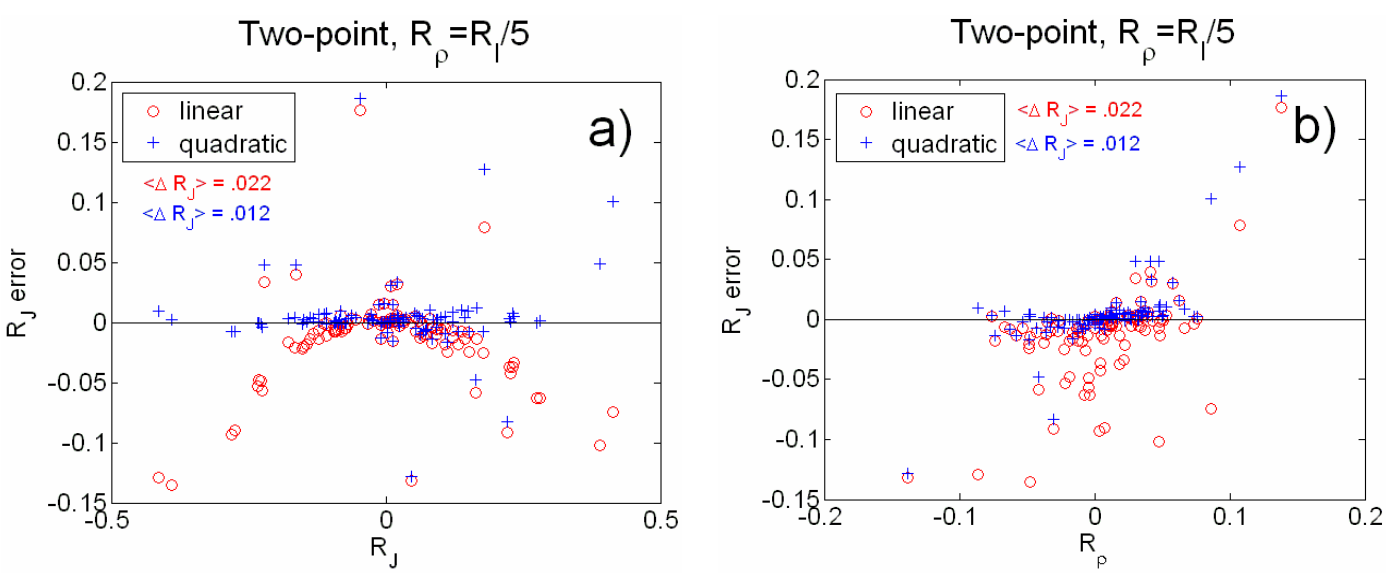

FIG. 1. A comparison of the 110 individual errors in R_J as predicted by Eqs (10) and (13). The results are plotted against the exact value of R_J in a), where it is clear that Eq. (10) removes a quadratic trend from the error. The same results are plotted against the exact value of R_ρ in b), where it is seen that a few values predicted poorly by Eq. (10) are generally associated with interfaces possessing a large value of R_ρ.

In Figure 2 we compare Eq. (10) against the two other R_ρ-term approximations of Eqs (9) and (11). Here we see again that approximations to R_ρ itself are superior to approximations to γ. For points with large R_ρ, employing the Gardner relation is preferable to simply neglecting R_ρ.

In Figure 3 we compare Eq. (10) to its expansion method analogue, Eq. (4). The two-point method appears superior for the main cluster of points, but the expansion method is better for some of the outliers.

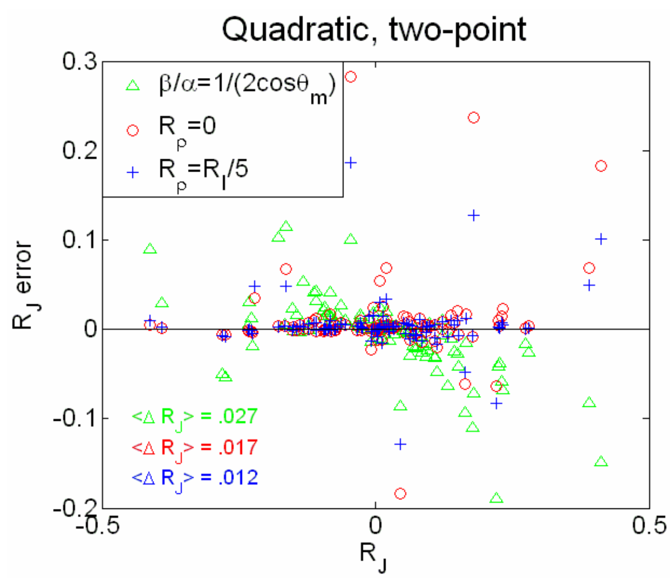

FIG. 2. A comparison of errors in R_J as predicted by Eqs (9)-(11).

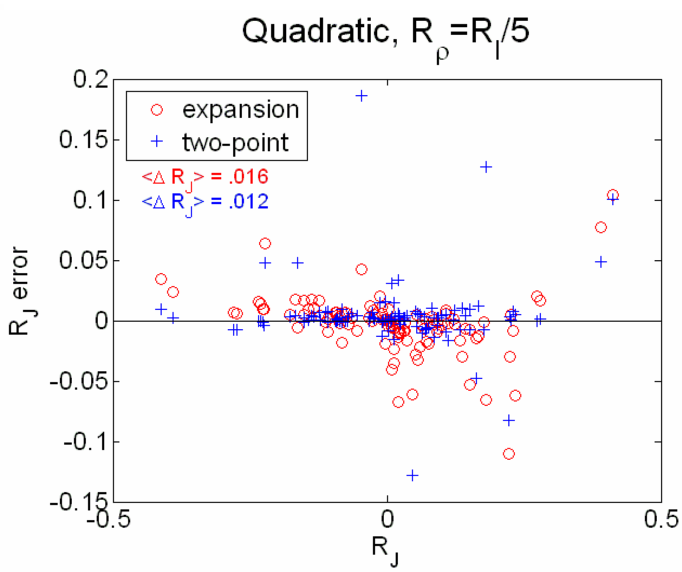

FIG. 3. A comparison of errors in R_J as predicted by Eqs (4) and (10).

Discussion and Conclusions

We have demonstrated two methods (the expansion method and the two-point method) for incorporating quadratic shear terms into the interpretation of intercept and gradient results. These corrections, however they are implemented, are seen to improve significantly over the standard linear theories. This improvement varies as R_J^2, so that even in the presence of noise it would be expected to be important for systems in which R_J is of sufficient magnitude.

The issues raised in Figure 2 relate to information that is not available and must normally be estimated. γ can be estimated from converted-wave seismic data, or from P-wave data along with a locally calibrated mudrock relation. One message of Figure 2 is that the R_J estimation is more sensitive to approximations in γ than in R_ρ, so one should make the best estimate one can of the former. Approximations to R_ρ are more difficult, but also more readily forgiven. A reasonable approach for estimating R_ρ would be to use a locally calibrated Gardner relation. For instance, if calibration yields $\rho = C\alpha^n$, then the appropriate expressions consist of Eq. (3) with A replaced by $(1 + 4n\gamma^2)\, A\, /\, (1+n)$, or Eq. (9) with A replaced by $(1 + 4n\gamma^2\cos^2\theta_{max})\, A\, /\, (1+n)$.

The comparison in Figure 3 is valid for the chosen θ_{max} of 30°, but the behaviour of both methods would vary with larger offsets. The expansion method would give less accurate results as higher orders of $\sin^2\theta$ become significant, especially when a critical point is approached. In such a case one should fit the data to, say, the three-term Shuey relation rather than the two-term, even if the third coefficient is simply thrown away after fitting. This would help maintain the integrity of the B coefficient. An advantage of using the two-point formula is that because the expressions are dependent on θ_{max}, they are able to compensate to some extent for the deviation from strict quadricity. Hence they perform better than the two-point expansion formula for this θ_{max}. For strong non-quadratic behaviour the two-point formula may prove insufficient. However it is straightforward to derive a three-point formula, so there is potential to use this method with larger offsets as well.

Acknowledgments

The author wishes to thank Dr. Jon Downton for numerous enlightening discussions related to this work.

References

Aki, K. and Richards, P. G., 1980, Quantitative Seismology: Theory and Methods: W.H. Freeman, 1980.

Castagna, J. P. and Smith, S. W., 1994, Comparison of AVO indicators: A modeling study: Geophysics, **59**, 1849-1855.

Fatti, J. L., Smith, G. C., Vail, P. J., Strauss, P. J., and Levitt, P. R., 1994, Detection of gas in sandstone reservoirs using AVO analysis: A 3-D seismic case history using the Geostack technique: Geophysics, **59**, 1362-1376.

Gardner, G. H. F., Gardner, L. W., and Gregory, A. R., 1974, Formation velocity and density – The diagnostic basics for stratigraphic traps: Geophysics, **39**, 770-780.

Shuey, R.T., 1985, A simplification of the Zoeppritz equations, Geophysics, 50, 609-614.

Ursenbach, C., 2004, Two new approximations for AVO inversion: CSEG Extended Abstracts.