Comparing three methods for inverse-Q filtering

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Summary
Three different approaches for inverse-Q filtering are reviewed and assessed in terms of effectiveness in correcting amplitude and phase, and numerical instability. The starting point for the three methods is the linear, frequency-independent Q theory, which needs two parameters to be completely defined: the attenuation parameter Q and a reference frequency ω_0 for which the phase velocity reaches a reference value. In Hale method, the unattenuated trace is found by inverting a matrix similar to the attenuation matrix with minimized nearly-singular characteristics. Wang method is based on the downward-continuation migration method and is a highly efficient and numerically stable method. PDO method uses the generalized nonstationary Fourier integrals, or pseudodifferential operators, to apply the inverse-Q filter and is also highly stable. The performance of the three kinds of filters is assessed using a synthetic trace, for different situations regarding uncertainty around Q and ω_0.

Introduction
Seismic waves travelling through inelastic media are attenuated by the conversion of elastic energy into heat. Upon being attenuated, the travelling wave changes: amplitude is reduced, travelling waveform is modified due to high-frequency content absorption, and phase is delayed. Attenuation is usually quantified through the quality factor Q: the ratio between the energy stored and lost in each cycle due to inelasticity. It is generally accepted that the Q-constant model (Kjartansson, 1979) is a good representation of the attenuation process, due to inelasticity, for most crustal rocks in the range of seismic frequencies. The Q-constant theory is a very simple and powerful representation of the attenuation phenomenon, it is based on the assumptions of linearity and causality and depends on two material-dependent parameters: the attenuation parameter, Q, and the phase velocity for a reference frequency, ω_0. When reliable estimations of Q are available, for example from VSP data, inverse-Q filtering is the natural method to compensate for the effects of the attenuation process on the seismic signal. Different methods to apply inverse-Q filtering have been developed. In inverse-Q filtering, besides the accuracy and the considerable computational cost, the numerical stability is an additional critical issue to consider. In this paper three different methods to apply inverse-Q filtering are reviewed and compared: inverting a Q matrix or an equivalent Q matrix (Hale, 1981); downward-continuation inverse-Q filtering (Hargreaves and Calvert, 1991) and (Wang, 2002) and, nonstationary inverse-Q filtering (Margrave, 1998). The three methods will be called henceforward, Hale, Wang, and PDO method respectively.

Constant-Q attenuation model
The constant-Q model theory (Kjartansson, 1979) predicts an amplitude loss given by

$$A(x) = A_0 \exp\left(-\omega x / 2\upsilon Q\right), \qquad (1)$$

where Q is the attenuation parameter, ω is the angular frequency, υ the velocity, A_0 the initial amplitude, and $A(x)$ the amplitude at the travelled distance x. A dispersion relation, for the velocity with respect to the frequency, is an essential element of the Q-constant theory. For the examples shown in this paper, the following dispersion relation (Aki and Richards, 2001) has been used,

$$\upsilon(\omega) = \upsilon(\omega_0)\left[1 + (1/\pi Q)\log(\omega/\omega_0)\right], \qquad (2)$$

which gives the phase velocity at any frequency, $\upsilon(\omega)$, in terms of the velocity at a reference frequency ω_0. A linear filter is entirely characterized by its impulse response. In the constant-Q theory the earth is considered a linear filter, the attenuating earth impulse response is a fundamental result. Kjartansson (1979) shows that the Fourier transform of the attenuating medium impulse response is

$$B(\omega) = \exp\left[-\omega x / 2\upsilon_\infty Q\right]\exp\left[-i\omega x/\upsilon(\omega)\right]. \qquad (3)$$

A nonstationary convolutional model for an attenuated seismic trace can be established by combining equations (2) and (3), by nonstationaryly convolving the attenuated impulse response with a reflectivity function and, finally, by convolving the result with an arbitrary wavelet. A such model has been used, for example by Margrave and Lamourex (2002), in the frequency domain,

$$\hat{s}(\omega) = \frac{1}{2\pi}\hat{w}(\omega)\int_{-\infty}^{\infty}\alpha_Q(\omega,\tau)r(\tau)e^{i\omega(t-\tau)}d\tau . \qquad (4)$$

where the 'hat' symbol indicates Fourier transform, r is the reflectivity function, w is the wavelet and $\alpha_Q(\omega,\tau)$ is the time-frequency exponential attenuation function,

$$\alpha_Q(\omega,\tau) = \exp\left(-\omega\tau/2Q + iH(\omega\tau/2Q)\right). \qquad (5)$$

in which the real and imaginary components in the exponent and connected through the Hilbert transform H, result that is consistent with the minimum phase characteristic of the attenuated pulse. A discrete formulation of the attenuated trace can be expressed as the following matrixes multiplication,

$$s = WQr , \qquad (6)$$

where the attenuated trace s is equal to the wavelet matrix W, in which each column is a shifted version of the source wavelet, multiplied by the attenuation matrix Q, which contains the attenuated impulse response, and finally multiplied by the reflectivity series.

Inverse-Q filtering by inverting the Q-matrix (Hale method)
In practice, as the wavelet is unknown and the procedures to remove the source signature such as spiking deconvolution work under the assuption tha the trace is stationary, the attenuation effects should be removed before the source signature removal. Under this practical constraint, the reflectivity *r* is solved in tems of the commutator between the matrixes Q^{-1} and W^{-1} from equation (6), as

$$r = W^{-1}Q^{-1}s + \left[Q^{-1}, W^{-1}\right]s \,. \tag{7}$$

In the simplest method for inverse-Q filtering, the commutator term is thrown away from equation (7), and the attenuation removed from the trace by computing the inverse of the Q matrix, Q^{-1}, and then multiplying the attenuated trace by , Q^{-1}, as expressed in equation (8).

$$r \approx W^{-1}Q^{-1}s \,. \tag{8}$$

The Q matrix becomes nearly singular for Q values below 70 (based on experiments), introducing numerical instability in its inverse estimation. Hale (1981) proposes a method to build an equivalent Q matrix, Q_e, and then to invert it. The Q_e matrix is generated by pre- and post-multiplying the Q matrix by an auxiliary matrix P, whose columns are the convolutional inverse of the Q matrix columns. The inverted trace is thus computed as

$$r \approx W^{-1}(PQ)^{-1}Ps \tag{9}$$

This procedure is as accurate as inverting the Q matrix itself and stable for Q values above 40, allthough it is computationally very expensive, due to the matrix inversion requirement.

Inverse-Q filtering by downward continuation (Wang method)
This method is based on a wave propagation approach in which deconvolution and inverse-Q filtering are processes closely related to inverse wave propagation or migration. Hargreaves and Calvert (1991) incorporate attenuation and dispersion effects into the downward-continuation operator of the Gazdag phase-shift method. This technique aims for phase compensation based on the 1-D (two-way propagation) wave Equation,

$$\frac{\partial^2 U(z,\omega)}{\partial z^2} + k^2 U(z,\omega) = 0 \,, \tag{10}$$

where z is the depth, U is the plane-wave of frequency ω, and k is the wavenumber. The inverse filter is obtained

$$U(t+\Delta t,\omega) = U(t,\omega)\exp\left(\frac{i\omega\upsilon(\omega_0)}{\upsilon(\omega)}\Delta t\right)\exp\left(\frac{\omega\upsilon(\omega_0)}{2Q\upsilon(\omega)}\Delta t\right). \tag{11}$$

The first exponential in Equation (11) compensates for the phase error introduced by velocity dispersion, and the second one for the amplitude decay due to energy absorption that occurs in anelastic attenuation. A velocity dispersion relation such as Equation (3) can be used to define $\upsilon(\omega)$ in Equation (11). Downward-continuation is applied to each monochromatic component of the signal according to Equation (11). The signal in the time domain is found by adding up the elementary plane-waves,

$$U(t+\Delta t) = \frac{1}{2\pi}\int U(t+\Delta t,\omega)\,d\omega \,. \tag{12}$$

Hargreaves and Calvert (1991) implement a solely phase compensation filter by ignoring the second exponent in Equation (11) which causes numerical instability for high values of t/Q. Wang (2001) tackles the stability problem introduced by the second exponent in the filter by limiting the frequencies contributing to the compensation to

$$\omega_q \le 2Q/t \,. \tag{13}$$

The frequency function in Equation (13) determines the upper limit of a time-varying filter. Equations (11) and (12) have to be applied alternatively for each interval Δt to get the filtered signal. Wang (2001) also introduces an efficient layered implementation of the method by averaging the second exponent in Equation (11) over the time, the frequency or both domains.

Inverse-Q filtering by pseudodifferential operators (PDO method)
Margrave (1998), in his theory of nonstationary linear filtering in the Fourier domain, defines two new operations, nonstationary convolution and combination, as nonstationary extensions of the stationary convolution operation. These new operations can be formulated in the time domain, in the frequency domain, which represent nonstationary extension of the convolution theorem, and in mixed time-frequency domains, where turn out to be generalized Fourier integrals. When the nonstationary convolution is expressed in the mixed time-frequency domain, equation (14), the input, *h(t),* is in the time domain and the output, $S(\omega)$, is in the frequency domain. For the nonstationary convolution in the mixed time-frequency domain, equation (15), the input H(Ω) is in the frequency domain, and the output $\tilde{s}(t)$ is in the time domain. In both expressions a time-frequency function, $\beta(\omega,\tau)$, called nonstationary transfer function, contains the essential characteristics of the nonstationary filter.

$$S(\omega) = \int_{-\infty}^{\infty} \beta(\omega,\tau)h(\tau)e^{-i\omega\tau}\,d\tau \,, \tag{14}$$

$$\tilde{s}(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \beta(\Omega,t)H(\Omega)e^{i\Omega t}\,d\Omega \,. \tag{15}$$

The filtering operators defined by equations (14) and (15) belongs to a general class of operators called pseudodifferential operators (e.g. Sant-Raymond, 1991), which are the entities resulting from the extension of Fourier analysis to inhomogeneous cases. In pseudodifferential operators language, the function β(ω,τ), corresponds to the pseudodifferential operator symbol. These equations may be used to apply nonstationary filtering, and particularly, to apply forward and inverse-Q filtering efficiently and with more stability. In the forward case the nonstationary transfer function is equal to α (ω,t) as defined in Equation (5). For the corresponding inverse-Q filter, a first approximation to the nonstationary transfer function is $\alpha^{-1}(\omega,t)$, the arithmetic inverse function of $\alpha(\omega,t)$. This method is highly stable and its computational speed is intermediate between the other two methods.

Examples

A two seconds long synthetic reflectivity series, 2 miliseconds sample rate, was used to test the three methods. To avoid mixing the error introduced by the source wavelet supression, the reflectivity series was not convolved with any sorurce wavelet, which is equivalent to use a pulse source. An attenuated trace was generated by applyng a forward-Q filter for Q values of 50, which can be roughly considered as a limit Q value below which Hale's method start to show stability problems, and 100, which is a representativily Q value for which Hale's method is highly stable. Four kinds of experiments were performed to test the sensibility of the methods to the uncertainty around the two parameters that detemine the Q-constant model: Q and the reference frequency, ω_0. The maximum coefficient of the crosscorrelation between the real and the expected output, MAXCORR, is used used as indicator of the similarity between the expected and the real output. The lag of MAXCORR is used as indicator of the phase restoration. Both indicators are plotted in the bottom of figures 1 to 8. In the first experiment, the two parameters Q and ω_0 are supposed to be known; figures 1 and 2. In this case Hale's method yields a practically perfect result whereas the other two methods produce just acceptable results. Then, uncertainty around ω_0 was considered in the second experiment, figures 3 and 4, where the inverse-Q filter is applied using a ω_0 value, equal to the Nyquist frequency, different from the one used for the forward-*Q* filter, 10000 Hz. Now, for Q=50, Hale's method turns unstable, and for Q=100, yields results similar to the obtained with the other two methods; all of the methods fail equally in recovering the drif introduced by the difference between the forward and inverse ω_0, which is called pedestal effect. In the third experiment, figures 5 and 6, uncertainty around Q is considered. Here, the inverse-Q filter was applied using a Q value 20% greater than the one used in the forward process. In this case the Hale's method performance suffers a quality loss similar to the previous case. Finally in figures 7 and 8, both ω_0 and *Q* used in the inverse-*Q* filter are different from the values used in the forward process. In this case Hale method deteriorates to become sligthly worse than the other two methods. In all the cases a sligthly advantage for the PDO method over the Wang method was observed.

Conclusions

Three different methods to apply inverse-Q filter has been reviewed, evaluated and compared using a synthetic trace. Hale's matrix inversion approach produces a filtered trace practically identical to the expected output for Q values greater than 40 when Q and ω_0 are perfectly known. Under the same contitions the performance of the other methods is excellent for phase restoration but just acceptable, and gets poorer for lower Q values, for amplitude recovering. When uncertainty in ω_0 is considered, Hale method losses its adavantage over the other two methods; the quality of the performance in the three methods become similar both in phase and amplitude recovery for Q=100, and Hale method becomes unstable for Q=50. When only uncertainty in Q is introduced, Hale method behaves better thant the other methods for Q=100 but turns unstable again for Q=50. When uncertainty both in Q and in ω_0 are combined, the other two methods perform better than Hale method. The results obained from PDO and Wang methods are very similar, but in most of the cases a sligthly adavantage in favor of PDO was observed.

Acknowledgements

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References

Aki, K., and Richards, P. G., 2002, Quantitative Seismology; Theory and methods: University Science Books.

Kjartansson, E., 1979, Constant-*Q* wave propagation and attenuation: J. Geophysics. Res., 84, 4737-4748.

Hale, D., 1981, An Inverse-*Q* filter: SEP Report 26.

Hargreaves, N. D., 1991, and Calvert A. J., Inverse-Q filtering by Fourier transform: Geophysics, 56, 519-527.

Margrave, G. F., 1998, Theory of nonstationary linear filtering in the Fourier domain with application to time-variant filtering: Geophysics, 63, 244-259.

Margrave, G. F., and Lamoureux, M. P. 2002, Gabor deconvolution: 2002 CSEG Annual Convention.

Saint-Raymond, X., 1991, Elementary introduction to the theory of pseudodifferential operators: CRC Press.

Wang, Y., 2002, A stable and efficient approach of inverse-*Q* filtering: Geophysics, 67, 657-663.

Figures

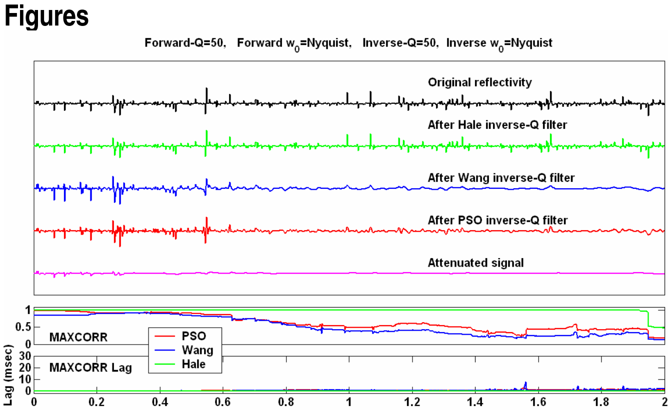

Figure 1. Using Q=50 and ω_0=250 in the forward filter and the same values in the inverse filter. Hale filter yields a practically perfect result both in amplitude and phase, MAXCORR is 1, and its lag is zero. The other two filters are excellent just in phase recovery, but its amplitude recovery is defficient.

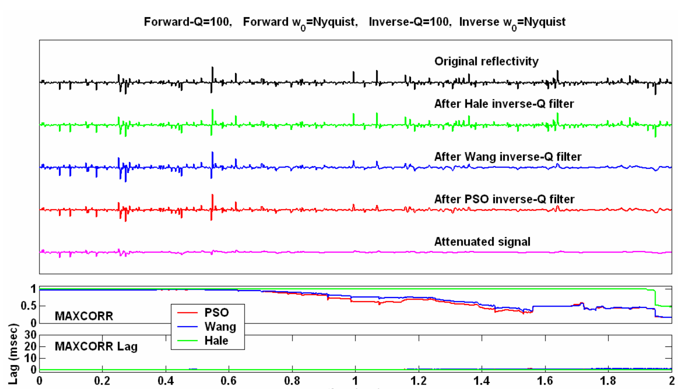

Figure 2. Same experiment as in figure 1 for Q=100. The same trends observed in the previous figure can be observed here, although the amplitude recovery has improved in Wang and PDO methods with respect to the case for Q=50, as can be easily observed by comparing the MAXCORR curve in both experiments.

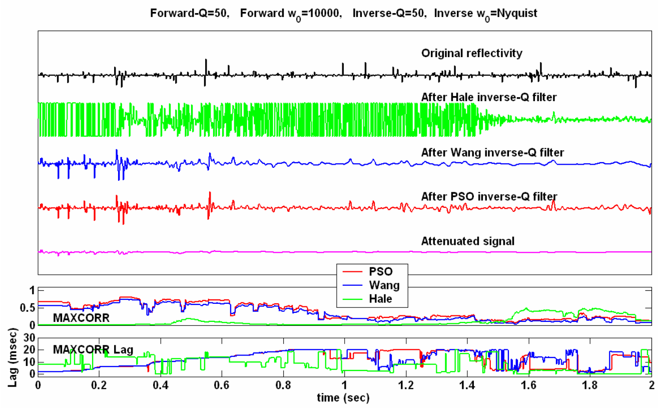

Figure 3. Using Q=50 and ω_0=10000 in the forward filter and Q=50 and ω_0=250 the in the inverse filter. Hale filter becomes unstable. PDO yields better result than Hang method in recovering amplitude; in phase restoration none of the methos is able to recover the pedestal effect phase lag.

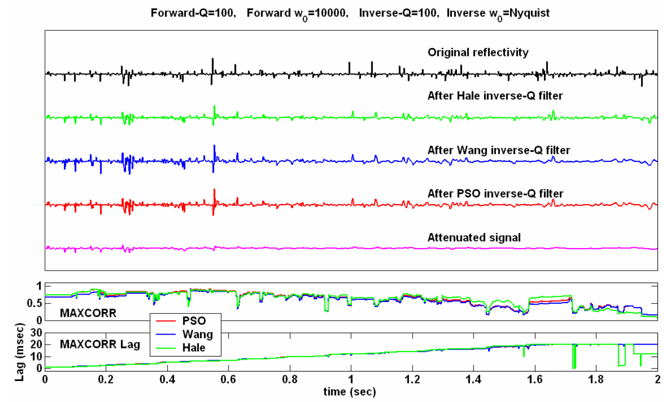

Figure 4. Same experiment as in figure 3, but for Q=100. Hale filter has lost its huge advantage over the other two methods. The amplitude and phase recovery of the three methods is similar. The growing phase lag due to the pedestal effect remains in the trace after applying inverse-Q filter using any method.

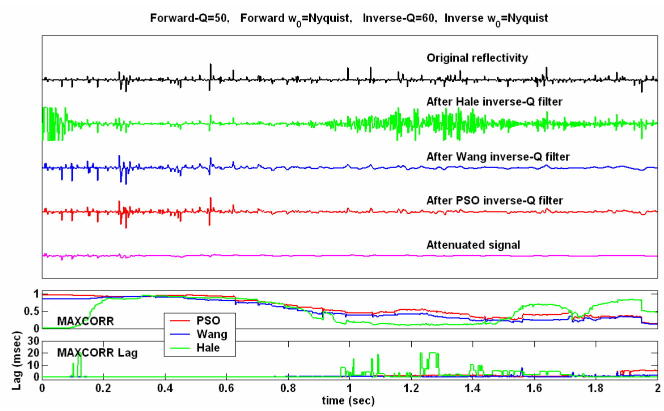

Figure 5. Using Q=50 and ω_0=250 in the forward filter and Q=60 and ω_0=250 the in the inverse filter. Hale filter becomes unstable. PDO yields better result than Hang method in recovering amplitude, and has a similar good performance in recovering phase.

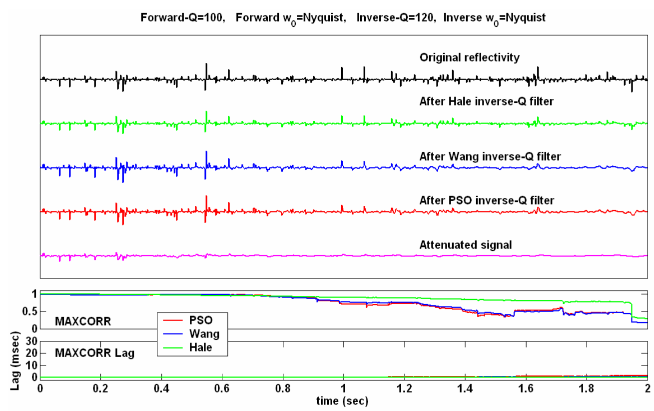

Figure 6. Same experiment as in figure 5, but for Q=100. Hale filter shows a considerable advantage over the other two methods in recovering amplitude. The amplitude recovery in the three methos is practically perfect as can be observed in the MAXCORR lag curve which is practcally zero for any method.

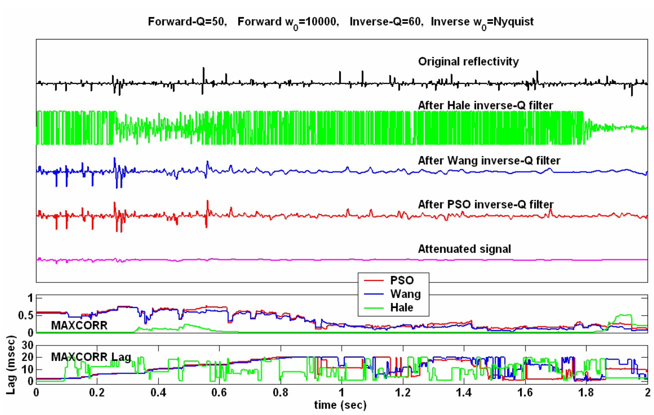

Figure 7. Using Q=50 and ω_0=10000 in the forward filter and Q=60 and ω_0=250 the in the inverse filter. Hale filter becomes unstable. PDO yields better result than Hang method in recovering amplitude, but none of the methods suceeds in recovering the phase lag due to the pedestal effect.

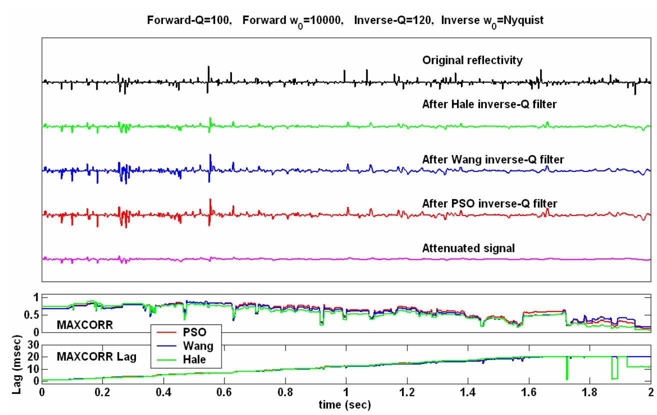

Figure 8. Same experiment as in figure 6, but for Q=100. PDO method becomes the best in recovering amplitude, followed by Wang. Hale method may become the worst method in recovering amplitude when considering uncertainty in Q and ω_0, as in this case.