NMO Inversion for Multilayer Subsurface with Horizontal Transverse Isotropic layers

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Introduction
The multilayered transverse isotropic model with horizontal anisotropy axis is used to study fractured reservoirs. Reflection moveout in fractured media is usually used to determine azimuth of main set of fractures and to estimate the crack density and to predict whether the cracks are fluid or dry (Contrera et al (1999)). In this paper, I use an inversion technique, originally developed by Blias (1983) for the layered medium with vertical symmetry axis (VTI media). This algorithm is utilized to determine an explicit formula for azimuthally-dependent NMO velocity in a HTI layered media with depth-varying orientation of the symmetry axis. The NMO inversion problem for the HTI medium was solved by Contrera et al (1999) using a technique, originally developed by Grechka and Tsvankin (1999). They derived a formula for azimuth-dependent NMO velocity using 2x2 matrices responsible for the 'interval' NMO ellipses in each layer. Here I suggest an alternative approach to NMO velocity derivation for the multilayered transverse isotropic model with depth-varying orientation of the symmetry axis. Explicit analytical formula for azimuth-dependent NMO velocity helps to develop insight on the anisotropy influence on reflection travelttime and to develop Dix-type inversion algorithm.

Method
Let’s consider a layered medium with horizontal boundaries and transverse isotropic layers with horizontal anisotropy axis in each layer. Suppose the traveltimes of a reflected wave have been recorded for a fixed midpoint but for different azimuthal orientation. Let d be the source-receiver distance and dx and dy be the x and y projection of the source-receiver segment, α is an azimuth, fig. 1. Then the traveltime t is a function of dx and dy. This function contains only even powers of dx and dy. For conventional spreadlengths close to the reflector depth, for the arbitrary 3-D subsurfaces, the traveltime function t(dx,dy) can be written in a form (Blias, 1988):

\[ t^2(d_x, d_y) = t_0^2 + w_{11}d_x^2 + 2w_{12}d_xd_y + w_{22}d_y^2 \]  

(1)

We can rewrite this equation using azimuth α (dx = d cosα, dy = d sinα)

\[ t^2(d_x, d_y) = t_0^2 + \left( w_{11} \cos^2 \alpha + 2w_{12} \sin \alpha \cos \alpha + w_{22} \sin^2 \alpha \right) d^2 = t_0^2 + \frac{d^2}{V_{NMO}^2(\alpha)} \]  

(2)
This representation for anisotropic media was proposed by Grechka and Tsvankin (1998). Let $h_k$ be the thickness of the $k$-th layer, $\varphi_k$ be the azimuth of the anisotropy axis in the same layer; $v_{0k}$ is a vertical velocity in the layer. For each layer, the HTI model is characterized by the stiffness tensor $c_{ijkl}$ that corresponds. P and SV phase velocities are described with formulas using Thomsen’s notations (Thomsen, 1986).

I am using the same approach that has been developed by Blias (1983) for VTI medium. The only difference is that in HTI layer, $\Theta=\pi/2$ corresponds to the vertical axis so instead of using angle $\theta$, I will use an angle $\alpha = \pi/2 - \theta$. For the phase P velocity $V_P(\alpha)$ and phase angle $\alpha$, we can write equations:

$$\rho V_P^2(\alpha) = \alpha_0^2 \left[ 1 + \epsilon \cos^2 \alpha + \hat{D}(\alpha) \right],$$

$$\psi(\alpha) = \frac{\pi}{2} - \alpha - \arctg \left( \frac{d \ln V}{d \alpha} \right),$$

where $D(\sin \theta) = \hat{D}(\cos \alpha)$, $\epsilon$ is Thomsen parameter (Thomsen, 1986). From these equations, we can find explicit connection between the group velocity $v_P$ and group angle $\psi$:

$$V_P(\psi) = V_0 + \frac{1}{2} \frac{\partial^2 V_P}{\partial \psi^2} \left( \frac{\pi}{2} - \psi \right)^2 + \frac{1}{24} \frac{\partial^4 V_P}{\partial \psi^4} \left( \frac{\pi}{2} - \psi \right)^4 + \ldots$$

A reflection traveltime $T$ can be calculated as a sum of traveltimes in each layer:

$$T = \sum_{k=1}^{n} t_k(x_k, y_k) = 2 \sum_{k=1}^{n} \frac{\sqrt{h_k^2 + x_k^2 + y_k^2}}{v_k(\varphi_k, x_k, y_k)}$$

(2)
Here $x_k$ and $y_k$ are X and Y the projections of the ray in the $k$-th layer,

$$\sum_{k=1}^{n} x_k = 2d_x,$$
$$\sum_{k=1}^{n} y_k = 2d_y,$$  \hfill (3)

$\theta_k$ is a ray angle, determined by the equation:

$$\theta_k = \arccos \left( \frac{x_k \cos \varphi_k + y_k \sin \varphi_k}{\sqrt{h_k^2 + x_k^2 + y_k^2}} \right)$$

To find minimum of function $T$ with respect to $x_k$, $y_k$, we use Lagrange function, which leads us to the system of equations:

$$\frac{\partial t_k(x_k, y_k)}{\partial x_k} = \lambda, \quad \frac{\partial t_k(x_k, y_k)}{\partial y_k} = \mu \quad k=1, 2, \ldots, n \hfill (4)$$

where $\lambda$ and $\mu$ are Lagrange multipliers (Blias, 1983). We have $2n+2$ equations (3) – (4) to determine $2n+2$ unknown $x_k$, $y_k$, $\lambda$ and $\mu$. First we find $x_k$ and $y_k$ as a solution of system (4) as functions of $\lambda$ and $\mu$ independently for each $k$:

$$x_k = \lambda x_{1,k} + \mu x_{2,k} + \lambda^3 x_{3,k} + \ldots \quad y_k = \lambda y_{1,k} + \mu y_{2,k} + \lambda^3 y_{3,k} + \ldots \hfill (5)$$

Then we substitute (5) into (2) and (1) and come to the parametric form of the time-distance relationship with $\lambda$ and $\mu$ as parameters:

$$T = T_0 + T_{2,0}\lambda^2 + T_{1,1}\lambda \mu + T_{0,2}\mu^2 + \ldots$$
$$d_x = d_{1,x}\lambda + d_{2,x}\mu + d_{3,x}\lambda^3 + d_{4,x}\lambda^2\mu + \ldots$$
$$d_y = d_{1,y}\lambda + d_{2,y}\mu + d_{3,y}\lambda^3 + d_{4,y}\lambda^2\mu + \ldots$$

From these equations we can determine coefficients $w_{ij}$ in formula (1):

$$w_{1,1} = \frac{1}{2} t_0 B_{11} \left( B_{11} B_{22} - B_{12}^2 \right), \quad w_{1,2} = -\frac{1}{2} t_0 B_{12} \left( B_{11} B_{22} - B_{12}^2 \right),$$
$$w_{2,2} = \frac{1}{2} t_0 B_{22} \left( B_{11} B_{22} - B_{12}^2 \right) \hfill (6)$$

where

$$B_{11} = \sum_{k=1}^{n} \Delta t_k v_{0,k}^2 \left( 1 + 2 \hat{\varphi}_k \sin^2 \varphi_k \right), \quad B_{12} = 2 \sum_{k=1}^{n} \Delta t_k v_{0,k}^2 \hat{\varphi}_k \cos \varphi_k \sin \varphi_k$$
$$B_{22} = \sum_{k=1}^{n} \Delta t_k v_{0,k}^2 \left( 1 + 2 \hat{\varphi}_k \cos^2 \varphi_k \right) \hfill (7)$$
Here $\hat{\beta}_P$ is a coefficient, similarly to Thomsen parameter $\beta$ and defined by formula:

$$2\hat{\delta}_P = \frac{(C_{13} - C_{11} + 2C_{44})(C_{13} + C_{11})}{(C_{11} - C_{44})C_{11}}$$

The same equation holds for SV waves, only $\hat{\delta}_{SV}$ is determined by formula:

$$2\hat{\delta}_{SV} = \frac{(C_{33} - C_{44})(C_{11} - C_{44}) - (C_{11} + C_{44})^2}{(C_{11} - C_{44})C_{44}}$$

If $\hat{\delta}_{P,k} = 0$ then formulas (6) – (7) are the same as for isotropic medium. Formulas (6) can be written in a matrix form:

$$\mathbf{B} = \mathbf{W}^{-1}$$

where $\mathbf{B} = ||B_{ij}||$, $\mathbf{W} = ||w_{ij}||$.

**Inverse Problem**

Now we describe Dix-type inversion to determine interval vertical velocities and anisotropic parameters $\hat{\beta}_P$ and $\hat{\beta}_{SV}$. From traveltime measurement we know coefficients $w_{11}$, $w_{12}$ and $w_{22}$. Then we consider equations (6) as a system with respect to $A$, $B$ and $C$. This system has a unique solution:

$$B_{11,n} = \frac{1}{2} \frac{t_{0n} w_{1,1}^{(n)}}{w_{1,1}^{(n)} w_{2,2}^{(n)} - (w_{1,2}^{(n)})^2}, \quad B_{12,n} = -\frac{1}{2} \frac{t_{0n} w_{1,2}^{(n)}}{w_{1,1}^{(n)} w_{2,2}^{(n)} - (w_{1,2}^{(n)})^2}, \quad B_{22,n} = \frac{1}{2} \frac{t_{0n} w_{2,2}^{(n)}}{w_{1,1}^{(n)} w_{2,2}^{(n)} - (w_{1,2}^{(n)})^2} \quad (8)$$

where $n$ is a reflector number. Taking into account (7), we can write system with unknown:

$$v_{0,n}^2 \Delta t_n \left(1 + 2\hat{\delta}_n \sin^2 \phi_n \right) = 2\Delta B_{11,n},$$

$$v_{0,n}^2 \Delta t_n \hat{\delta}_n \cos \phi_n \sin \phi_n = \Delta B_{12,n},$$

$$v_{0,n}^2 \Delta t_n \left(1 + 2\hat{\delta}_n \cos^2 \phi_n \right) = 2\Delta B_{22,n}$$

where $\Delta A_n = A_n - A_{n-1}$, $\Delta B_n = B_n - B_{n-1}$, $\Delta C_n = C_n - C_{n-1}$. This system can be easily solved for the vertical velocity $v_{0n}$, azimuth $\phi_n$ and anisotropic parameter. After replacing $\hat{\beta}_P$ by $\hat{\beta}_{SV}$, formulas (6) – (7) hold for SV waves. It directly implies that we can find vertical SV velocity and anisotropic coefficient $\hat{\beta}_{SV}$ by solving system (8) where the matrix $\mathbf{B}$ is calculated for SV waves.

**Conclusions**

A new representation of NMO velocity has been derived for the multilayered transverse isotropic model with horizontal anisotropy axis in each layer. Dix’s type of inversion is applied to azimuth-dependent NMO velocity to determine vertical P and S velocities and anisotropic parameters $\hat{\beta}_P$ and $\hat{\beta}_{SV}$. 
References


Contreras, P., Grechka, V. and Tsvankin, I., Moveout inversion of P-wave data for horizontal transverse isotropy: Geophysics, 64, 1219-1229.


