3D Seismic Data Interpolation by Frequency Domain Local 2D Prediction

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Introduction

I continue to work on a scheme for interpolating one seismic trace at a time. This output-driven gives rise to the ultimate in flexibility for specifying every shot & receiver positions of the interpolated prestack data volume. The user first inspect the input data geometry and typically suggests a shot ordered output data structure with desired shot/receiver grid spacing/extent. For each interpolated trace, I select a group of input traces best serving the interpolation method. Any imperfections in input data sampling (with respect to the requirments of the interpolation algorithm) are analyzed and kept in interpolated trace headers for later QC.

My last work in this area was a time domain interpolation which involved fitting each time sample by a polynomial (Wang, 2004). This approach is fast and avoids data gridding, but cannot interpolate highly structured data.

In this work, I develop a new interpolation kernel in the frequency domain which is capable of interpolating aliased data.

Prestack seismic data can be fully described in five dimensions: for example, source point x & y, receiver point x & y, and time, or alternatively, CMP x & y, offset x & y, and frequency. I will work in the latter combination.

First the seismic trace is FFT'ed from time to the frequency domain. Spatial prediction can then be done in any number of dimensions, at most in 4D or at least in 1D. The higher the dimension count the larger the filter size and the more costly. My present approach is to fix offset x & y, and do 2D prediction in CMP x & y. For any target prestack trace to be interpolated, input data usually do not exist exactly where I want them to be. Such deviation from "perfection" is recored in trace headers.

2D Frequency Domain Prediction

Let $A(f,x_k,y_k,x_d,y_d)$ denote a trace at mid-point (x_k,y_k) with offset vector $\mathbf{v}=(x_d,y_d)$ and at frequency f. For locally linear seismic signals, it can be approximated by 2D prediction in the CMP (x_k,y_k) plane:

$$A(f,x_k,y_k,x_d,y_d)=\sum_{i=-n_x}^{n_x}\sum_{j=-n_y}^{n_y}F_{ij}A(f,x_k-id_x,y_k-jd_y,x_d,y_d)\,, \tag{1}$$

where F_{ij} is the 2D prediction filter (and therefore F_{00} is not included in the summation), d_x & d_y are CMP intervals in x & y direction and n_x & n_y are one-sided maximum filter lags in the x & y directions, respectively.

Eqn.(1) forms the basis of the Spitz (1991) approach to interpolation. His method is global, and exploits the regular sampling of the input data volume in order to make it denser by an integer factor, resulting in an efficient but less flexible algorithm. His algorithm also needs good quality data at low frequencies. By contrast, my approach needs good *local* data distribution, is flexible but less efficient, and has no special demand for low frequency data.

In order to use the above scheme to interpolate a trace at $A(f,x_m,y_m,x_d,y_d)$ we need known data with the same offset (x_d,y_d) and CMP locations around (x_m,y_m) on the CMP grid of $(x_m\pm N_x,\ y_m\pm N_y)$, where N_x, & N_y define the input data block size ($N_x\geq n_x$ & $N_y\geq n_y$ to guarantee an overdetermined least squares system in solving the filter coefficients), and the maximum number of equations (corresponds to the case where input data exists at all CMP locations within the grid) is $N_{eq}=(2N_x+1)(2N_y+1)$. Setting up equation (1) with data CMP index k going through all the input data grid of N_{eq} points, we get N_{eq} equations to solve for the $N_{flt}=(2n_x+1)(2n_y+1)-1$ filter coefficients. With the filter solved, we then substitute (x_m,y_m) into equation (1) in place of (x_k,y_k), and interpolate the data at one frequency. Missing input data on the grid will degrade the filter solution, and this gets worse when input data becomes one-sided in any direction; again, the QC trace headers closely watch the situation and “red flag” any interpolated output of questionable reliability.

Input Trace Selection for Prediction & QC

We will seldom get input data exactly as desired. The imperfection of input data sampling in the neighborhood of the interpolated trace is measured by two QC quantities.

First, the CMP misposition of one input trace relative to the local CMP grid is given by

$$\Delta_{cm}^2=[x_k-(x_m-id_x)]^2+[y_k-(y_m-jd_y)]^2\,, \tag{2}$$

and secondly, the discrepancy between the offset vectors of input and interpolated traces, the *offset vector focusing factor,* is given by

$$Q_{fcs}=1-\frac{|\mathbf{v}\bullet\mathbf{v}_k|}{\max(|\mathbf{v}|^2,|\mathbf{v}_k|^2)}\,, \tag{3}$$

in which, $\mathbf{v}$ & $\mathbf{v}_k$ are the offset vectors of the interpolated and the input trace, respectively.

The definition of CMP misposition is straightforward, but the defination of Q_{fcs} needs some clarification.

The absolute value of the dot product in the numerator of eqn. (3) will honour the reciprocity principle, since exchanging the source & receiver location of an input trace simply reverses the offset vector, which in turn will not change Q_{fcs}. Note also that the denominator of the ratio in (3) is always greater than or equal to the numerator, and therefore the dimensionless Q_{fcs} is always non-negative and between 0 (corresponding to $\boldsymbol{v}=\pm\boldsymbol{v}_k$) & 1 (corresponding to $\boldsymbol{v}\perp\boldsymbol{v}_k$). Just as in the case of Δ^2_{cm}, the smaller the value for Q_{fcs} the better, and the interpolation is "perfect" when both Δ^2_{cm} & Q_{fcs} are zero .

Input traces are selected on the $(2N_x+1)$ by $(2N_y+1)$ grid centered around the interpolated trace CMP position, and at each input grid point, the trace with the smallest Δ^2_{cm} & Q_{fcs} is chosen. The average of Δ^2_{cm} and that of Q_{fcs} over the $(2N_x+1)(2N_y+1)$ input grid points are noted in the trace header of this interpolated trace.

Synthetic & Real Data Tests

Model data contains aliased events in both *x* & *y* directions. Fig.1 show an interpolated partial shot gather after NMO. Interpolation by time domain method (fig.1t) fails somewhat and frequency domain (fig.1f) method is better.

A data set collected by a potash mine with known mine entry location offers a good test for resolution of the interpolation. Prediction methods offers better perservation of mine locations than time domain methods in our decimation test. Details will be discussed at the oral presentation.

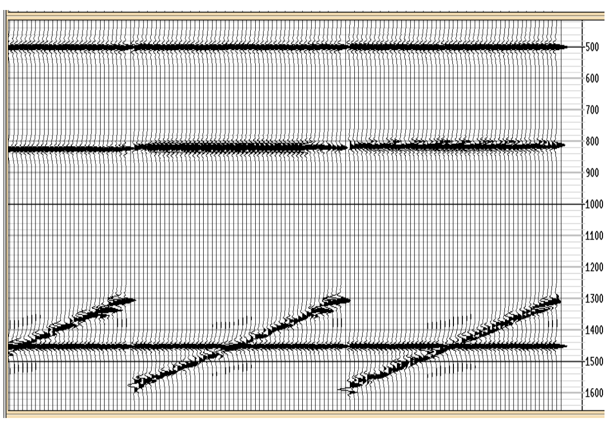

Figure 1t

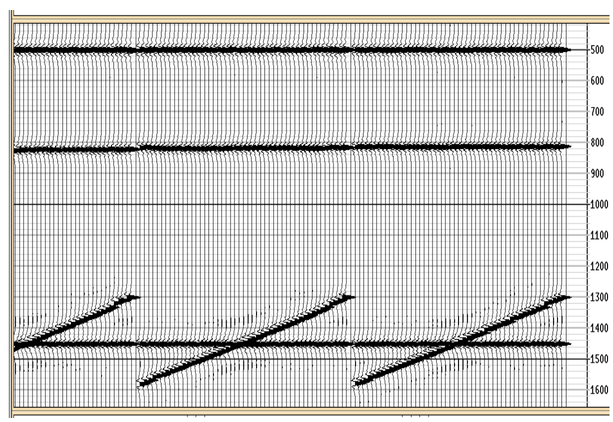

Figure 1f

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References

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