Fast Gabor Imaging with Spatial Resampling

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Summary

The Gabor depth imaging method has been made much more efficient thanks to adaptive partitioning algorithms. Moreover, as introduced in the FOCI space-frequency domain imaging method wavefield spatial sampling at low frequencies can be decreased, resulting in a lower computation cost without loss of imaging accuracy. This spatial resampling technique can be implemented in the Gabor imaging method because this imaging method also works in the space-frequency domain. In this paper, we demonstrate the application of spatial resampling in the Gabor imaging method and demonstrate substantial reduction in computational effort.

Introduction

Gabor imaging theories have been described by Grossman et al. (2002) and Ma and Margrave (2006). In the Gabor imaging method, wavefields are localized with spatial windows, and then Fourier transformed across windows and extrapolated to new depths. A simple way to implement Gabor imaging is to use evenly distributed small windows across the lateral coordinate. These small windows (called *atomic* windows) are usually chosen to address the most rapidly varying velocities in the lateral direction at a depth step. As a result of this setting, there is redundancy in these atomic windows. Adaptive partitioning algorithms (Grossman et al., 2002; Ma and Margrave, 2007) have been introduced to eliminate redundant windows. Gabor imaging deals with the wavefields that are dependent on temporal frequency.

In a simple case of extrapolation, we have a thin slab without any lateral velocity variation, a homogeneous layer. The adaptive partitioning algorithm automatically uses one wide window across the lateral coordinate. Therefore, we need one forward and one reverse spatial Fourier transforms for each frequency of the source field and the recorded data field, consequently; at each depth step, four spatial Fourier transforms are needed to transform seismic data back and forth between the spatial and wavenumber domains. These processes have to be done frequency by frequency. In a general case, we usually expect lateral velocity variations and more windows at a depth step, meaning computation cost is several times higher.

The number of depth steps in a typical depth marching migration can be several hundred or even a thousand. The number of frequencies usually is from several hundred to more than a thousand. The product of the number of Fourier transforms related to a single window and those of depths, frequencies and windows at depths gives a very large number of Fourier transforms during the extrapolation process. Any reduction in those numbers will make Gabor imaging more efficient. Usually numbers of depths, frequencies in seismic depth migrations are set. However, we can eliminate redundant windows and alter spatial sample rates at low frequencies. We have implemented the former, and we describe the application of spatial resampling (Margrave et al., 2006) to reduce wavefield size in Gabor depth imaging, which will be demonstrated in the following sections using the Marmousi synthetic data sets (Bourgeois et al., 1991).

Spatial Resampling Theory

Margrave et al. (2006) introduced spatial resampling in the forward operator and conjugate inverse (FOCI), a space-frequency domain imaging method. Many wavefield extrapolation methods use operators of a fixed number of samples, which would run into problems at low frequencies. That is because if both the number of operator samples and the wavefield sample rate are constant, then as frequency decreases, the operator spans fewer wavelengths. In fact, at low frequencies, wavelengths are large and typical operators span only a small fraction of a wavelength. The result is, the fixed-length operator incurs poor phase and instability when working with low frequencies.

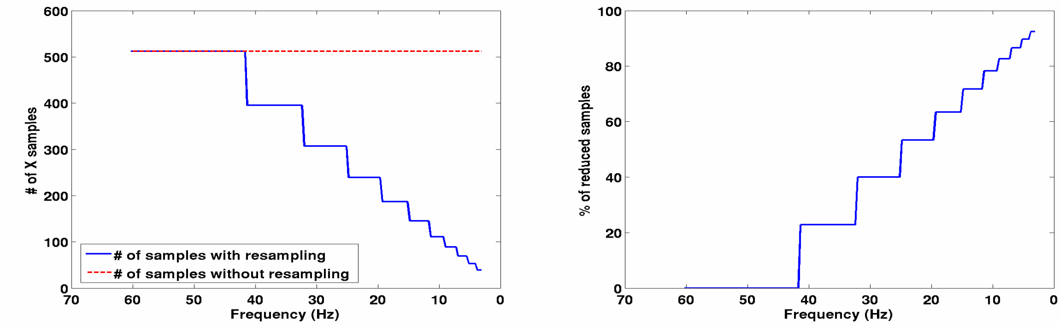
To address this problem, the spatial resampling technique was suggested by Margrave et al. (2006). In their paper, they mentioned the advantage of using this technique. That is, by resampling the lower frequencies to a larger sample size, an operator with a fixed number of points actually spans a greater physical length. Resampling can improve the extrapolation speed because the lower frequencies of the wavefield have fewer samples and can therefore be processed more quickly. While the Gabor method uses a phase shift rather than a spatial convolution operator, we can still use spatial resampling to increase efficiency because the Fourier transforms require less effort as frequency decreases. The first step of the implementation is to break the frequency range into small bands as (Margrave et al., 2006)

$$\left[\omega_{\min},\omega_{\max}\right]=\left[\omega_{\min},\omega_1\right]\cup\left[\omega_1,\omega_2\right]\cup\ldots\cup\left[\omega_{n-2},\omega_{n-1}\right]\cup\left[\omega_{n-1},\omega_{\max}\right], \qquad (1)$$

where $\omega_{\min}$ and $\omega_{\max}$ are the minimum and maximum frequencies of interest (i.e. signal frequencies, respectively, from band-limited seismic data. Suppose we have the original (fixed) spatial sampling interval as Δx; we resample it to new ones, Δx_j, according to various frequency bands, which are defined as

$$\alpha\left(\frac{\pi}{\Delta x_j}\right)\le\frac{\omega}{v_{crit}}\le\beta\left(\frac{\pi}{\Delta x_j}\right),\ \alpha<\beta\in\left[0,1\right],\ \omega\in\left(\omega_{j-1},\omega_j\right), \qquad (2)$$

where v_{crit} is the velocity chosen to define the highest evanescent boundary. Using spatial resampling, we always have $\Delta x_j>\Delta x$, which means that the wavefield at low frequencies always has fewer spatial samples than the input wavefield. Fewer samples give faster Fourier transforms. Hence, we have a faster Gabor imaging than the one with fixed lengths of operators for all frequencies. Figures 1 (a) and (b) show the statistics of samples in x using the spatial resampling method.



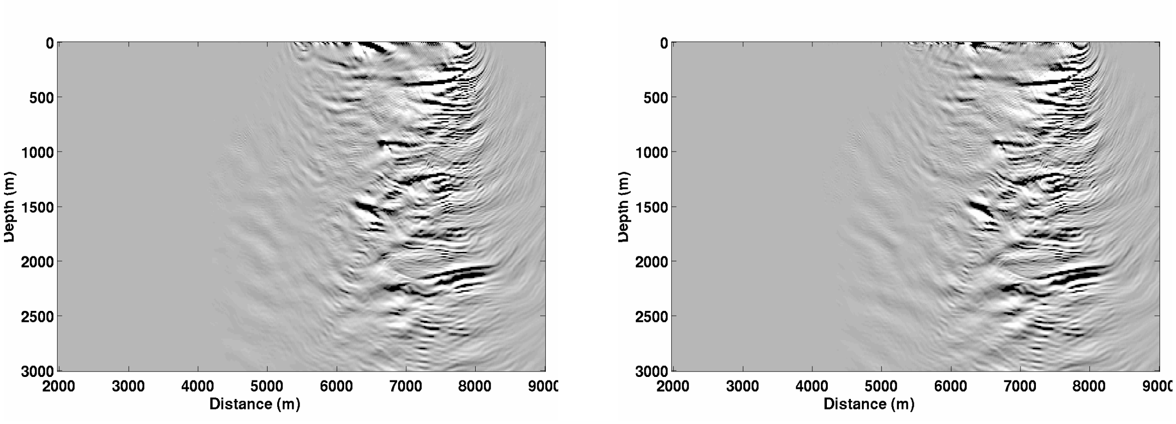
(a) (b)

Figure1: (a) Samples versus frequency: the lower the frequency, the fewer the samples in x; (b) percentage of change in samples (compared to no spatial resampling) versus frequency. Each step (in blue) in (a) or (b) corresponds to a frequency band.

It is imperative that the spatial resampling be done as accurately as possible, and for this purpose, we use a Fourier method. Essentially, for each frequency, we apply a sharp lowpass filter to zero the wavefield at all wavenumbers greater (in absolute value) than ω / v_{crit}. We then shift the wavenumber Nyquist from $\pi / \Delta x$ to $\pi / \Delta x_j$, thereby truncating in the wavenumber domain most of the samples that were zeroed by our lowpass filter. Finally, we apply an inverse wavenumber Fourier transform.

Gabor Imaging Examples with Spatial Resampling

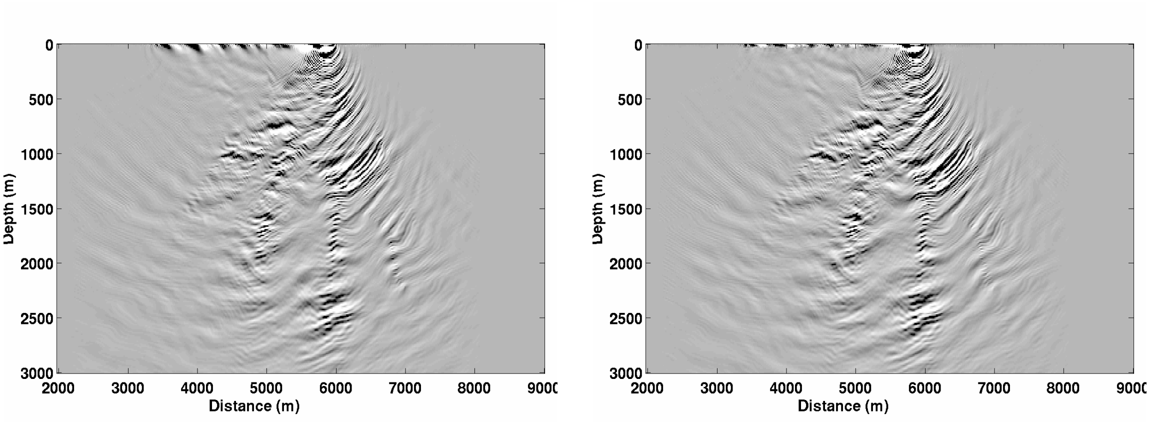
In this section, we show some imaging examples with the application of spatial resampling in the Gabor imaging method. Figure 2 (a) shows depth imaging of the 200th shot record in the Marmousi data sets using the fixed-length Gabor imaging method. Figure 2 (b) shows the image of the same shot record but using the Gabor imaging method with the spatial resampling. All other imaging parameters such as accuracy criteria are the same. Total reduction of the computation effort in percentage for this shot is accumulated as 35.72% (see also Figure 1 (b)).



(a) (b)

Figure2: Gabor imaging of the 200th shot record from the Marmousi data sets without (a) and with (b) spatial resampling

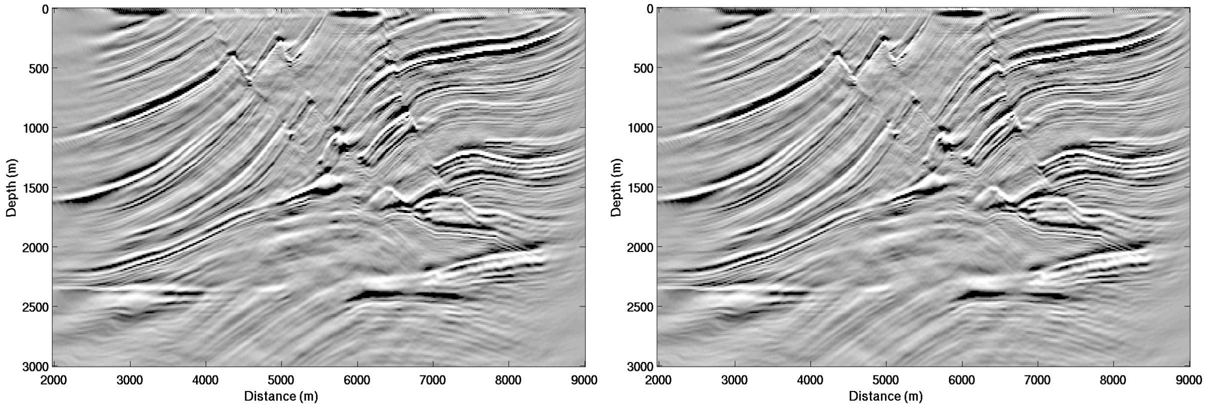
From visual comparison, we can see that these two images are similar to each other but slight difference can be seen with careful observation. If we look at the run time, image in Figure 2 (a) consumed about 366 s, while the one in Figure 2 (b) used about 127 s. Figures 3 (a) and (b) show another two shot migrations from shot 120. We see images are very similar to each other again. The image in Figure 3 (a) used 395 s CPU time, and the one in Figure 3 (b) used 137 s.



(a) (b)

Figure3: Gabor imaging of the 120th shot record from the Marmousi data sets without (a) and with (b) spatial resampling

For migration of all 240 shot records, we show imaging results in Figures 4 (a) and (b). We can hardly tell the difference between the two. However, some subtle difference between the two images can still be noticed with careful inspections (e.g., see the small areas, (x,z) = (5500, 1600), close to the centers of the images). For the run time, Gabor imaging without the spatial resampling used 19 hours. The one with the spatial resampling used 7 hours. This shows that the Gabor imaging speed has been made roughly 3 times faster. These migrations were performed on a PC with a single CPU of 3.0 GHz.



(a) (b)

Figure4: Gabor imaging of the Marmousi data sets (all 240 shot records) without (a) and with (b) spatial resampling

Conclusions

Spatial resampling method reduces samples of wavefield at low frequencies, which makes shorter Fourier transforms during wavefield extrapolations. As a result, the efficiency of wavefield extrapolations is accelerated. This is critical to seismic depth migration methods such as the Gabor depth migration, where large amount of Fourier transforms are employed. The application of spatial resampling in Gabor depth imaging improves imaging speed to about 3 times faster for the Marmousi synthetic data sets.

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References

Bourgeois, A., Bourget, M., Lailly, P., Poulet, M., Ricarte, P. and Versteeg, R., 1991, Marmousi, Model and data, *in* R. Versteeg and G. Grau, eds., The Marmousi experience: Proceedings of the 1990 EAGE workshop on practical aspects of seismic data inversion, 5–16.

Grossman, J. P., Margrave, G. F. and Lamoureux, M. P., 2002, Fast wavefield extrapolation by phase-shift in the nonuniform Gabor domain: CREWES Research Report, **14**.

Ma, Y., and Margrave, G. F., 2006, Prestack depth imaging with the Gabor transform: 76th Annual International Meeting, SEG, Expanded Abstracts, 2504–2507.

Ma, Y., and Margrave, G. F., 2007, Adaptive Gabor imaging using the lateral position error criterion: 77th Annual International Meeting, SEG, Expanded Abstracts.

Margrave, G. F., Geiger, H. D., Al-Saleh, S. M. and Lamoureux, M. P., 2006, Improving explicit seismic depth migration with a stabilizing wiener filter and spatial resampling: Geophysics, **71**, S111–S120.