Frontiers + Innovation
2009 CSPG CSEG CWLS CONVENTION • CALGARY, ALBERTA, CANADA

The Application of Extreme Attributes of PS Converted Wave

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Summary

The analysis of amplitude variation with offset (AVO) of P-wave reflection is a very popular and powerful tool for detecting the presence of gas sands. However, the AVO attributes of PS-wave are not studied as much as for P-wave. Using the empirical Gardner equation describing the relationship between density and P-wave velocity, we propose converted wave reflection coefficient extreme attributes for AVO analysis and derive relations between the extreme position and amplitude, average velocity ratio across the interface, and shear wave reflection coefficient. The extreme position is a monotonically decreasing function of average velocity ratio, and the extreme amplitude is a function of average velocity ratio and shear wave reflection coefficient. For theoretical models, the average velocity ratio and shear wave reflection coefficient are inverted from the extreme position and amplitude obtained from fitting a power function to converted wave AVO curves. At last the method is applied to real CCP gather to invert average velocity ratio, shear wave reflection coefficient and Poisson's ratio. The prediction is in accordance with indicator of oil and gas got from statistic of drilling.

Introduction

Amplitude versus Offset (AVO) is used to analyze the relationship between seismic amplitude, lithology and pore fluid properties in order to predict hydrocarbon distributions. The foundation of the AVO technique is based on the Zoeppritz equations (Zoeppritz, 1919), but though these equations can describe the variation of PP (incident P wave, reflected P wave) and PS (incident P wave, reflected S wave) reflection coefficients, they are mathematically complex and are usually not applied directly in practice.

With the development of converted wave exploration, AVO analysis of PS waves is extensively used to describe and predict reservoirs. Currently, one common simplification for P-wave AVO analysis is to use a two-term approximation which consists of an intercept and gradient, which satisfies the inversion precision only at small incidence angles. However, this may not be the optimum approach for converted-wave AVO analysis since the signal to noise (S/N) ratio is often very low for converted waves at small incidence angles, and the concept of intercept and gradient is not really applicable. At middle to far offset the converted wave amplitude reaches an extreme, and the S/N ratio is higher. Therefore, we propose to make use of the extreme position and amplitude to perform converted-wave AVO analysis. This utilizes the best of high S/N ratio data within the middle offset range, and makes it possible to invert for the average velocity ratio and shear wave reflection coefficient from the extreme position and amplitude.

AVO Attributes of Converted waves

On the assumption of small contrasts in elastic properties across an interface,the reflection coefficient of a PS wave is given by (Aki and Richards,1980)

$$R_{ps}(\theta) = -\frac{v_p}{2v_s}\tan\varphi\left\{\left[(1-2\frac{v_s^2}{v_p^2}\sin^2\theta+2\frac{v_s}{v_p}\cos\theta\cos\varphi\right]\frac{\Delta\rho}{\rho}-\left[4\frac{v_s^2}{v_p^2}\sin^2\theta-4\frac{v_s}{v_p}\cos\theta\cos\varphi\right]\frac{\Delta v_s}{v_s}\right\} \quad (1)$$

Using $J=\rho v_s$, then $\Delta J/J=\Delta\rho/\rho+\Delta v_s/v_s$ .So equation 1 can be rewritten as

$$R_{ps}(\theta)=\left[-\frac{\sin\theta}{2\sqrt{1-(\frac{v_s}{v_p}\sin\theta)^2}}-\frac{\frac{v_s^2}{v_p^2}\sin^3\theta}{\sqrt{1-(\frac{v_s}{v_p}\sin\theta)^2}}+\frac{v_s}{v_p}\sin\theta\sqrt{1-\sin^2\theta}\right]\frac{\Delta\rho}{\rho}+\left[\frac{2\frac{v_s^2}{v_p^2}\sin^3\theta}{\sqrt{1-(\frac{v_s}{v_p}\sin\theta)^2}}-2\frac{v_s}{v_p}\sin\theta\sqrt{1-\sin^2\theta}\right]\frac{\Delta J}{J} \quad (2)$$

Using Gardner’s relation, the following approximation is obtained (Wei et al.2008):

$$R_{ps}(x)=\left[-\frac{1}{5}\frac{x}{\sqrt{1-(\gamma x)^2}}+\frac{18}{5}\frac{\gamma^2x^3}{\sqrt{1-(\gamma x)^2}}\quad-\frac{18}{5}\gamma x\sqrt{1-x^2}\right]R_{ss0} \quad (3)$$

Where $R_{ss0}=0.5\Delta J/J$, $x=\sin\theta$ and $\gamma=v_s/v_p$。 If we keep three orders of x, and make $dR_{ps}(x)/dx=0$, extreme position x_{ext} and the extreme amplitude $R_{ps,ext}$ is:

$$x_{ext}=\sqrt{\frac{2(1+18\gamma)}{105\gamma^2+54\gamma}} \quad (4) \qquad \text{and} \qquad R_{ps,ext}=-\frac{R_{ss0}}{5}\left[\frac{(70\gamma+46)(1+18\gamma)}{105\gamma+54}\sqrt{\frac{2(1+18\gamma)}{105\gamma^2+54\gamma}}\right] \quad (5)$$

Figure 1 shows the variation of PS wave reflection coefficient with incidence angle. It is evident that the extreme (peak) position and amplitude are two important attributes for PS AVO analysis. We see from equation 5 that the extreme position x_{ext} is only a function of the average velocity ratio νp/νs and increases with that ratio. The extreme amplitude is a nonlinear function of the average velocity ratio νp/νs and the linear decreasing function of shear wave reflection coefficient Rss0. When the average velocity ratio is constant the extreme amplitude $R_{ps,ext}$ decreases with increasing Rss0. Furthermore, smaller average velocity ratios result in a larger gradient. For negative Rss0 , $R_{ps,ext}$ is positive and decreases with increasing average velocity ratio. For positive Rss0 , $R_{ps,ext}$ is negative and increases with average velocity ratio.

AVO fitting and inversion of converted wave

Because the average velocity ratio and shear wave reflection coefficient are directly inverted from the converted wave extreme attributes, it is necessary to measure the amplitude and position of the extreme from real pre-stack converted wave gathers. Equation 4 can be represented as:

$$y=ax^3+cx \quad (6)$$

Assuming that the PS wave AVO curve satisfies the power function, Equation 6 can be generalized to:

$$y=ax^b+cx \quad (7)$$

When coefficient b is different from three, Equation 7 includes effects caused by higher terms. To verify the accuracy of equation 7, the six models listed in table 1 are used to compare their goodness of fit with equation 1 and to analyze the inversion accuracy of reflection coefficient and average velocity ratio. Rock properties of models 1 and 4 are taken from Figure 2 and 3 of Ostrander (1984). For models 2 and 5, the relation of density and P wave velocity follows the Gardner relationship and the other parameters are the same as models 1 and 4. For models 3 and 6, Poisson’s ratios across the interface are the same and the other parameters are the same as models 2 and 5. The coefficient b in equation 7 considers the effects of the higher order terms in equation 3, so equation 7 not only improves the fitting precision using three coefficients instead of high order coefficients but also avoids multiple extremes due to the effect of noise on fitting high order polynomials.

The extreme position x_{ext} and amplitude $R_{ps,ext}$ for six models computed from (1) (Aki and Richard's exact expression) are fitted to the cubic function 6 and the power function 7 giving inversion results of average velocity ratio vp/vs and shear wave reflection coefficient, and the fitting error of the cubic and power functions. It is evident that: 1) the fit of extreme positions are all slightly small, but the fitting error of the power function is smaller than the cubic function (see Figure 2a); 2) the extreme amplitude fit to the power function is approximately equal to the accurate value, and the fitting errors are smaller than those of the cubic function (Figure 2b); and 3) the curve fitting error of the power function is less than that of the cubic function by at least one order of magnitude (Figure 2c).

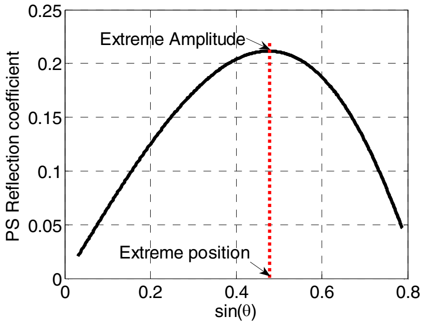
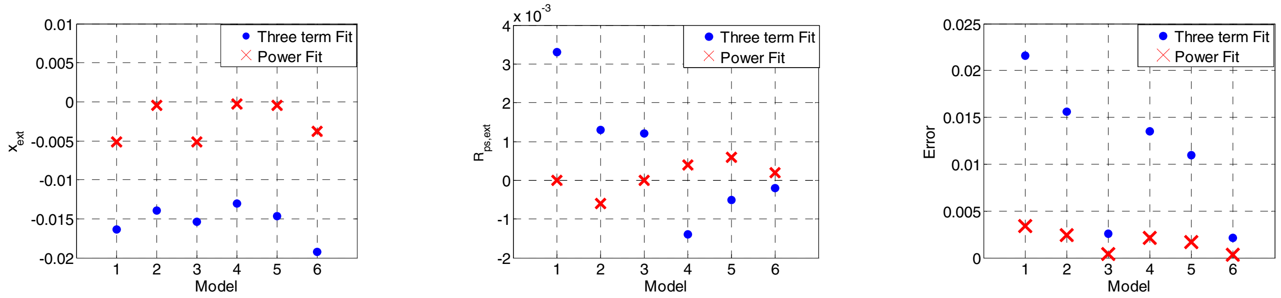


Figure 1. Variation of converted PS wave reflection coefficient with incidence angle.

Table 1 model parameters

	v_{p2}/v_{p1}	ρ_2/ρ_1	σ_1	σ_2
Model 1	1.25	1.25	0.3	0.1
Model 2	1.25	$1.25^{0.25}$=1.0574	0.3	0.1
Model 3	1.25	$1.25^{0.25}$=1.0574	0.3	0.3
Model 4	0.9	0.9	0.1	0.3
Model 5	0.9	$0.9^{0.25}$=0.9740	0.1	0.3
Model 6	0.9	$0.9^{0.25}$=0.9740	0.1	0.1



(a) (b) (c)

Figure 2. Comparison of the fit errors of extreme position (a), extreme amplitude (b), and curve (c) obtained from three term polynomial and power functions.

Inversion of Real data

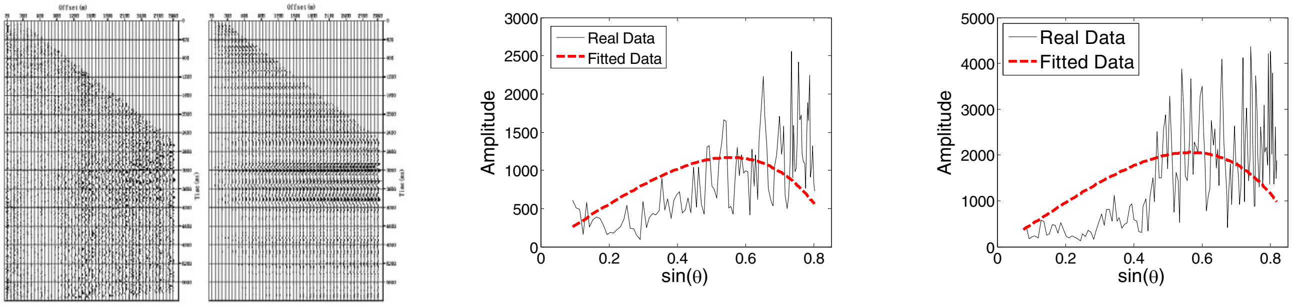
To verify the feasibility of power function fits to the converted wave AVO curve and inversion of average velocity ratio, real AVO curves of converted wave CCP gathers at a borehole position are analyzed. For near offsets, the CCP gather has small amplitudes, but for middle offsets (<2000 m) the CCP gather has large amplitudes and high S/N ratio (Figure 3a). First, data extracted from CCP gathers are transformed to angle functions. Then the power function is applied to fit these data. Figures 3c and 3d show the real data and fitted curves for layers N (2600 ms) and D (3100 ms) at the well (Figure 4a). Power function fitting not only smoothes the AVO curve, but also improves the seismic data resolution (Figures 3a and 3b). For layer N, the inverted average velocity ratio is 1.8355 compared to the well log value of 1.85. For layer D, the inverted average velocity ratio is 1.9 compared to the well log value of 1.9117, as shown in Figure 4. So the inversion results are viable. At last the method is applied to whole area for layer D. Figure 5 shows the areal distributive characteristics of Poisson ratio. According to the ratio of oil, gas and water, those wells are divided into four classes: water(blue),oil-water(gray),oil(red) and gas(green). The predicted Poisson's ratio map is in accordance with indicator of oil and gas got from statistic of drilling.

Conclusions

We propose the use of extreme attributes of position and amplitude for converted-wave AVO analysis as an alternative to the traditional method of intercept and gradient. Average velocity ratio and shear-wave reflection coefficient can be inverted from the extreme attributes, as demonstrated using real data.

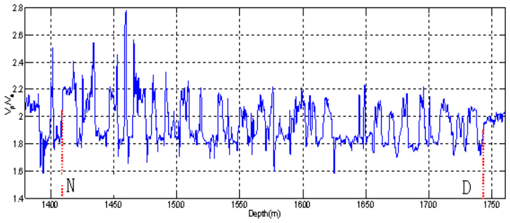
Acknowledgements

This work is funded by National 973 Basic Research Development Program of China (No.2005CB422104), 863 National High Technique Research Development Project of China(No. 2006AA06Z203 and No. 2007AA060505), and the SinoPec international collaboration program through the Edinburgh Anisotropy Project.

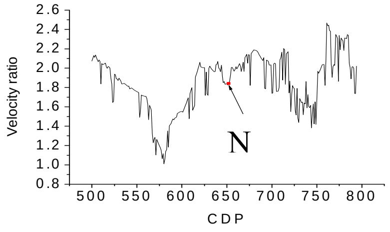


(a) Raw gather (b) Power function fit (c) Layer N (t = 2600 ms) (d) Layer D (t = 3100 ms)

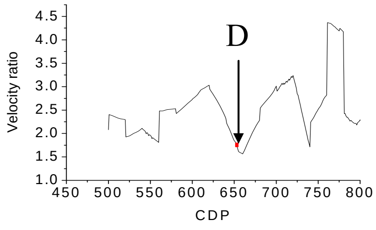
Figure 3. Comparison between a raw gather and the power function fit for a real CCP gather at well A.



(a) Velocity ratio computed from well logs at well A.



(b) Layer N (t = 2600 ms)



(c) Layer D (t = 3100 ms)

Figure 4. The inverted average velocity ratio for layers N and D across well A.

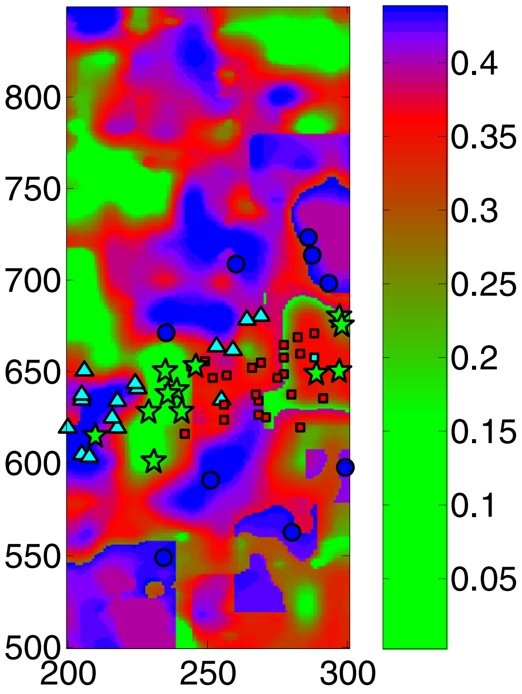


Figure 5. The distributive characteristics of Poisson's ratio inversion for layer D

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