Modelling Near-field Effects in VSP-based Q-estimation
Part 1: Theoretical Developments

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Summary

The complete spherical wavefield emanating from a P-wave point source surrounded by a homogeneous isotropic medium is computed with the aid of Weyl/Sommerfeld integrals. In a resulting synthetic VSP, we observe a near-field, a far-field and a 90° phase rotation between the two. Depth dependence of magnitude spectra in these two depth regions is distinctly different. Log magnitude spectra show a linear dependence on frequency in the far-field but not in depth regions where the near-field becomes significant. Near-field effects are one possible explanation for large positive and even negative Q-factors in the shallow section that may be estimated from VSP data when applying the spectral ratio method. A near-field compensation method for Q-estimation in homogeneous models reduces errors in Q-values except in the immediate vicinity of the source.

Introduction

Why is attenuation (Q) estimation still a research topic after all these years of investigation? Seismic quality factors Q are sought because Q-compensation increases resolution and Q could serve as a seismic attribute describing rock properties. What, then, are the practical difficulties encountered in attenuation (Q) estimation? The three main hurdles mentioned by Best (2007) are still with us. They are

1) restricted bandwidth,
2) poorly constrained multiple scattering, and
3) geometric spreading losses.

By contrast, the popular spectral ratio method of Q-estimation is derived for plane-waves propagating in a homogeneous medium. The seismic wavefield is composed of near-field as well as far-field contributions. We expect plane-wave theory to break down in the near-field, but what is the extent of this region and what is the size of Q-estimation errors there? It has been observed that VSP-based Q-estimates can give negative values for the near surface. For example Raikes and White (1984) noted negative Q and attributed their result to a change in geophone coupling. This possibility is included in our own list (Haase and Stewart, 2006) which comprises 1) acquisition issues, 2) analysis problems and 3) lithology model inadequacies.

In pursuing the topic of stratigraphic attenuation, we recently computed synthetic VSP data with the aid of Weyl/Sommerfeld integrals (Haase and Stewart, 2007) and noted some peculiar behavior of magnitude spectra as a function of depth in near-surface situations. The cause for this strange departure from far-field expectations turns out to be, not surprisingly, a near-field contribution to the total wave field of a P-wave point source. Also interesting is the behaviour of near-field magnitude spectra as a function of frequency. Kjartansson (1979) mentions that Q is mildly frequency dependent but then proceeds with a constant Q assumption. Frequency dependent near surface Q-factors have also been observed in the Alberta oil sands (K. Hedlin, 2007, personal communication).

The aim of this study is to show that near-field behavior is another possible explanation for negative Q-estimates and frequency-dependent Q in the near-surface. Part 1 presents theoretical developments. Modelling results are introduced in Part 2.

Sommerfeld Integral Review

From a wide variety of Q-estimation techniques developed over the years, the spectral ratio method (SRM) appears to be the most commonly used approach. The derivation of Q-estimation methods is usually based on the definition of the quality factor Q. For the spectral ratio method Tonn (1991) gives

$$\ln\left[\frac{A_1(\omega)}{A_0(\omega)}\right] = -\frac{\omega t}{2Q} \quad , \tag{1}$$

where the geometric spreading ratio is ignored and Q >> 1 is assumed.

In case of a homogeneous medium, Q-factors can be recovered exactly (within numerical accuracy) under far-field conditions when the source-receiver separation is large. What are these far-field conditions and when do they break down? Aki and Richards (1980) use the phrase "*many wave lengths*" when describing the far-field distance from a point source. They give an equation for the P-wave potential caused by a point source (Weyl/Sommerfeld integral approach) in a 3D homogeneous medium. P-wave displacements can be obtained from such potentials by computing gradients, which for the direct wave gives (Haase and Stewart, 2007)

$$u_P(\omega) = i\omega e^{-i\omega t}\int_0^\infty \left[\frac{p^2}{\xi}J_1(\omega pr)\sin(\theta) - ipJ_0(\omega pr)\cos(\theta)\right]e^{i\omega z\xi}\,dp \quad , \tag{2}$$

where $u_P(\omega)$ is the P-wave displacement along the ray at the current receiver and ω,

- ω is the frequency in radians,
- t is the time,
- p is the horizontal slowness,
- ξ is the vertical slowness,
- J_0 and J_1 are zero-order and first-order Bessel functions of the first kind,
- r is the range (horizontal offset between source and receivers),
- z is the source-to-receiver depth interval (to the current receiver) and
- θ is the incidence angle from the vertical (at the current receiver).

The P-wave displacements along the ray, computed with Equation 2, are obtained by integration over (always) real horizontal slowness p, one frequency point at a time. Because of finite Q-factors all velocities are complex; also the vertical slowness ξ is complex, even outside the evanescent region. The Bessel functions in Equation 2 are the result of, firstly, integrating over azimuth when deriving the Sommerfeld-integral from the Weyl-integral (describing the 3D nature of the problem by J_0) and, secondly, taking the gradient of potentials to obtain displacements (adding a J_1-term). The last term in Equation 2, that is $e^{i\omega z\xi}$, represents a travel-time term. As we will see below, displacements computed with Equation 2 are composed of near-field and far-field contributions. Plane-wave approximations are accurate only in the far-field; close to a source we encounter the near-field term and spherical spreading as well as distance dependent phase changes because of a near-field to far-field change over as demonstrated in Figure 1. The earth model parameters used for Figure 1 are $V_p = 2000$m/s, $V_s = 879.88$m/s, $\rho = 2400$kg/m^3 and $Q_p = 100$.

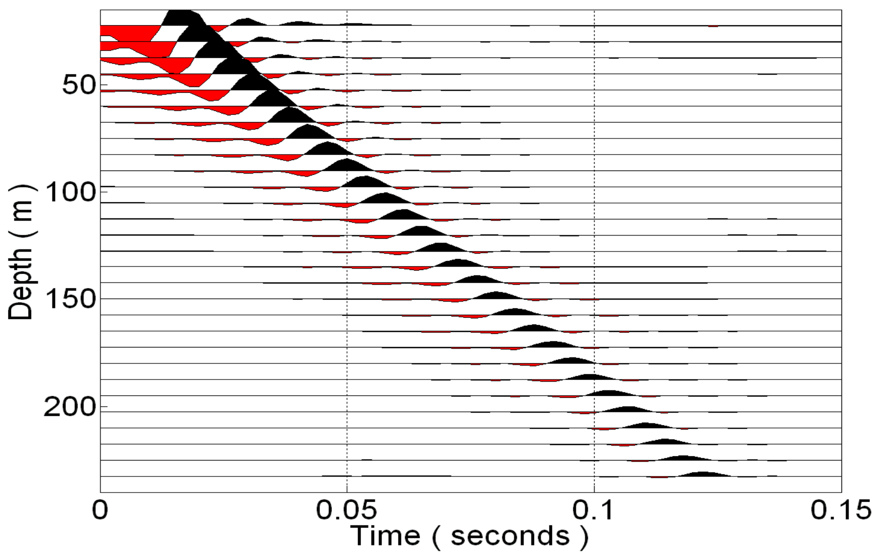

Figure 1: Zero-offset synthetic VSP computed with Equation 2 for a zero-phase (non-causal) 5/15-80\100 Hz Ormsby wavelet.

The zero-phase (non-causal) Ormsby wavelet employed for these computations has the parameters 5/15-80\100 Hz. Amplitude decay with increasing depth is clearly visible. Also note the phase rotation between shallower and deeper traces where deep traces are roughly zero-phase but shallow traces are ~90° phase rotated. As the near-field decays with $1/z^2$ (in contrast to $1/z$ for the far-field) its influence quickly disappears with increasing depth.

Zero-offset Simplifications

For the zero-offset (*zo*) VSP, the range *r* in both Bessel function arguments is zero. With $J_0(0) = 1$, $J_1(0) = 0$ and introducing $[p\, dp] = [-\xi\, d\xi]$ (obtained from $\xi^2 = 1/V_p^2 - p^2$ and $d\xi/dp = -p/\xi$, where V_p is P-wave velocity) ,we find

$$u_{P-zo}(\omega) = -\omega e^{-i\omega t} \int_{1/V_p}^{i\infty} \xi e^{i\omega z\xi} d\xi \quad , \tag{3}$$

which can be integrated analytically giving

$$u\Big|_{r=0} = \frac{i\omega}{V_p} e^{-i\omega t} \left[\frac{1}{z} + \frac{iV_p}{\omega z^2} \right] e^{i\omega z/V_p} = \frac{i\omega}{zV_p} e^{-i\omega t} \left[1 + \frac{iV_p}{\omega z} \right] e^{i\omega z/V_p} \,. \tag{4}$$

The first term in the brackets of Equation 4 is the familiar far-field term. It decays with *1/z*. The second term in brackets decays with $1/z^2$. Because of this more rapid decay, it is noticeable only in the near-field where it can dominate the far-field term at low frequencies thereby mimic a frequency dependent Q. For $\infty > Q >> 1$, there is an almost 90° phase difference between these two terms which should allow separation at least in this homogeneous modeling situation. Note that P-wave velocity V_p is complex for finite Q.

Near-field Compensation of Q-estimates

From Equation 4 it is clear that all amplitudes are multiplied by a factor of $(1+iV_p/\omega z)$. In the far-field where depth *z* is large, and/or at higher frequencies *ω*, this multiplier approaches unity. As a first step we try to precondition the input spectra prior to Q-estimation through division by this factor. Since the spectral ratio method operates on amplitudes, and ignores phase information, we proceed with the magnitude of this multiplier. Spectra thus compensated are introduced in Part 2 of this contribution.

Because the near-field multiplier of Equation 4 is frequency dependent, a more accurate approach to near-field Q-estimation involves non-linear curve fitting of log spectral ratios. This is in contrast to the straight line fitting normally applied for SRM Q-estimation. On introducing the near-field multiplier of Equation 4 into Equation 1 we obtain the modified log spectral ratio as

$$\ln\left[\frac{A_1(\omega)}{A_0(\omega)}\right] = (const.) - \frac{\omega t}{2Q} - \ln\left[\omega^2 z_0^2 z_1^2 + c^2 z_1^2\right]/2 + \ln\left[\omega^2 z_0^2 z_1^2 + c^2 z_0^2\right]/2 \qquad (5)$$

which will be used for non-linear curve fitting in Part 2 of this contribution.

Conclusions

The near-field of a P-wave point source decays faster with distance from the source than the far-field ($1/z^2$ versus $1/z$). It also decays with frequency. The combined near-field factor is $V_p/(\omega z)=\lambda/(2\pi z)$ and for $V_p/(\omega z)<<1$ far-field conditions apply. Near-field Q-estimates can be improved by near-field compensation which extends the useful Q-estimation range closer to the source point.

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