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Finite-difference Elastic Modelling Below a Structured Free Surface

P. M. Manning*
CREWES project, University of Calgary
pmmannin@ucalgary.ca

Summary

This paper shows experiments using a unique method of implementing a structured free surface boundary for a finite-difference model. The method employs the same concepts as the implicit solutions used for finite-difference formulations posed in a recursive manner. The displacements required across the free surface boundary are then found by a deterministic method where structure is locally flat, and by an implicit method where the structure is sloped, proceeding from low to higher elevations. The limitations to this technique are shown, and a practical example is given.

Introduction

When a finite-difference model is begun, one of the first choices that must be made is where the boundaries of the model space will be. With the exception of well to well models, most choices will have the surface of the earth as a top boundary. This is in contrast to the other three boundaries, which we wish would go away, and would put at infinity if we could.

The top boundary of the model must then represent a 'real' boundary, which is almost perfectly represented as a free surface because the air above the surface can transmit only tiny amounts of energy in the range characterizing seismic waves. This condition can be simulated by assuming displacements across the boundary which are not zero, but which when used in equations spanning the boundary result in zero stresses. This means zero shear stress and zero compressional stress. These conditions ensure that body waves are reflected from the surface accurately, but more noticeably, that the surface can propagate surface 'Rayleigh' waves.

An improvement in the realism of a finite-difference model would allow a structured free surface, simulating particular acquisition conditions. Here, the actual top of the model could still be flat, but the boundary conditions on the top would be irrelevant because the structured free surface would prevent any energy from entering and leaving the real model area.

Free surface boundary conditions, flat case

The two templates of the staggered grid displacements which will contribute to the accelerations toward the next model time step are shown in Figure 1. The calculated points will apply at the circled displacements at the centers of the templates.

An example of a staggered grid model with a flat free surface upper boundary is given in Figure 3. Here the area to be modelled is that covered by the black displacement arrows, where the surface is considered to be along the blue line at depth zero, and where the surface displacements in this case have been specified as vertical.

Placing the right (vertical) template of Figure 1 on these surface displacements shows that computations for the next time step will require inputs from two further rows of 'ghost' displacements above the actual surface. These displacements are shown in red.

The zero stress equations which apply to the two templates in Figure 2 are (from Levander, 1988)

$$\sigma_{xz} = \mu\left(\frac{\partial U_z}{\partial x} + \frac{\partial U_x}{\partial z}\right) = 0, \quad \sigma_{zz} = (\lambda + 2\mu)\frac{\partial U_z}{\partial z} + \lambda\frac{\partial U_x}{\partial x} = 0 ,$$

with the left equation applying to the left template, and the right equation to the right template.

Referring to Figure 3, a value for a ghost displacement on the lower row (at -2.5 m.) can be determined by using the left equation/template, where the shear stress must be zero. The set of values here may be combined with the surface displacements to find the vertical displacements (at -5.0 m.) by using the right equation/template, where the compressional stress must be zero.

Free surface boundary conditions, structured case

An example of a structured free surface boundary is shown in Figure 4. The black displacements cover the interior of the model, below the blue line representing the surface. Again, the red arrows show the ‘ghost’ displacements required to do time stepping near the surface. For this case, though, the calculation of these extra displacements must be done in an order dependent on the details of the structure, starting from the lower elevations, and progressing to higher elevations.

For an example from Figure 4, the left equation/template may be used to calculate the horizontal displacements at particular points, for example at offsets 2.5 and 87.5 metres. Similarly, the right equation/template may be used to project the vertical displacements at offsets of 75 and 80 metres. Once the lower level displacements have been calculated, the values there may be used within calculations for the next level of displacements, and so continue upwards until all the required displacements have been determined. This type of implicit solution is described in Strikwerda (2004), section 3.5.

Limitations of the method

Figure 5 shows the technique used on a model with the free surface sloped at 1 part in 6. Although there is some high frequency noise on the section, it shows a quite satisfactory wave oriented perpendicular to the surface. Figure 6 has a free surface sloped at 1 part in 5, and the noise here has become quite obtrusive.

The effects of a surface wave propagating on a flat versus a step-up in topography

Figure 7 shows a snapshot at 150 milliseconds after an explosive source was initiated on the left boundary of a flat free surface model, and it shows the common features seen in these cases. The P wave directly from the explosion advances along the surface and coincides there with a converted shear wave. At an offset about half the distance to this converted event (for Vp/Vs of 2) a compact surface wave progresses.

To contrast with this, Figure 8 shows events at the same time, with the same initiation, but with a free surface that rises 5 metres across an offset range of 50 to 60 metres. The surface wave has been broken up at deeper levels, but appears to maintain most of its energy at the more shallow levels. The unexpected event is the lesser but definite surface wave propagating at a position advanced from that seen in Figure 7.

These cases may also be compared by using the simulated seismic data collected on the model surface. Figure 9 shows the traces collected on the flat free surface where, aside from the first breaks, the only event is a surface wavelet similar to the explosive source propagating with almost no attenuation. Figure 10 shows the traces collected on the ‘step-up’ free surface, and this minor topographic feature has added two consistent new events to the record.

Discussion

The suggested technique as used in Figure 6 shows that a free surface with a long consistent slope can generate noise from the evenly spaced elevation steps. Despite this, some useful model studies can be done, as the later examples show.

From the flat free surface in Figure 9, projection of the surface wave back to zero offset shows that it originated within a very narrow offset range near the source. This also explains why the generated surface wavelet is almost as compact as the first break wavelet. Figure 7 shows that near the surface there is a consistent and stable relationship between the top of the original pressure wave and an advancing shear wave.

Figure 8 shows that the free surface ‘step-up’ disrupted the association of the pressure and shear body waves at that point, and new waves were generated before the association was re-established. Figure 10 shows that the new waves were a lagged set of first breaks, and an advanced set of surface waves.

Conclusions

The technique of defining zero stress conditions on a structured free surface in an implicit way is effective, but has limitations.

The particular example shown is an example of the possible studies using this technique.

References

Levander, A. R., 1988, Fourth-order finite-difference P-SV seismograms: Geophysics, 53, No. 11, 1425-1436.

Strikwerda, J. C., 2004, Finite difference schemes and partial differential equations: Society for Industrial and Applied Mathematics, Philadelphia, second edn.

Figures

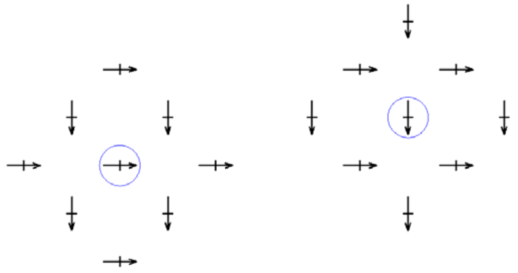

Figure 1: The templates of the displacements which contribute to the next time stepped displacements of: Ux (left), and Uz (right).

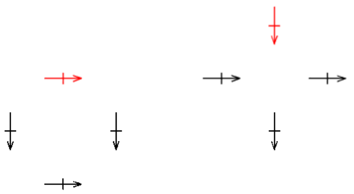

Figure 2: The templates showing displacements related to zero stress conditions: zero shear stress (left), and zero compressional stress (right). The red displacements are calculated.

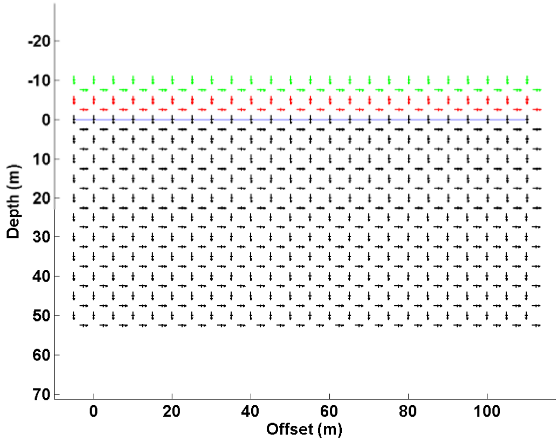

Figure 3: The flat free surface. The red displacements must be projected from the internal displacements using the zero stress equations and templates from Figure 2. The zero shear stress condition is applied first, then the zero compressional stress condition.

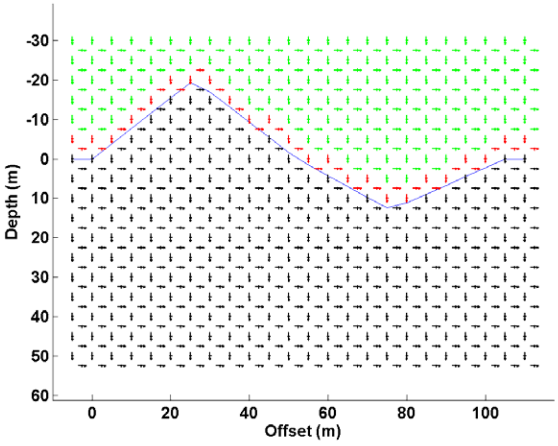

Figure 4: An example of a structured free surface, given by the blue line. The internal displacements are shown in black, and the required external displacements are in red. These must be determined in sequences dependent on the detailed topography.

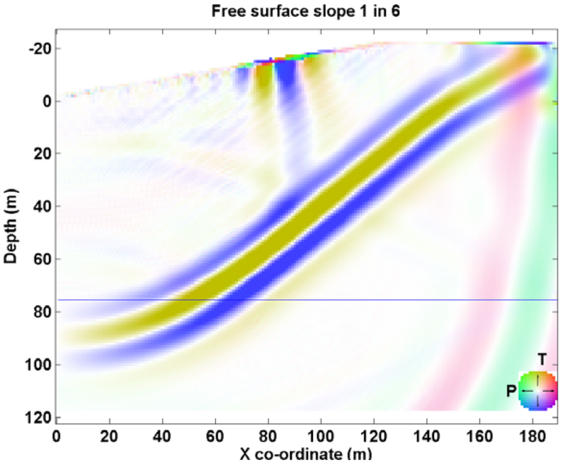

Figure 5: This Figure shows a satisfactory model for a free surface sloped by 1 part in 6. The irrelevant displacements above the free surface have been zeroed out. The surface wave at top-centre is perpendicular to the surface.

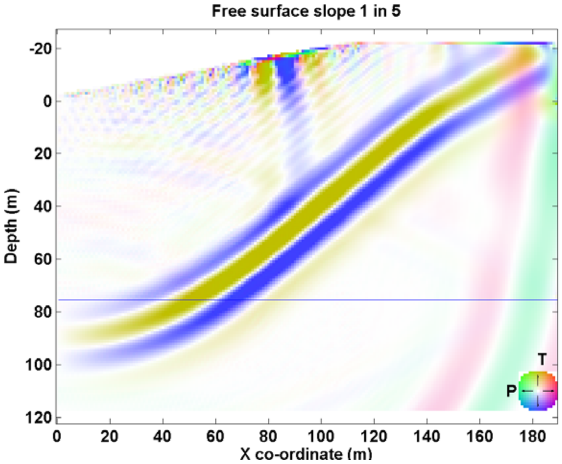

Figure 6: A model with a free surface slope of 1 part in 5. This shows that the algorithm introduced here has some limitations. The periodic nature of the constant slope seems to reinforce the instabilities generated from the surface.

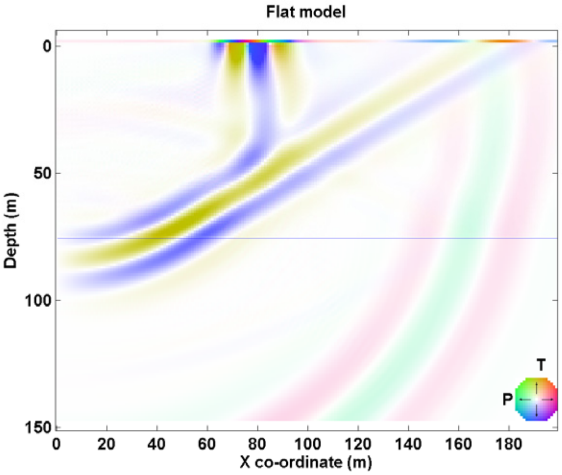

Figure 7: A snapshot of the flat free surface model at 150 ms. The smooth relationship between the shear wave and the surface wave may be seen.

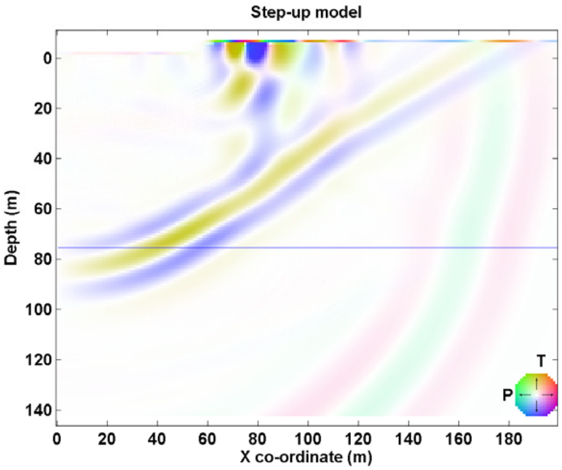

Figure 8: A snapshot of the step-up free surface model at 150 ms. The uppermost part of the surface wave includes the wave from Figure 5, but an additional leading wave has appeared.

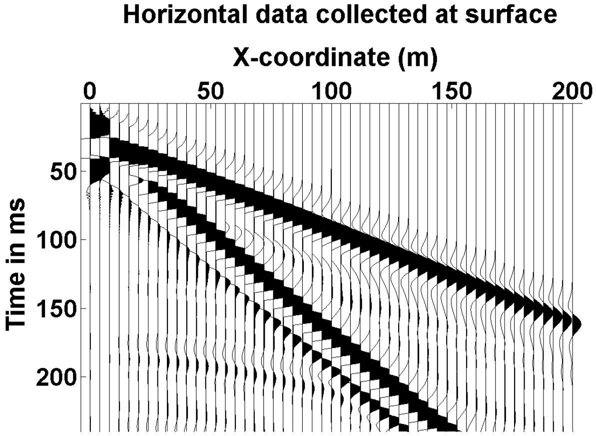

Figure 9: A seismic record collected on the free surface of the flat model.

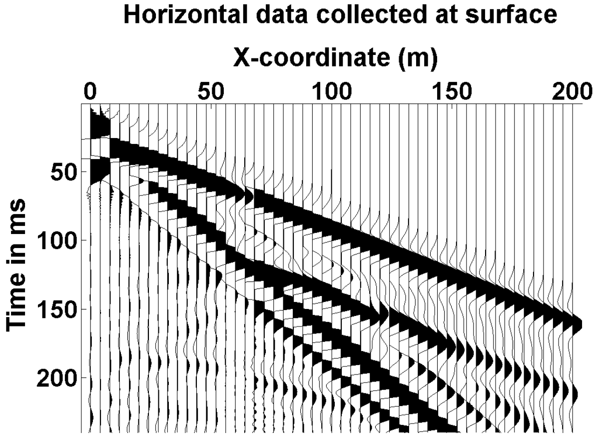

Figure 10: A seismic record collected on the free surface of the step-up model. The break at the surface has created a second set of first breaks and surface waves.