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Interpolation of Nonstationary Seismic Records using a Fast Generalized Fourier Transform

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Summary

We introduce a fast and efficient method for the interpolation of nonstationary seismic data. The method uses the fast generalized Fourier transform FGFT to identify the space-wavenumber evolution of nonstationary spatial signals at each temporal frequency. The nonredundant nature of FGFT renders a big computational advantage to this interpolation method. A least-squares fitting scheme is used next to retrieve the optimal FGFT coefficients representative of the ideal interpolated data. For randomly sampled data on a regular grid, we seek a sparse representation of FGFT coefficients to retrieve the missing samples. In addition, to interpolate the regularly sampled seismic data at a given frequency, we use a mask function derived from the FGFT coefficients of the low frequencies. Synthetic and real data examples can be used to examine the performance of the method.

Introduction

The problems of seismic data reconstruction and interpolation have attained a special stature in the seismic data processing community in recent years. Reconstruction methods use available seismic traces, measured on irregular and/or coarsely sampled grids in space, to estimate data on a regularly and sufficiently sampled grid. An effective solution can open the door to the application of multidimensional wave-equation imaging and demultipling algorithms, without having had to acquire the data sets with the completeness these methods demand. In a useful method of interpolation/reconstruction, we look for speed, stability in the presence of noise and aliasing, and the ability to manage complex events. In this paper, we propose an interpolation/reconstruction methodology that provides a constructive mixture of the above properties, and can be used for interpolation of nonstationary seismic events

In seismic data processing, nonstationarity means that the frequency/wavenumber content of the signal varies in time/space. For instance, an absorptive medium causes nonstationarity in the time dimension by making the frequency content of a seismic pulse a function of path length. In addition, seismic sections that contain hyperbolic and parabolic or any nonlinear events produce nonstationary spatial signals in the frequency-space f-x domain at a given frequency. Interpolation/reconstruction methods typically cope with nonstationary signals through spatial windowing. Inside sufficiently small spatial windows, nonlinear seismic events appear linear or stationary. Hence, methods that assume stationarity, such as those referenced above, might be applied.

The S-transform (Stockwell et al., 1996) is a type of short-time Fourier transform in which the window size is frequency dependent. The S-transform can be effectively utilized for the analysis of nonstationary data. Recent work on the S-transform has led to a fast, nonredundant algorithm (Brown et al., 2010), renewing the possibility of developing an efficient and effective interpolation/ reconstruction approach based on S-transform theory. In this paper, we develop such an approach and examine its behavior when applied to synthetic and field data sets. Because the transform algorithm is new, we will first describe a straightforward and intuitive frequency-domain computation of the fast generalized Fourier transform (FGFT). We will then combine the FGFT with a least-squares fitting principle to formulate our FGFT interpolation method. Synthetic and field data examples are examined.

Fast Generalized Fourier Transform

The details of FGFT can be found in Brown et. al. (2010) and Naghizadeh and Innanen (2011). The FGFT is considered as a non-redundant S-transform. Figure 1 illustrates schematically an application of the FGFT algorithm. Figure 1a represents a time signal containing 16 samples. A fast Fourier transform is applied to the time signal to obtain the frequency-domain representation illustrated in Figure 1b. In the frequency domain, the signal is dyadically segmented (dashed boxes), and within each segment inverse Fourier transforms are applied to the data. The output is illustrated in Figure 1c. Each individual inverse Fourier transform is represented by a particular symbol in Figure 1c (square, diamond, triangle, circle). Next, to represent properly the time-frequency behavior of the data, the underbraced FGFT coefficients must be arrayed in a 2D plot. Figure 1d illustrates this arrangement of FGFT coefficients. Each element of a given inverse Fourier transform is distinguished from the others in that group via size. Hence each FGFT coefficient is uniquely represented by a symbol type and size. Figure 1d illustrates how these outputs are distributed. We note that there is better time resolution in the high frequencies and better frequency resolution in the low frequencies.

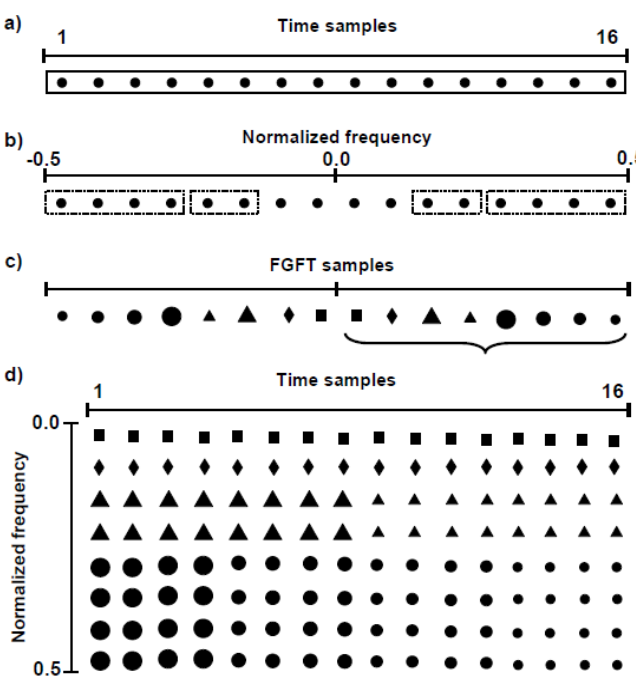

Figure 1: Graphic representation of implementing FGFT. a) Original signal with 16 time samples. b) Fourier transform of original data in a. c) The FGFT representation of data after applying inverse Fourier transform on each dashed box of data in b. d) The time-frequency interpretation of FGFT coefficients in c only for positive frequencies.

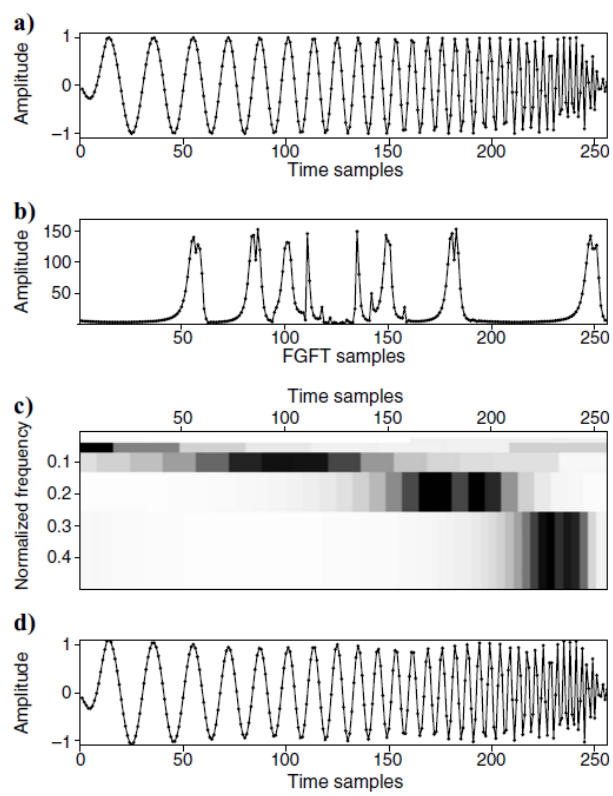

Figure 2: a) Original chirp signal. b) The FGFT coefficients of (a). c) A 2D plot of FGFT coefficients clearly showing the time-frequency distribution of the chirp signal. d) The recovered chirp signal by applying inverse FGFT on b.

Figure 2a shows the a chirp function. Figure 2b shows the FGFT of Figure 2a. Figure 2c illustrates the 2D array of FGFT coefficients after proper upscaling, i.e., the time-frequency decomposition of the chirp. Because the original chirp input is real, we include only the positive frequencies for this example. The FGFT evidently captures the nonstationary nature of the chirp function. The low frequencies predominate at the beginning of the signal and high frequencies predominate at the end. Figure 2d illustrates the adjoint FGFT acting on the FGFT coefficients of the chirp function. The adjoint FGFT recovers the original data within a small error level produced by the windowing step.

FGFT interpolation of seismic data

The interpolation problem is underdetermined, and hence to solve it we require some prior information. To provide this, let us consider the FGFT coefficients **g** of the desired, fully sampled signal. These must be related to **d** by

$$\mathbf{g} = \mathbf{G}\mathbf{d}, \qquad [1]$$

where **G** represents the forward FGFT operator. The adjoint FGFT operator **G** can furthermore be used to express the desired interpolated data **d** in terms of **g** as follows:

$$\mathbf{d} \approx \mathbf{G}^T\mathbf{W}\mathbf{g}, \qquad [2]$$

where we have introduced a diagonal weight function **W** that preserves a subset of FGFT coefficients. The desired data **d** is related to observed data by sampling function **T**

$$\mathbf{d}_{\text{obs}} \approx \mathbf{T}\mathbf{G}^T\mathbf{W}\mathbf{g}. \qquad [3]$$

The system of equations in equation 3 is underdetermined , and therefore it admits an infinite number of solutions. A stable and unique solution can be found by minimizing the following cost function

$$J = \|\mathbf{d}_{\text{obs}} - \mathbf{T}\mathbf{G}^T\mathbf{W}\mathbf{g}\|_2^2 + \mu^2\|\mathbf{g}\|_2^2, \qquad [4]$$

where the cost function J is minimized using the method of conjugate gradients.

For FGFT interpolation of regularly sampled seismic data, the following steps are in order:

1. Transform the original data from the times-pace (t-x) to the f-x domain.

2 Compute the FGFT of the data at a given frequency $d(f)$ along all spatial axes, to obtain $g(f)$.

3 Create the weight function $\mathbf{W}(f)$ with one for coefficients larger than a threshold value and zero elsewhere.

4 For the frequency $f'=2f$, interleave zero values between available spatial samples to obtain $\mathrm{d}_{\text{dec}}(f')$.

5 Upscale the weight function $\mathbf{W}(f)$ to fit the size of $\mathrm{d}_{\text{dec}}(f')$ and create the new weight function$\mathbf{W}'(f')$. The upscaling operator is a simple nearest-neighbor interpolation scheme.

6 Use equation 4 to reconstruct the missing samples of $\mathrm{d}_{\text{dec}}(f')$.

7 Repeat steps 2-6 for all frequencies.

8 Transform the reconstructed f-x data to the t-x domain.

Example

To exemplify the procedure, in Figure 3a we illustrate a synthetic seismic section composed of three hyperbolic events with 81 traces. Next we decimate the original data to obtain the decimated seismic section in Figure 3b with 41 traces. The FGFT interpolation of the decimated data is shown in Figure 3c. Figures 3d-f represents the f-k panels of data in Figure 11a-c, respectively. This underscores an important property of the FGFT interpolation method, which is the ability to cope with severely aliased energy in interpolating the decimated data. The f-k spectra of the interpolated data contain some artifacts because of the compromise made in the FGFT method to gain speed instead of resolution.

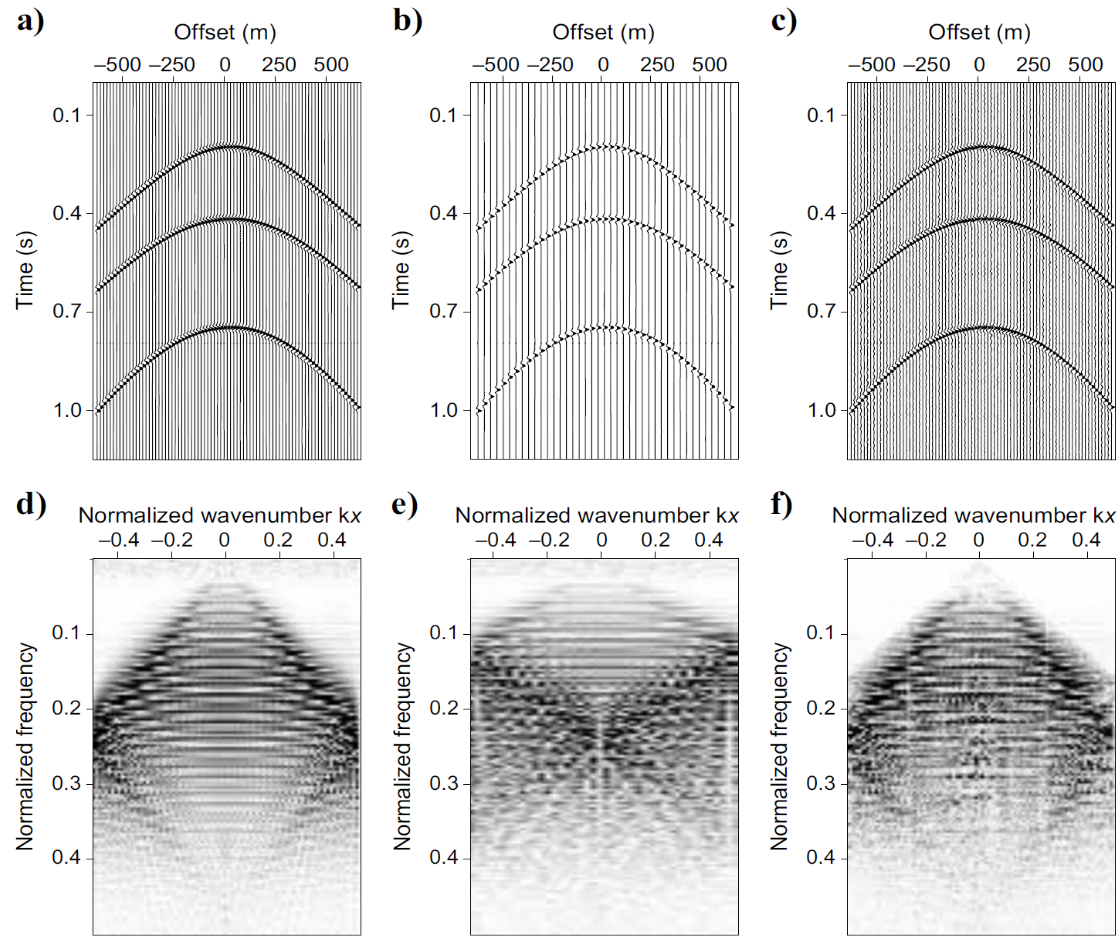

Figure 3: a) Original synthetic seismic section with three hyperbolic events. b) The seismic section after decimating every other trace. c) Reconstructed data using the FGFT interpolation method. d–f) The *f-k* representations of (a) through (c).

Conclusions

The FGFT is a fast and efficient way of analyzing nonstationary signals and identifying their time-frequency evolution. We use the FGFT inside a least-squares fitting algorithm to interpolate nonstationary seismic data. The method has the ability to cope with rapid and local changes of dip information in the seismic data. For regularly sampled data, the FGFT interpolation method uses the low-frequency portion of data for beyond-alias reconstruction of the high frequencies. The proposed method is very fast and less demanding on computational memory compared to the alternative methods.

Acknowledgements

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