

Investigating physical mechanisms of seismic anelasticity

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Summary

Anelasticity in rocks is a very important property responsible for seismic attenuation. Currently attenuation is quantified by the rocks quality factor Q , which is defined empirically and is often frequency dependent. It is also often difficult to compare when inferred from different types of experiments or for different wave types. Here, we propose an explanation of anelasticity based on real, intrinsic physical parameters that are not frequency dependent by nature. The frequency dependence of Q is explained by non-linear solid viscosity, thermoelasticity, as well as other effects. The model is developed by using a mechanical approach based on Lagrangian mechanics and thermodynamics. To illustrate the model, we use recent stress/strain phase-lag data for Plexiglas. The absorption peak measured in these data is explained as caused by comparable contributions from solid viscosity combined with thermoelastic effects.

Introduction

Measurement of seismic-wave attenuation is one of the most important geophysical approaches for characterization of the composition and physical state of the rock. Granularity, fluids, fractures, and a number of other internal factors cause anelastic behaviour of Earth materials (e.g. Nowick and Berry, 1972; Mavko and Nur, 1975). However, the relation of the observed wave attenuation to the *in situ* anelasticity is neither simple nor straightforward.

The goal of this research is to accurately describe the physical processes required to explain some results seismic attenuation observations in the lab, such as strain-stress phase lags and creep. The basic idea is to use first-principle physics to analytically and numerically simulate several typical attenuation measurements in the lab. Such experiments often include two cylinders placed in series, one being an anelastic rock sample and the other being a known elastic standard (Figure 1). A torsional or axial sinusoidal force is applied to the standard and through it, to the rock sample. Due to the anelasticity of the rock sample, its response will lag the elastic standard by a certain phase angle, ϕ . This phase lag will give us insight into the nature of the anelasticity of the rock sample. A recent example of such phase lags in a compression experiment (Tisato et al., 2009) is shown in Figure 2.

The conventional interpretation of phase lags consists in viewing $\tan\phi$ as an intrinsic measure of rock anelasticity (Jackson and Paterson, 1993; Lakes, 2009). However, in this study, we note that ϕ is still a phase lag between two parts of the apparatus, and attempt explaining it in terms of basic physical phenomena, such as viscosity and thermoelasticity. We attempt explaining two key observations from Figure 2: 1) the nearly frequency-independent $\tan\phi$ with frequency; and 2) the ~20% “absorption peak” in $\tan\phi$ near ~3 Hz.

Theory

Seismic anelasticity are conventionally described by the quality factor of the material, usually denoted Q . This factor was introduced from an analogy with a mechanical (acoustical) or electric resonator (Knopoff, 1964). However, there exist several ways to carry out this analogy, and also several ways to measure the Q . For forced oscillations, this factor is defined so that its inverse, Q^{-1} , gives the relative

amount of damping in the system. In seismology Q is usually defined as a ratio of peak elastic energy in the system E to energy dissipated during one cycle ΔE (Aki and Richards, 2002),

$$Q = 2\pi \frac{E}{\Delta E} . \quad (1)$$

However, neither E nor ΔE are directly observable, and several methods of arriving at Q from observational parameters exist. The following method is derived from the phase lag ϕ between the driving force and response in forced oscillation experiments conducted far from resonance, typically at $\omega \ll \omega_0$ (e.g., Jackson and Paterson, 1993):

$$Q_{phase} = \frac{1}{\tan \phi} . \quad (2)$$

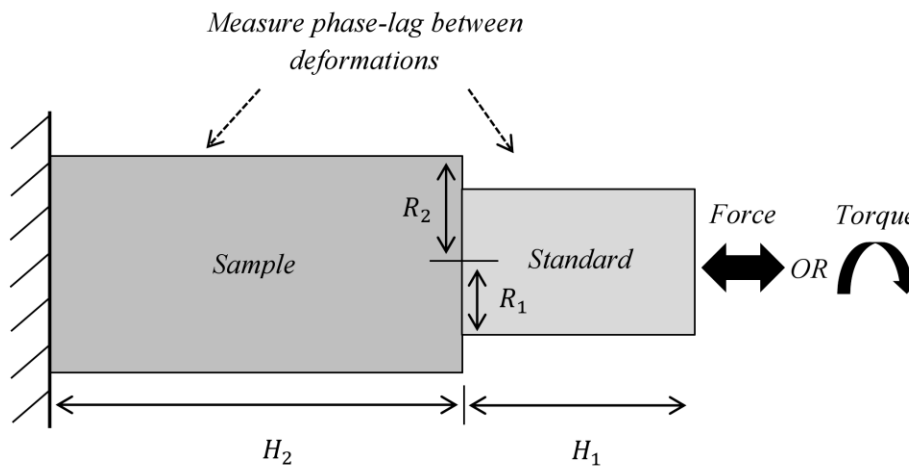


Figure 1: Scheme of an apparatus for measuring phase-lag Q .

Below, we outline different, rigorous relations of the above quantities with physical properties of realistic mechanical systems.

Solid viscosity

To describe the viscosity of solid rocks, we use the Lagrangian method of analytical mechanics. This approach is based on the Lagrangian given by $L = T - V$, where T and V are the kinetic and potential energies of the system, respectively. For an extended elastic system, such as shown in Figure 1, the Lagrangian is (Landau and Lifshitz, 1976b):

$$L = \int_V \left(\frac{\rho}{2} \dot{u}_i \dot{u}_i - \frac{K}{2} \Delta^2 - \mu \tilde{\varepsilon}_{ij} \tilde{\varepsilon}_{ij} \right) dV , \quad (3)$$

where \mathbf{u} is the displacement vector, $\tilde{\varepsilon}$ is the deviatoric strain tensor, Δ is the dilatational strain, K and μ are bulk and shear moduli, and over-dots represent time derivatives. Energy dissipation is described by the dissipation function:

$$D = \int_V \left(\frac{\eta_\Delta}{2} \dot{\Delta}^2 + \eta_\epsilon \dot{\epsilon}_{ij} \dot{\epsilon}_{ij} \right) dV, \quad (4)$$

where η_Δ and η_ϵ are the bulk and shear “solid viscosities” of the medium (Landau and Lifshitz, 1976a). The quadratic form of the dissipation function is necessary to produce linear differential equations; however, geophysical observations indicate otherwise. The above dissipation function can simply be generalized as follows:

$$D = \int_V \left[\eta_\Delta \left(\frac{\dot{\Delta}^2}{2} \right)^\nu + 2\eta_\epsilon \left(\frac{\dot{\epsilon}_{ij} \dot{\epsilon}_{ij}}{2} \right)^\nu \right] dV. \quad (5)$$

When $\nu=1$, the dissipation function reduces to the linear case in eq. (4). A power of 1.0 corresponds to fluid viscosity found in a Newtonian fluid, while a power of 0.5 corresponds to dry friction. Results from free oscillations of the Earth suggest a power between 0.5–0.6 for Earth materials (Morozov, unpublished). Results closer to dry friction appear intuitive as solids are “drier” than liquids.

Thermoelasticity

Thermoelastic effects represent another important cause of energy dissipation. Compression of the cylinder will cause a reduction in volume and an increase in temperature. Relaxation will cause an increase in volume and a decrease in temperature. During each of these time intervals, the heat will be redistributed, adding to the mechanical dissipation described above. Even for a perfectly uniform sample, the associated heat should release into the environment during low-frequency cycling of deformation. Two end-member regimes of thermal relaxation in heterogeneous bodies are relevant to

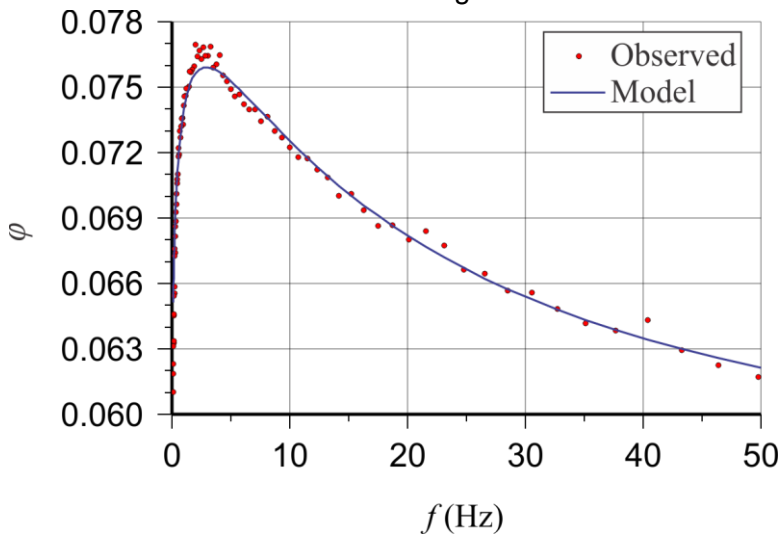


Figure 2: Observed (Tisato et al., 2009) and our modeled phase lag results for Plexiglas.

lab measurements (Landau and Lifshitz, 1976a). For very low frequencies, the temperature equilibrates within each period of oscillation, and the oscillation occurs nearly isothermally. In this case, $\tan \phi \propto \omega$, similarly to the case of regular linear viscosity. For intermediate frequencies, equilibration takes place by means of “temperature waves” across the boundaries of heterogeneities (such as the contact between the specimen and standard in Figure 1). In such cases, $\tan \phi \propto 1/\sqrt{\omega}$.

Results

Because of its sensitivity to heterogeneity, thermoelastic dissipation is the most likely cause of the absorption peak in Figure 2. At higher frequencies, it produces a $\tan \phi \propto 1/\sqrt{\omega}$ decay, and at lower frequencies, viscosity dominates and gives $\tan \phi \propto \omega^{2\nu-1}$ for dissipation function (5). A combination of

Table 1: Model parameters and resulting viscosity values.

Standard (Aluminum)		Sample (Plexiglas)	
Parameter	Value	Parameter	Value
H_1	0.080m	H_2	0.250m
R_1	0.033m	R_2	0.076m
ρ_1	2700kg/m ³	ρ_2	1180kg/m ³
γ_1	0.334	γ_1	0.35
μ_1	26.0GPa	μ_2	1.15GPa
E_1	70.0GPa	E_2	3.10GPa
η_1	0 Pa s	η_2	650 Pa s
ν_1	0	ν_2	0.5618

See Figure 1 for parameter information.

non-linear viscosity and thermoelastic effects, with parameters shown in Table 1, yielded the fit to experimental data shown in Figure 2. The contribution from both thermoelasticity and rheology (anelasticity) in phase lags are nearly equally significant, as shown in Figure 3.

Modeling results also show that $\tan \phi$ does not depend on the dimensions of the specimen while at the same time, $\tan \phi$ is proportional to the oscillation frequency for linear dissipation. This parameter is generally proportional to the viscosity and inversely proportional to the elastic modulus of the specimen.

These observations should be significant for interpreting the Q data measured in the lab, which are often taken as directly related to the *in situ* Q of the materials.

A comparison of these results to literature Q^{-1} data (weakly frequency dependent) suggests that dissipation in most solids is non-linear and involves non-mechanical, thermoelastic effects.

Conclusions

Instead of a frequency dependent Q value commonly used to characterize anelasticity of materials, we propose two physical parameters - the solid viscosity η and power-law exponent of non-linear viscosity ν . Fitting with a non-linear dissipation function and including a thermoelastic peak produced results closely corresponding to lab Q^{-1} data in Plexiglas. The resulting value of ν is near 0.56, suggesting a “near-dry” internal friction within the material. A significant portion of the phase lag results, and particularly the absorption peak, are interpreted as due to thermoelastic effects.

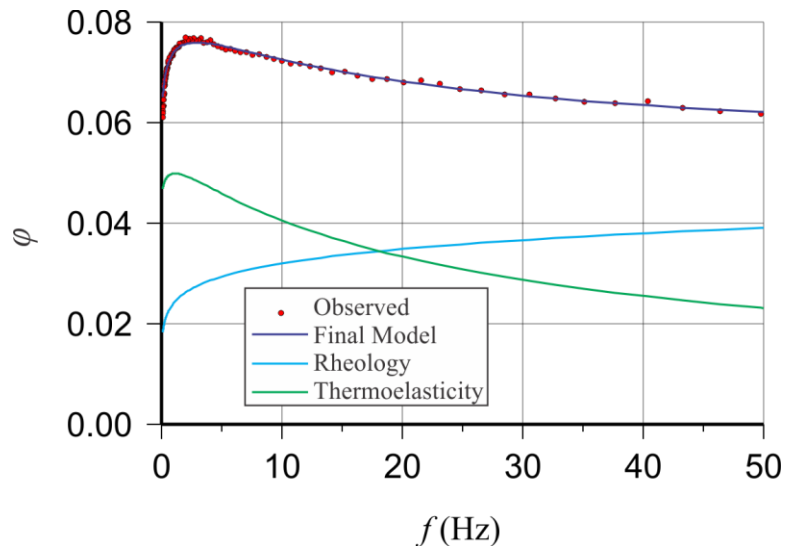


Figure 3: Contributions of the viscous and thermoelastic dissipations in modeled phase lag.

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