Exact, linear and nonlinear AVO in terms of poroelastic parameters

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Summary

AVO is a tool that can model amplitudes of wave signatures that reflect across an interface. These amplitudes can be modeled using the Zoeppritz equations for reflected P- and S-waves that are exact for a two-layer elastic model. In this paper, we will compare a derivation for poroelastic reflection coefficients due to Russell et al. (2011) with an approach based on a modification of the exact Zoeppritz equations. Before this comparison is made, a brief discussion of our method will be provided, in which we focus on how linear and nonlinear poroelastic reflection coefficients for P-waves are derived. Numerical examples will also be explored with these approximations.

Introduction

Russell et al's. (2011) research introduces a linearized poroelastic approximation that uses reflectivity models in $\Delta f/f$, $\Delta\mu/\mu$, and $\Delta\rho/\rho$. We will refer to this equation as Russell and Gray's formula. As explained in the report, this approximation is derived from the Aki and Richards' approximation. Our research has also proposed a method to derive an approximation with poroelastic models as mentioned above, as well as to extend the approximation into nonlinearity. We will show our derivation beginning from the Zoeppritz equations by re-creating a first-order poroelastic approximation. This approximation will then be compared to Russell and Gray's formulation for consistency. We will then show the second and third-order approximations using examples.

Poroelasticity: Review

Poroelastic terms distinguish between geological layers that contain fluids and those which are fluid-free. For instance, $(V_P)_{\text{dry}}$ describes the compression wave speed of a medium that contains no fluid. This will allow us to differentiate between poroelastic velocities (saturated) and elastic velocities (dry). The theory of poroelasticity due to Biot (1949) and Gassmann (1951) leads to the forms for the equations for $(V_P)_{\text{dry}}$ and $(V_S)_{\text{dry}}$. A fluid term (f) accounts for the difference between poroelastic and elastic velocities. The equations for $(V_P)_{\text{sat}}$ and $(V_S)_{\text{sat}}$ are

$$(V_P)^2_{\text{sat}} = \frac{K_{\text{dry}} + (4/3)\mu_{\text{dry}} + f}{\rho_{\text{sat}}}, \tag{1}$$

and

$$(V_S)^2_{\text{sat}} = \frac{\mu_{\text{sat}}}{\rho_{\text{sat}}}, \tag{2}$$

where λ is one of Lamé's constants, μ represents shear modulus, K represents bulk modulus, ρ is density, and f is the fluid term. Note that shear velocity does not include the fluid term. Without going into too much detail, this occurs due to the fact that the fluid term does not affect shearing motion and also knowing that $\mu_{\mathrm{sat}} = \mu_{\mathrm{dry}}$. The derivation of this occurrence is better illustrated by Russell et al. (2011). This fluid term is measured by the Biot coefficient α and a poroelastic constant M where

$$f = \alpha^2 M \,, \tag{3}$$

$$\alpha = 1 - \frac{K_{\mathrm{dry}}}{K_{\mathrm{m}}} \,, \tag{4}$$

and

$$M = \left(\frac{\alpha - \phi}{K_{\mathrm{m}}} + \frac{\phi}{K_{\mathrm{fl}}} \right)^{-1} \,, \tag{5}$$

where K_{dry} is the drained bulk modulus, K_{m} is the mineral bulk modulus, K_{fl} is the fluid bulk modulus, and ϕ is the porosity.

Exact, linear, and nonlinear poroelastic AVO

Our purpose is to re-derive AVO expressions along the lines of those of Russell et al. (2011), and then analyze any differences. To do so, we define model parameters that measure the contrast of a two-layer model. These will be perturbations in fluid, shear modulus, and density and will be defined as

$$a_f = 1 - \frac{f_0}{f_1}, \qquad a_\mu = 1 - \frac{\mu_0}{\mu_1}, \qquad a_\rho = 1 - \frac{\rho_0}{\rho_1} \quad , \tag{6}$$

These perturbations will then be substituted into ratios found in the Zoeppritz equations (ratios of V_P, V_S, and ρ across the reflecting boundaries). Using equations (1), (2), and (6), we have

$$\left(\frac{V_{S_1}^2}{V_{S_0}^2} \right) = \left(\frac{\mu_1}{\rho_1} \right) \left(\frac{\rho_0}{\mu_0} \right) = \left(1 - a_\mu \right)^{-1} \left(1 - a_\rho \right), \tag{7}$$

for the shear wave velocity ratio,

$$\left(\frac{\rho_1}{\rho_0} \right) = \left(1 - a_\rho \right)^{-1}, \tag{8}$$

for the density ratio, and

$$\left(\frac{V_{P_1}^2}{V_{P_0}^2} \right) = \left(1 - a_\rho \right) \left[\left(\frac{\gamma_{\mathrm{dry}}^2}{\gamma_{\mathrm{sat}}^2} \right) \left(1 - a_\mu \right)^{-1} + \left(1 - \frac{\gamma_{\mathrm{dry}}^2}{\gamma_{\mathrm{sat}}^2} \right) \left(1 - a_f \right)^{-1} \right], \tag{9}$$

for the compression wave velocity ratio where γ_{dry} and γ_{sat} represent the dry and saturated V_P/V_S ratios respectively. These ratios are then substituted into the Zoeppritz equations but let us write these equations first before making the substitution. We can observe density and velocity ratios in the Zoeppritz equations. With equations (7)-(9), we can make substitutions into the Zoeppritz equations and present these equations as defined by Keys (1989) in their substituted form

$$\begin{bmatrix} A_{11} & A_{12} & A_{13} & A_{14} \\ A_{21} & A_{22} & A_{23} & A_{24} \\ A_{31} & A_{32} & A_{33} & A_{34} \\ A_{41} & A_{42} & A_{43} & A_{44} \end{bmatrix} \begin{bmatrix} R_{\mathrm{PP}} \\ R_{\mathrm{PS}} \\ T_{\mathrm{PP}} \\ T_{\mathrm{PS}} \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix} \tag{10}$$

where the elements A_{ij} for the first row are defined as

$$A_{11} = -\sin\theta ,$$

$$A_{12} = -\left[1 - \frac{1}{\gamma_{\text{sat}}^2}\sin^2\theta\right]^{1/2} ,$$

$$A_{13} = \left(\frac{\gamma_{\text{dry}}^2}{\gamma_{\text{sat}}^2}\frac{\left(1-a_\rho\right)}{\left(1-a_\mu\right)} + \left(1-\frac{\gamma_{\text{dry}}^2}{\gamma_{\text{sat}}^2}\right)\frac{\left(1-a_\rho\right)}{\left(1-a_f\right)}\right)^{1/2} \sin\theta ,$$

$$A_{14} = -\left[1 - \frac{\left(1-a_\mu\right)^{-1}\left(1-a_\rho\right)}{\gamma_{\text{sat}}^2}\sin^2\theta\right]^{1/2} ,$$

the elements of the second row are

$$A_{21} = \left[1-\sin^2\theta\right]^{1/2} ,$$

$$A_{22} = -\left(\frac{1}{\gamma_{\text{sat}}}\right)\sin\theta ,$$

$$A_{23} = \left[1 - \left\{\left(1-a_\rho\right)\left[\left(\frac{\gamma_{\text{dry}}^2}{\gamma_{\text{sat}}^2}\right)\left(1-a_\mu\right)^{-1} + \left(1-\frac{\gamma_{\text{dry}}^2}{\gamma_{\text{sat}}^2}\right)\left(1-a_f\right)^{-1}\right]\right\}\sin^2\theta\right]^{1/2} ,$$

$$A_{24} = \frac{\left(\left(1-a_\mu\right)^{-1}\left(1-a_\rho\right)\right)^{1/2}}{\gamma_{\text{sat}}}\sin\theta ,$$

the elements of the third row are

$$A_{31} = \frac{2}{\gamma_{\text{sat}}^2}\sin\theta\left[1-\sin^2\theta\right]^{1/2} ,$$

$$A_{32} = \left(\frac{1}{\gamma_{\text{sat}}}\right)\left[1-2\left(\frac{1}{\gamma_{\text{sat}}^2}\right)\sin^2\theta\right] ,$$

$$A_{33} = 2\left(1-a_\rho\right)^{-1}\left(\frac{\left(1-a_\mu\right)^{-1}\left(1-a_\rho\right)}{\gamma_{\text{sat}}^2}\right)\sin\theta\left[1-\left(\left(1-a_\rho\right)\left[\left(\frac{\gamma_{\text{dry}}^2}{\gamma_{\text{sat}}^2}\right)\left(1-a_\mu\right)^{-1} + \left(1-\frac{\gamma_{\text{dry}}^2}{\gamma_{\text{sat}}^2}\right)\left(1-a_f\right)^{-1}\right]\right)\sin^2\theta\right]^{1/2}$$

,

$$A_{34} = -\left(1-a_\rho\right)^{-1}\left(\frac{\left(1-a_\mu\right)^{-1}\left(1-a_\rho\right)}{\gamma_{\text{sat}}^2}\right)^{1/2}\left[1-2\left(\frac{\left(1-a_\mu\right)^{-1}\left(1-a_\rho\right)}{\gamma_{\text{sat}}^2}\right)\sin^2\theta\right] ,$$

the elements of the fourth row are

$$A_{41} = -\left[1-2\left(\frac{1}{\gamma_{\text{sat}}^2}\right)\sin^2\theta\right] ,$$

$$A_{42}=2\left(\frac{1}{\gamma_{\text{sat}}^{2}}\right)\sin\theta\left[1-\left(\frac{1}{\gamma_{\text{sat}}^{2}}\right)\sin^{2}\theta\right]^{1/2},$$

$$A_{43}=\left(1-a_{\rho}\right)^{-1}\left\{\left(1-a_{\rho}\right)\left[\left(\frac{\gamma_{\text{dry}}^{2}}{\gamma_{\text{sat}}^{2}}\right)\left(1-a_{\mu}\right)^{-1}+\left(1-\frac{\gamma_{\text{dry}}^{2}}{\gamma_{\text{sat}}^{2}}\right)\left(1-a_{f}\right)^{-1}\right]\right\}^{1/2}\left[1-2\left(\frac{\left(1-a_{\mu}\right)^{-1}\left(1-a_{\rho}\right)}{\gamma_{\text{sat}}^{2}}\right)\sin^{2}\theta\right],$$

$$A_{44}=2\left(1-a_{\rho}\right)^{-1}\left(\frac{\left(1-a_{\mu}\right)^{-1}\left(1-a_{\rho}\right)}{\gamma_{\text{sat}}^{2}}\right)\sin\theta\left[1-\left(\frac{\left(1-a_{\mu}\right)^{-1}\left(1-a_{\rho}\right)}{\gamma_{\text{sat}}^{2}}\right)\sin^{2}\theta\right]^{1/2},$$

and the elements of the vector on the right-hand side are

$$b_{1}=\sin\theta,$$

$$b_{2}=\left[1-\sin^{2}\theta\right]^{1/2}$$

$$b_{3}=2\left(\frac{1}{\gamma_{\text{sat}}^{2}}\right)\sin\theta\left[1-\sin^{2}\theta\right]^{1/2},$$

$$b_{4}=\left[1-2\left(\frac{1}{\gamma_{\text{sat}}^{2}}\right)\sin^{2}\theta\right].$$

Using Cramer's rule, we may solve for any of the reflection or transmission coefficients in equation (10). Our research focuses on R_{PP} but we may use Cramer's rule to solve for R_{PS}, T_{PS}, or T_{PP}. Cramer's rule is the determinant of the augmented matrix divided by the pre-augmented matrix or

$$R_{PP}=\frac{\det(A_{\text{aug}})}{\det(A)}. \tag{11}$$

Performing this operation will result in a series of weighted perturbations such that

$$R_{PP}=R_{PP}^{(1)}+R_{PP}^{(2)}+\ldots. \tag{12}$$

By truncating the expression in equation (12) such that only first order terms remain is

$$R_{PP}^{(1)}(\theta)=\left[\frac{1}{4}\left(1+\sin^{2}\theta\right)-\frac{\gamma_{dry}^{2}}{4\gamma_{sat}^{2}}\left(1+\sin^{2}\theta\right)\right]a_{f}+\left[\frac{\gamma_{dry}^{2}}{4\gamma_{sat}^{2}}\left(1+\sin^{2}\theta\right)-\frac{2}{\gamma_{sat}^{2}}\sin^{2}\theta\right]a_{\mu}+\left[\frac{1}{4}-\frac{\sin^{2}\theta}{4}\right]a_{\rho}. \tag{13}$$

The equivalence of the approximations at small angle and contrast

Russell et al. (2011) have derived and used a linearized approximation for PP reflection coefficients that is comparable to that of Aki and Richards (2002), but expressed in poroelastic terms. This derivation is comparable to other previously derived AVO formulas (Shuey, 1985; Smith and Gidlow, 1987; Fatti et al., 1994) Russell et al. (2011) argue that the poroelastic form,

$$R_{PP}^{(RG)}(\theta')\approx\left[\left(1-\frac{\left(\gamma'_{\text{dry}}\right)^{2}}{\left(\gamma'_{\text{sat}}\right)^{2}}\right)\frac{\sec^{2}\theta'}{4}\right]\frac{\Delta f}{f'}+\left[\frac{\left(\gamma'_{\text{dry}}\right)^{2}}{4\left(\gamma'_{\text{sat}}\right)^{2}}\sec^{2}\theta'-\frac{2}{\left(\gamma'_{\text{sat}}\right)^{2}}\sin^{2}\theta'\right]\frac{\Delta\mu}{\mu'}+\left[\frac{1}{2}-\frac{\sec^{2}\theta'}{4}\right]\frac{\Delta\rho}{\rho'}, \tag{14}$$

where the superscript ($'$) signifies average properties, and θ is the ray path angle with respect to the interface, is a more effective way of detecting fluid in the target medium. Equation (14) is referred to as the f-m-r equation.

In contrast, our approximation starts from the Zoeppritz equations. For small angles and small contrasts, it is equivalent to that of Russell and Gray, but it differs when contrasts are large, it adapts the linear term with a series of nonlinear corrections. Let us first demonstrate their equivalence. The average angle can be replaced with the incidence angle (which we use) when they are both small. Next let us compare our fluid perturbation with Russell and Gray's fluid reflectivity. We have

$$\frac{\Delta f}{f}=\frac{2\left(f_1-f_0\right)}{f_1+f_0}=2\frac{1-\dfrac{f_0}{f_1}}{1+\dfrac{f_0}{f_1}}=2\frac{a_f}{1+\left(1-a_f\right)}=\frac{a_f}{1-\dfrac{1}{2}a_f}=a_f\left[1+\left(\frac{1}{2}a_f\right)+\left(\frac{1}{2}a_f\right)^2+\ldots\right]\approx a_f\,. \tag{15}$$

Similarly $\Delta\mu/\mu\sim a_\mu$ and $\Delta\rho/\rho\sim a_\rho$. Finally, the $(1+\sin^2\theta)$ terms in equation (13) are equal to $\sec^2\theta$ in equation (14) for small angles. We may show that

$$\sec^2\theta=\frac{1}{\cos^2\theta}=\frac{1}{1-\sin^2\theta}=1+\sin^2\theta+\sin^4\theta+\ldots\,. \tag{16}$$

We conclude that for small contrasts and small angles, Russell and Gray's approximation and our approximation are equivalent.

Nonlinear poroelastic AVO

With our method of deriving the poroelastic AVO equation, we may respond to larger contrast situations by including to higher orders, to account for nonlinearity. Our first order approximation contains three weighting coefficients, and provides an interpretable representation of a P-wave reflection. The second-order corrections contain six terms and take the form

$$R_{PP}^{(2)}\left(\theta\right)=W_{a_4}a_f^2+W_{a_5}a_\mu^2+W_{a_6}a_\rho^2+W_{a_7}a_f a_\mu+W_{a_8}a_f a_\rho+W_{a_9}a_\mu a_\rho\,, \tag{17}$$

where the weighting coefficients are found in the appendix. The third-order correction contains ten terms and is written as

$$\begin{aligned}R_{PP}^{(3)}\left(\theta\right)=&W_{a_{10}}a_f^3+W_{a_{11}}a_\mu^3+W_{a_{12}}a_\rho^3+W_{a_{13}}a_f^2a_\mu+W_{a_{14}}a_f^2a_\rho+W_{a_{15}}a_\mu^2a_f+W_{a_{16}}a_\mu^2a_\rho+W_{a_{17}}a_\rho^2a_f\\&+W_{a_{18}}a_\rho^2a_\mu+W_{a_{19}}a_f a_\mu a_\rho\end{aligned}\,, \tag{18}$$

where the weighting coefficients are also found in the appendix.

Numerical Results

Lastly we examine the relative importance of the first-order, second-order and third-order components of our approximate poroelastic AVO expressions. In Figure (1) the R_{PP} values derived from solution of the exact Zoeppritz equations, with the first and second-order poroelastic approximations.

We do so by plotting, in Figure (1), exact vs. approximate R_{PP} curves arising from four reflectors of increasing contrast, from 10% through 50%. We conclude that geophysically realizable (large) contrasts the nonlinear corrections can supply a significant up-tick in accuracy.

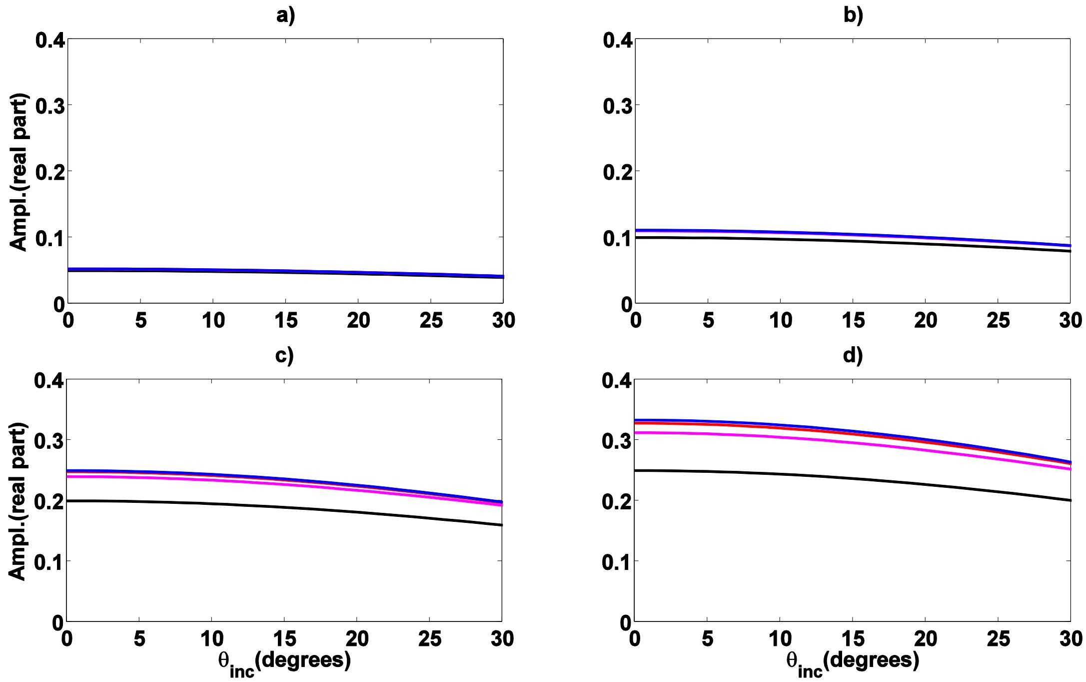

Figure 1: The relative importance of first, second and third order approximations as contrasts increase. In each panel, the blue curve is the exact R_{PP} (real part), the black curve is the first order approximation, the magenta curve is the second order approximation, and the red curve is the third order approximation. The panels illustrate the approximations for increasing contrasts: (a) 10%, (b) 20%, (c) 40%, (d) 50%.

Conclusions

This research shows a method in which to derive a first, second, and third-order poroelastic AVO approximations. This derivation begins from the Zoeppritz equations where poroelastic perturbation parameters are defined and substituted back into the Zoeppritz equations. Using the Zoeppritz equations provided by Keys (1989), we may use Cramer's rule to solve for any of the four coefficients found in the column vector in the left-hand side of equation (10). We chose to solve for R_{PP} for purposes of surface seismic experiments and strong reflections. After deriving a first-order approximation, we compared it to Russell and Gray's formula as shown by Russell et al. (2011), to show that these two forms are equivalent which we have shown. We have also shown the nonlinear correcting terms up to third order and demonstrate the ability of the nonlinear terms numerically with a few examples. As property contrasts become large, the third-order correcting terms provide a significant change in the R_{PP} curves.

For future work, we would like to explore the effects of linear and nonlinear inversion, apply our methods to field data, and to incorporate our tool into geophysical software.

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Appendix

The weighting coefficients for the nonlinear correcting terms in equations (17) and (18) are as follows:

$$W_{a_4}=\left(1-\frac{\gamma_{\mathrm{dry}}^2}{\gamma_{\mathrm{sat}}^2}\right)\frac{\sin^2\theta}{4}+\frac{1}{8}\left(1-\frac{\gamma_{\mathrm{dry}}^4}{\gamma_{\mathrm{sat}}^4}\right),$$

$$W_{a_5}=\frac{\gamma_{\mathrm{dry}}^2}{4\gamma_{\mathrm{sat}}^2}\left(\sec^2\theta-\frac{\gamma_{\mathrm{dry}}^2}{2\gamma_{\mathrm{sat}}^2}\right)-\frac{2}{\gamma_{\mathrm{sat}}^2}\left(1-\frac{1}{2\gamma_{\mathrm{sat}}}\right)\sin^2\theta,$$

$$W_{a_6}=\frac{1}{8}\left(1-\frac{2}{\gamma_{\mathrm{sat}}}\sin^2\theta\right),$$

$$W_{a_7}=-\frac{\gamma_{\mathrm{dry}}^2}{4\gamma_{\mathrm{sat}}^2}\left(1-\frac{\gamma_{\mathrm{dry}}^2}{\gamma_{\mathrm{sat}}^2}\right),$$

$$W_{a_8}=-\left(1-\frac{\gamma_{\mathrm{dry}}^2}{\gamma_{\mathrm{sat}}^2}\right)\frac{\sin^2\theta}{4},$$

$$W_{a_9}=\frac{1}{\gamma_{\mathrm{sat}}^2}\left(1-\frac{\gamma_{\mathrm{dry}}^2}{4}\right)\sin^2\theta,$$

$$W_{a_{10}}=\frac{1}{64}\left[5\left(1+3\sin^2\theta\right)\left(1-\frac{\gamma_{\mathrm{dry}}^4}{5\gamma_{\mathrm{sat}}^4}\right)+\frac{\gamma_{\mathrm{dry}}^2}{\gamma_{\mathrm{sat}}^2}\left(1-13\sin^2\theta\right)-5\frac{\gamma_{\mathrm{dry}}^6}{\gamma_{\mathrm{sat}}^6}\left(1-\frac{\sin^2\theta}{5}\right)\right],$$

$$W_{a_{11}}=\frac{5}{64}\frac{\gamma_{\mathrm{dry}}^6}{\gamma_{\mathrm{sat}}^6}\left(1-\frac{\sin^2\theta}{5}\right)+\frac{\gamma_{\mathrm{dry}}^2}{4\gamma_{\mathrm{sat}}^2}\sec^2\theta+\frac{7}{4\gamma_{\mathrm{sat}}^3}\left(1-\frac{2}{7}\frac{\gamma_{\mathrm{dry}}^2}{\gamma_{\mathrm{sat}}^2}\right)\sin^2\theta-\frac{2}{\gamma_{\mathrm{sat}}^2}\left(1-\frac{1}{16}\frac{\gamma_{\mathrm{dry}}^4}{\gamma_{\mathrm{sat}}^4}\right)\sin^2\theta-\frac{1}{4}\frac{\gamma_{\mathrm{dry}}^4}{\gamma_{\mathrm{sat}}^4},$$

$$W_{a_{12}}=\frac{5}{64}\left(1+\frac{\sin^2\theta}{5}\right)-\frac{3}{16\gamma_{\mathrm{sat}}}\sin^2\theta,$$

$$W_{a_{13}}=\frac{15}{64}\frac{\gamma_{\mathrm{dry}}^6}{\gamma_{\mathrm{sat}}^6}\left(1-\frac{\sin^2\theta}{5}\right)-\frac{\gamma_{\mathrm{dry}}^2}{64\gamma_{\mathrm{sat}}^2}\left(1+3\sin^2\theta\right)-\frac{7}{32}\frac{\gamma_{\mathrm{dry}}^4}{\gamma_{\mathrm{sat}}^4}\left(1-\frac{3}{7}\sin^2\theta\right)-\frac{\gamma_{\mathrm{dry}}^2}{4\gamma_{\mathrm{sat}}^4}\left(1-\frac{\gamma_{\mathrm{dry}}^2}{2\gamma_{\mathrm{sat}}^2}\right)\sin^2\theta$$
$$+\frac{1}{8\gamma_{\mathrm{sat}}^2}\sin^2\theta,$$

$$W_{a_{14}}=\frac{\gamma_{\mathrm{dry}}^2}{32\gamma_{\mathrm{sat}}^2}\left(1+9\sin^2\theta\right)-\frac{\gamma_{\mathrm{dry}}^4}{64\gamma_{\mathrm{sat}}^4}\sec^2\theta-\frac{1}{64}\left(1+17\sin^2\theta\right),$$

$$W_{a_{15}}=\left(1-\frac{\gamma_{\mathrm{dry}}^2}{\gamma_{\mathrm{sat}}^2}\right)\left(\frac{\gamma_{\mathrm{dry}}^2}{4\gamma_{\mathrm{sat}}^4}-\frac{1}{2\gamma_{\mathrm{sat}}^3}\right)\sin^2\theta-\frac{\gamma_{\mathrm{dry}}^2}{4\gamma_{\mathrm{sat}}^2}+\frac{31}{64}\frac{\gamma_{\mathrm{dry}}^4}{\gamma_{\mathrm{sat}}^4}\left(1-\frac{3}{31}\sin^2\theta\right)-\frac{15}{64}\frac{\gamma_{\mathrm{dry}}^6}{\gamma_{\mathrm{sat}}^6}\left(1-\frac{\sin^2\theta}{5}\right),$$

$$W_{a_{16}}=\frac{5}{4\gamma_{\mathrm{sat}}^2}\left(1+\frac{\gamma_{\mathrm{dry}}^2}{5\gamma_{\mathrm{sat}}^2}\right)\sin^2\theta-\frac{3}{4\gamma_{\mathrm{sat}}^3}\left(1+\frac{\gamma_{\mathrm{dry}}^2}{3\gamma_{\mathrm{sat}}^{-1}}\right)\sin^2\theta-\frac{\gamma_{\mathrm{dry}}^4}{64\gamma_{\mathrm{sat}}^4}\sec^2\theta,$$

$$W_{a_{17}}=-\left(1-\frac{\gamma_{\mathrm{dry}}^2}{\gamma_{\mathrm{sat}}^2}\right)\left(\frac{1}{8\gamma_{\mathrm{sat}}}+\frac{\sec^2\theta}{64}\right),$$

$$W_{a_{18}} = \frac{3}{8\gamma_{\text{sat}}^2}\left(1 - \frac{\gamma_{\text{dry}}^2}{3\gamma_{\text{sat}}}\right)\sin^2\theta - \frac{1}{16\gamma_{\text{sat}}}\sin^2\theta - \frac{\gamma_{\text{dry}}^2}{64\gamma_{\text{sat}}^2}\sec^2\theta,$$

$$W_{a_{19}} = \left(1 - \frac{\gamma_{\text{dry}}^2}{\gamma_{\text{sat}}^2}\right)\left(\frac{1}{4\gamma_{\text{sat}}^2}\sin^2\theta - \frac{\gamma_{\text{dry}}^2}{32\gamma_{\text{sat}}^2}\sec^2\theta\right).$$

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