Time-lapse AVO inversion

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Summary

We proposed new three time-lapse AVO inversion algorithms: 1) total inversion of the differences, 2) inversion of seismic difference only and 3) sequential reflectivity-constrained inversion. The proposed methods were implemented using synthetic data that simulate a time-lapse model of a heavy oil reservoir. Elastic physical parameters of the time-lapse model were chosen to represent reservoir conditions at pre-production and post-production periods after reservoir depletion.

The time-lapse AVO inversion schemes simultaneously invert baseline and monitor seismic P-P & P-S data in order to estimate the change of model parameters. The proposed algorithms have proven their robustness in terms of computation time as well as stability in presence of noise in order to ensure smooth changes in estimating reservoir attributes from time-lapse inversion.

Introduction

In time-lapse AVO inversion, we seek to estimate elastic parameters changes for baseline and monitor seismic surveys of a hydrocarbon reservoir after depletion. Successful estimation of these elastic differences can further assist in delineation of fluid saturation and pressure changes (LandrØ, 2001) in the reservoir due to production processes.

Practical inversion techniques that simultaneously invert seismic data of different vintages (Saeed et al., 2011) are used in this study. The objectives from proposed inverse techniques are to improve model parameters estimations in the presence of noise, and to prove their robustness in terms of accuracy and computation time.

Theory of Time-lapse AVO inversion

For two given data sets (base, $\mathbf{d_0}$, and a monitor, $\mathbf{d_1}$), reflectivity data can be written as:

$$\mathbf{d_0}=\mathbf{G_0}\,\mathbf{m_0} \quad \text{for base line} \qquad (1)$$

$$\mathbf{d_1}=\mathbf{G_1}\,\mathbf{m_1} \quad \text{for monitor line} \qquad (2)$$

where $\mathbf{d}$ is seismic data, $\mathbf{G}$ is forward operator, and $\mathbf{m}$ is unknown model parameters sought.

Conventional time-lapse AVO inversion is accomplished by inverting time-lapse data individually to obtain elastic model parameters. The low frequency component of well logs is then added to estimated model parameters (Ferguson and Margrave, 1996). The estimated model parameters ($\mathbf{m_0}$ and $\mathbf{m_1}$) resulting for inverting seismic data vintages separately are then illustrated either as differences or as percentage of changes between base and monitoring model parameters.

In this paper, we present applications of three different time-lapse AVO inversion schemes that simultaneously invert baseline and monitor seismic data to estimate the change of model parameters. The obtained model parameters can also be presented as percentage of changes.

Method 1: Total inversion of differences

The total inversion of differences to estimate model parameters change of time-lapse data is carried out by simultaneously inverting baseline and monitor data. Thus, equations (1 and 2) are then re-arranged as

$$\mathbf{G_1}\,\mathbf{m_1} - \mathbf{G_0}\,\mathbf{m_0} = \mathbf{d_1} - \mathbf{d_0} \qquad (3)$$

Based on model parameters sought in the cost functions from total inversion of differences, this method gives two options:

a) Inverting for model parameters of monitoring survey $(\frac{\Delta \mathbf{I}}{\mathbf{I}}, \frac{\Delta \mathbf{J}}{\mathbf{J}} \text{ and } \frac{\Delta \boldsymbol{\rho}}{\boldsymbol{\rho}})_{\mathbf{monitor}}$, and change of model parameters of baseline and monitoring survey $[\Delta(\frac{\Delta \mathbf{I}}{\mathbf{I}}), \Delta(\frac{\Delta \mathbf{J}}{\mathbf{J}}) \text{ and } \Delta(\frac{\Delta \boldsymbol{\rho}}{\boldsymbol{\rho}})]$.

By using ($\mathbf{\Delta G} = \mathbf{G_1} - \mathbf{G_0}$), substituting for $\mathbf{G_1}$in equation (3) and re-arranging, the cost function of time-lapse AVO inversion for estimating the model parameters of monitor line ($\boldsymbol{m_1}$) and the model parameter changes ($\boldsymbol{\Delta m}$) can be written in augmented matrix form as:

$$J(\boldsymbol{m_1}, \mathbf{\Delta m}) = \left\| \begin{bmatrix} \Delta\mathbf{G} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{G_0} \end{bmatrix} \begin{bmatrix} \boldsymbol{m_1} \\ \Delta\mathbf{m} \end{bmatrix} - \begin{bmatrix} \boldsymbol{d_1} \\ \boldsymbol{d_0} \end{bmatrix} \right\|^2 + \begin{bmatrix} \lambda^2 \boldsymbol{R_1^T R}_1 & \mathbf{0} \\ \mathbf{0} & \lambda^2 \boldsymbol{R_0^T} R_0 \end{bmatrix} \begin{bmatrix} \boldsymbol{m_1} \\ \Delta\mathbf{m} \end{bmatrix} \qquad (4)$$

Where, **R** and **λ** are the regularization operator and parameter (Constable et al., 1987) respectively. Model parameters for the base model can also be estimated from the outputs of equation (4) are $\mathbf{m_1}$ and $\mathbf{\Delta m}$ respectively using relation ($\mathbf{m_0} = \mathbf{m_1} - \mathbf{\Delta m}$).

b) Inverting for model parameters of baseline survey $(\frac{\Delta \mathbf{I}}{\mathbf{I}}, \frac{\Delta \mathbf{J}}{\mathbf{J}} \text{ and } \frac{\Delta \boldsymbol{\rho}}{\boldsymbol{\rho}})_{\mathbf{base}}$, and change of model parameters of baseline and monitoring survey $[\Delta(\frac{\Delta \mathbf{I}}{\mathbf{I}}), \Delta(\frac{\Delta \mathbf{J}}{\mathbf{J}}) \text{ and } \Delta(\frac{\Delta \boldsymbol{\rho}}{\boldsymbol{\rho}})]$.

Inverting for the base line, $\mathbf{m_0}$ and the time-lapse reflectivity model parameter changes *($\boldsymbol{\Delta m}$)* requires using relation ($\boldsymbol{m_1} = \boldsymbol{\Delta m} + \boldsymbol{m_0}$) to substitute for model parameters of the monitor model $\boldsymbol{m_1}$. Then by re-arranging equation (3), the cost function of time-lapse AVO inversion for estimating the model parameters of the base line ($\mathbf{m_0}$) and the model parameter changes ($\boldsymbol{\Delta m}$) can be written in augmented matrix form as:

$$J(\boldsymbol{m_0}, \mathbf{\Delta m}) = \left\| \begin{bmatrix} \Delta\mathbf{G} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{G_1} \end{bmatrix} \begin{bmatrix} \boldsymbol{m_0} \\ \Delta\mathbf{m} \end{bmatrix} - \begin{bmatrix} \boldsymbol{d_0} \\ \boldsymbol{d_1} \end{bmatrix} \right\|^2 + \begin{bmatrix} \lambda^2 \boldsymbol{R_0^T R}_0 & \mathbf{0} \\ \mathbf{0} & \lambda^2 \boldsymbol{R_1^T} R_1 \end{bmatrix} \begin{bmatrix} \boldsymbol{m_0} \\ \Delta\mathbf{m} \end{bmatrix} \qquad (5)$$

Method 2: Inversion of seismic differences data $(\mathbf{\Delta d})$only

Time-lapse inversion of differences data only is a quick inverse scheme used to estimate the change in elastic model parameters. When $\mathbf{\Delta G} \approx \mathbf{0}$, the cost functions of time-lapse inversion (4 and 5) of total inversion of the differences given in the previous section will be reduced and re-written as:

$$J(\Delta\mathbf{m}) = \|\boldsymbol{G}_{\mathbf{i}}\Delta\mathbf{m} - \Delta\mathbf{d}\|^2 + \lambda^2\|\boldsymbol{G}_{\mathbf{i}}\Delta\mathbf{m}\|^2 \qquad (6)$$

where, $\boldsymbol{G}_{\mathbf{i}}$ contains ray traced coefficients of the Aki-Richards (1980) linearized equation of AVO inversion for the baseline or the monitoring surveys. Thus, we are inverting for Δm using the difference of seismic data between baseline and monitoring survey data, **Δd**, as an input to the inversion scheme.

Method 3: Sequential reflectivity-constrained inversion

The sequential reflectivity-constrained inversion in equation (7) is one form of robust time-lapse inversion method. In this inverse scheme, estimated model parameters of the base survey are used to constrain the inversion of the monitoring model in order to ensure smooth variation in estimated elastic model parameters of the monitoring model.

$$[(\boldsymbol{G^T G} + \boldsymbol{\lambda_1 W_m^T W_m} + \boldsymbol{\lambda_2 V^T V})]\boldsymbol{m_i} = \left[\boldsymbol{G^T d} + \boldsymbol{\lambda_2 V^T V}(\boldsymbol{m_{i-1}^M} - \boldsymbol{m_0^B})^{\boldsymbol{T}}\right] \qquad (7)$$

Where $\boldsymbol{V_{monit.}} = diag\left[abs\left(\boldsymbol{m_{i-1}^M} - \boldsymbol{m_0^B}\right)\right]$

Estimated model parameters of the base model will act as prior information in the inversion. Because $\boldsymbol{V}$ and $(\boldsymbol{m_{i-1}^M} - \boldsymbol{m_{i-1}^B})^{\boldsymbol{T}}$are functions of unknown base and monitor model parameters, this is a non-linear system, and iterative approach must be used. This is referred to as iteratively re-weighted least-squares, IRLS in Saeed et. al., (2010).

In the inversion scheme, we set $\boldsymbol{V}$ and $(\boldsymbol{m_{i-1}^M} - \boldsymbol{m_{i-1}^B})^{\boldsymbol{T}}= \boldsymbol{I}$ for the first iteration, which result in a traditional least-squares solution. The estimation of $\boldsymbol{m_i^B}$ for i=1 is then subsequently substituted again in equation (7) to obtain new $\boldsymbol{m_{i+1}^M}$. The procedure is repeated until the estimated model parameters of the monitoring survey between successive IRLS iterations becomes less than the predefined user tolerance value.

Examples

A time-lapse model that simulates a heavy oil reservoir of Pikes Peak oil field is given in figure (1). Synthetic data were generated for the baseline and monitoring models. Figures (2 and 3) show the differences of PP- and PS- data for the baseline and monitor models respectively. Note that amplitudes of seismic data sections are scaled to the seismic amplitudes of the base line survey. Figure (4) shows the change of elastic impedances of the baseline and monitor surveys from implementing equation (4) of total inversion of differences, is in agreement with actual impedances (black dash lines) calculated from well logs. Figure (5) shows the change in elastic impedances of baseline and monitor survey as result of implementing the inversion of the difference data only, equation (6). Figure (6) shows the elastic impedances of the monitoring survey as a result of sequential reflectivity-constrained inversion given in equation (7).

Conclusions

We have developed three new methods for time-lapse AVO inversion. The elastic impedances obtained from application of proposed inverse schemes for time-lapse AVO inversion are consistent with actual elastic impedances calculated from well logs. The developed codes were optimized to perform inversion in less time, and shows fast convergence with a small number of iterations for robust time-lapse AVO inversion.

Acknowledgements

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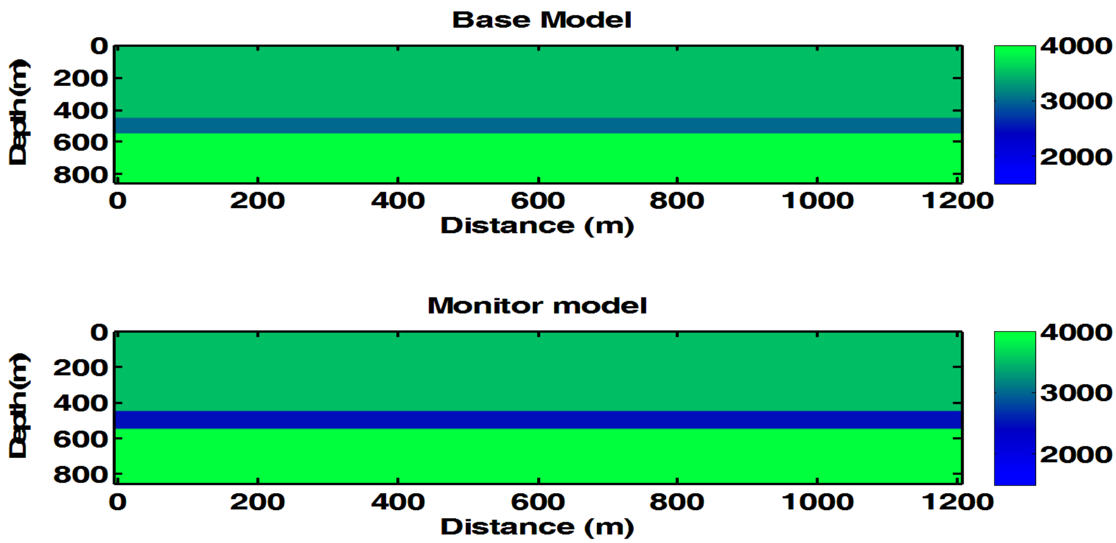

Figure 1: Time-lapse model for Pikes-Peak time-lapse survey.

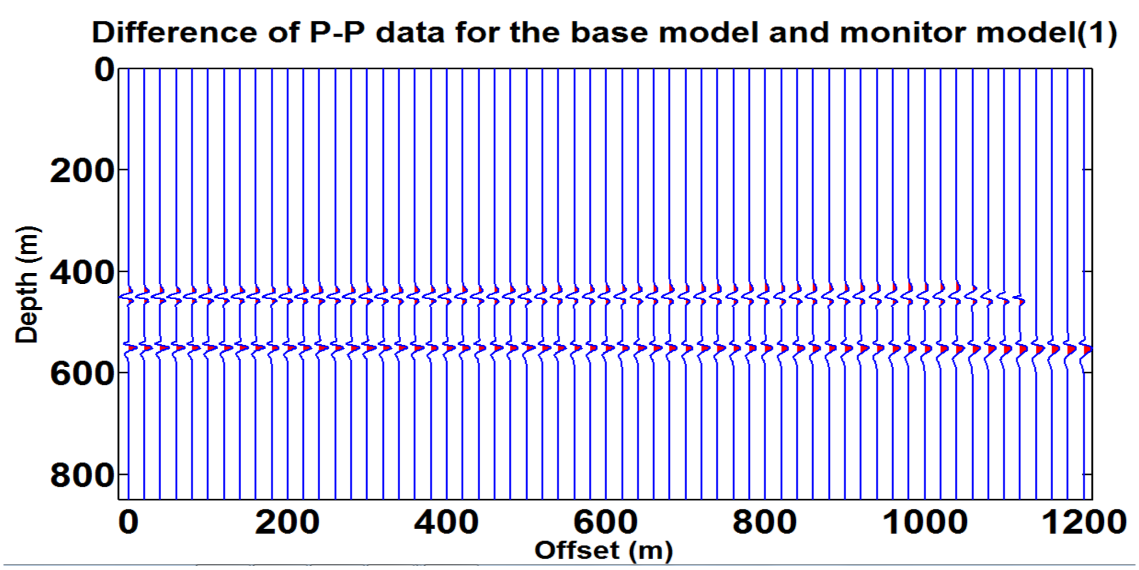

Figure 2: The difference of synthetic P-P data for the base and monitoring models.

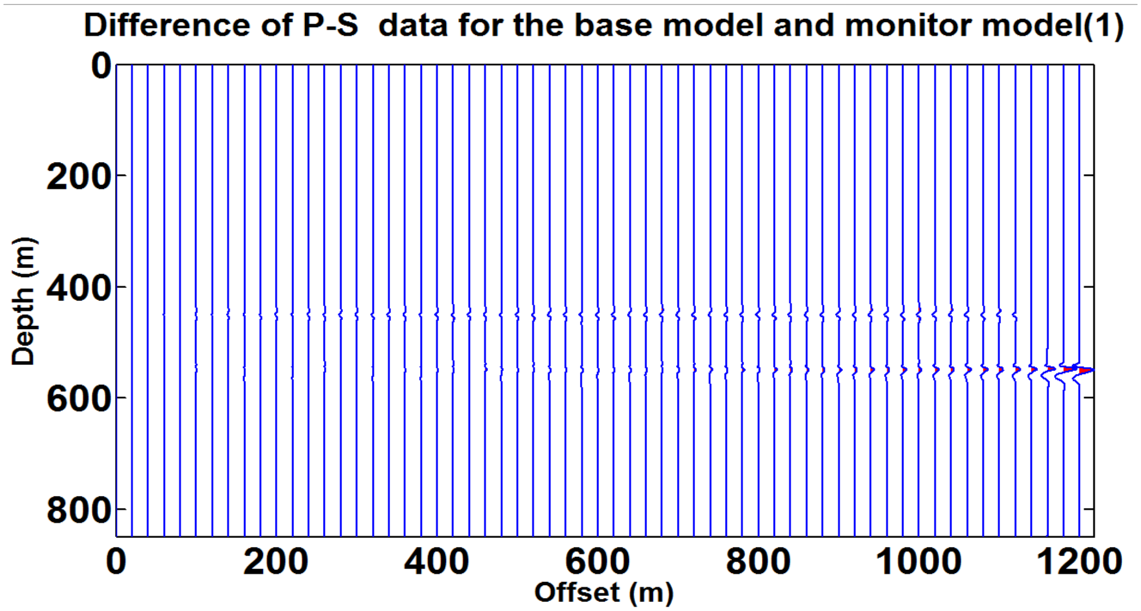

Figure 3: The difference of synthetic P-S data for the base and monitoring models.

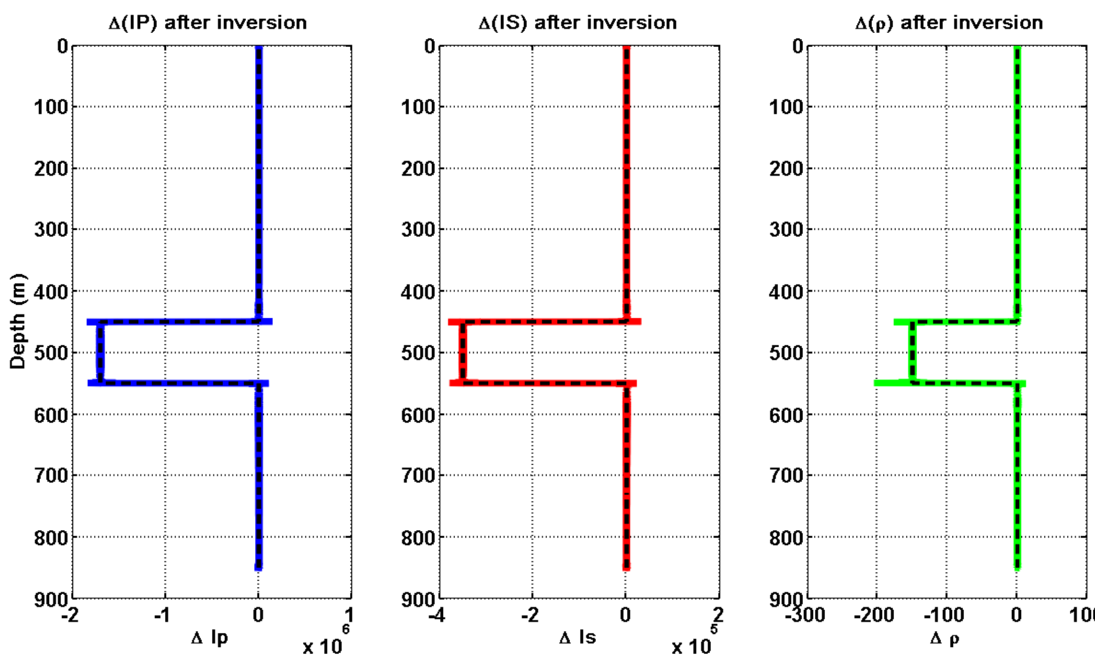

Figure 4: Elastic parameter changes (**ΔIP**, **ΔIS** and **Δρ**) of the time-lapse model **from total inversion of differences** (equation 4). Embedded graphs in bold black dotted lines represent actual elastic parameter changes calculated from logs.

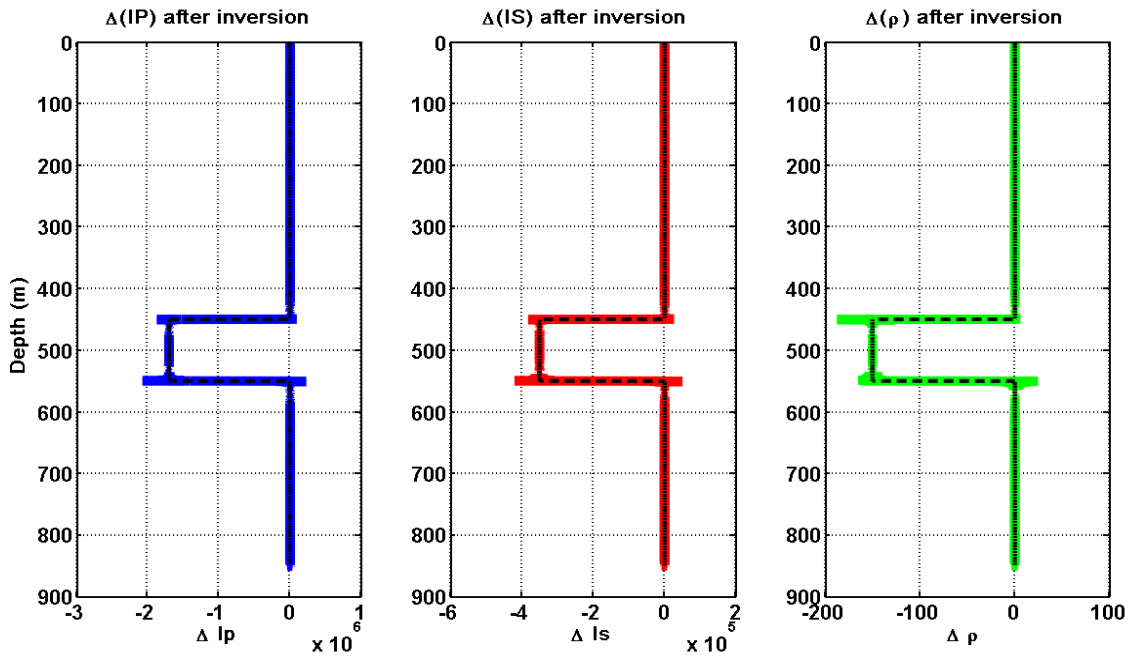

Figure 5: Change of elastic parameters (**ΔIP**, **ΔIS** and **Δρ**) from **inversion of data differences only** (equation 6). Embedded graphs in bold black dotted lines represent actual elastic parameter changes calculated from logs.

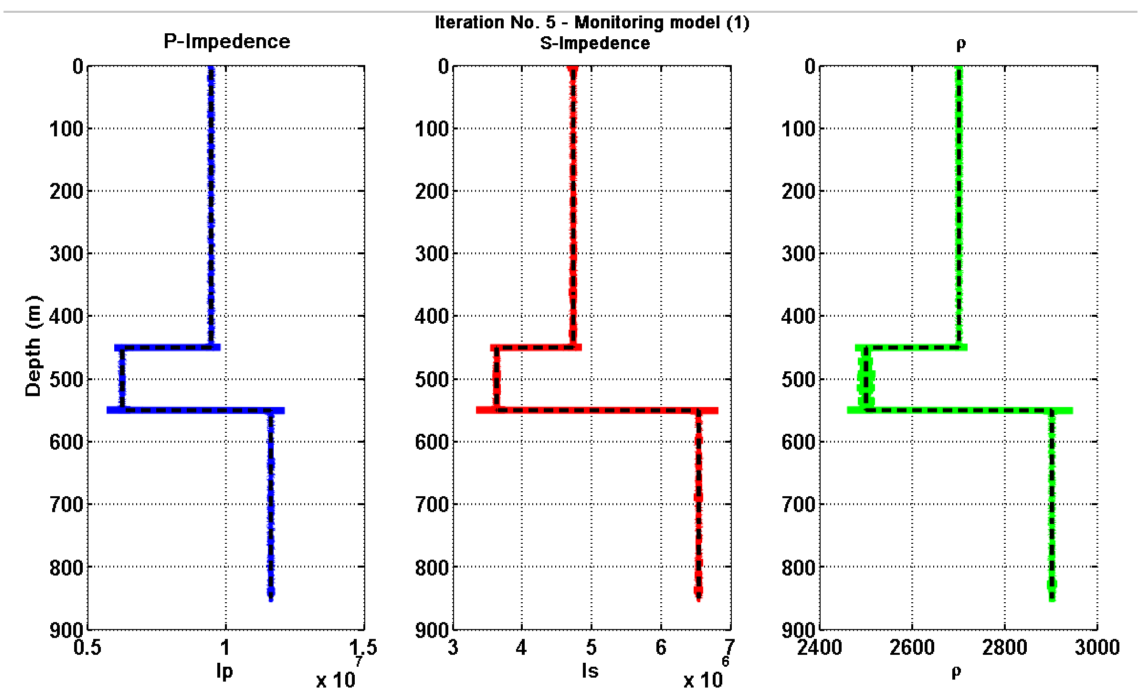

Figure 6: Elastic parameters (**IP**, **IS** and **ρ**) using sequential reflectivity-constrained inversion of the noisy (0.01) monitor model.