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Inference of 2D and 3D Locally Varying Anisotropy Fields

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Abstract

There has been a lot of interest to incorporate non-stationarity into geostatistical modeling. While first order non-stationarity can be easily considered using data transformations or considering a locally varying mean, the association of second order stationarity is less straightforward and few practical techniques exist. Current methodologies such as MPS with local pattern orientation, local kriging search orientation and kriging with LVA all deal with a varying anisotropy field. However, these techniques can be employed provided a map of local orientations and magnitude of anisotropy are exhaustively defined, called an LVA field map. The focus of this paper is to highlight a host of methodologies that can generate the required LVA map from various data sources.

Introduction

Traditional geostatistical techniques rely on the assumption of stationary anisotropy. This assumption is often relaxed and anisotropy is defined locally. Algorithms use this local definition of stationarity when calculating estimates or simulations. This work is concerned with obtaining this locally varying anisotropy (LVA) field. Non-stationary conditions in geostatistics can be easily incorporated though an LVA field that completely describes the magnitude and direction of anisotropy. Several techniques are explored for LVA field generation from mapped or exhaustive data, axial data in the form of point orientation measurements and centerlines of complex objects from geological modeling. Orientation is obtained from exhaustive data by calculating the orientation of local windows; a technique is presented to optimize the window definition to obtain better LVA definition. Point data are uniquely treated as they represent axial data, i.e. the angle is equivalent to 180°+angle; a specialized kriging program is provided to deal with this class of data. Finally, orientation is obtained from geological objects by thinning the object and calculating the centerline. This centerline is used as the local orientation.

Several reasonable methodologies are available to incorporate the assumption of second order non stationarity into geostatistical modeling, but require a well-defined map of deposit orientations and magnitude. Kriging with LVA (Boisvert and Deutsch, 2011) uses the shortest nonlinear paths to

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incorporate the locally varying anisotropy; local anisotropy kriging (Stroet and Snepvangers, 2005) calculates local orientation automatically; and local variogram reorientations (Deutsch and Lewis, 1992; Xu, 1996) are some of the methods that avail the use of an LVA map.

In most cases the conditioning data is not dense enough to show non-stationary features. Evaluation of exhaustive secondary data and geologic interpretation may indicate LVA features that are not represented by conditioning data. An LVA field must be constructed that honors these underlying non-stationary features in order to generate numerical property models with the correct anisotropy.

1. LVA from Point Source Data

Direct angle measurements are available from outcrops, borehole cameras and full-bore formation micro imagers (FMI). Data on formation dip can be directly measured from dipmeter. Inference of LVA from such point data, however, is not straightforward and requires preprocessing since data is axial in nature.

In standard geostatistical algorithms anisotropy is assumed axial (or undirected), strike (θ) is identical to θ+180°. In dealing with directional data in two dimensions a standard procedure is to decompose the angles into circular coordinate vectors (C=cosθ and S=sinθ) pertaining to angles that wrap on the unit circle, i.e. 0° is the same as 360°. This convention cannot be applied directly as axial data is 0≤θ≤180°.Mardia and Jupp (2000) suggest doubling the data such that observations are 0≤θ≤360° and through correct standardization data wraps at 2π. Prior to doubling, data are restricted to [0, π]. For example a value of 270° is recoded to 90°, then doubled to be 180°. Standard circular transformations can then be applied to the doubled data (Figure 1).

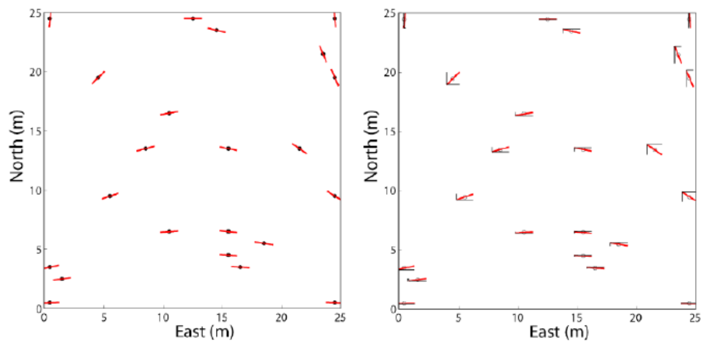

Figure 1 : Synthetic dipmeter data (orientations) and associated decompositions. (Boisvert,2010)

Once the observations are in circular components estimation (or simulation) is done using kriging (or SGS). It is necessary to reproduce the correct correlation between vector components C and S, and a strong relationship may warrant cokriging or cosimulation (Boisvert, 2010).

Three dimensional data is more complex as angle ‘doubling’ is not sufficient to consider the axial nature of data defined by a strike and dip. The method is outlined in Mardia and Jupp (2000), the need for such processing can be motivated similar to the limitations mentioned in the 2D case. Each input is defined by strike (θ) and dip (Φ) and is decomposed into spherical polar coordinates. The direction X is decomposed as:

$$X\ (z,x,y) = (\cos\theta,\ \sin\theta\cos\Phi,\ \sin\theta\sin\Phi)$$

Now using kriging to interpolate an exhaustive LVA field would give erroneous results as points with strike 45° and 225° are recognized opposite whereas they lie in the same axis and are continuous. There is a need to identify which orientations (positive or negative axis) in the sampled points would give the best estimate at any unknown location. Descriptive statistics such as variance of a set of angles provides a good measure in choosing axial orientations. For any number of sampled points our aim is to (i) align data on the same axis or along axes with small deviation and (ii) provide continuity of features among the known points and estimate. This by definition is given by the minimum variance of a cluster of axial data.

Consider three sample points being used to estimate at an unknown location (Figure 2). The proposed technique is to determine which axial orientation would be consistent for all three locations. There are a total of 3 different situations (Figure 2) and the variance of the angle data can be used to determine which of the three orientations should be averaged (equation 2). As indicated on Figure 2, the combination where location u_3 is 'flipped' (consider the opposite axis) has the minimum variance (Table 1). It is important to distinguish this angle variance from the typical kriging variance used in estimation. This angle variance is only used to determine which orientation results in the most consistent estimate for a given location. For any number of conditioning data, the aim is to determine the combination of data that results in the minimum variance of the angles. Because estimation proceeds sequentially, from location to location the input data is not significantly different when reasonable search ranges are selected. This will allow for the quick calculation of appropriate sample orientations if the orientations used in the previously estimated location are known.

Consider data on S^3, $x_{1\ldots}x_n$ the sample mean is :

$$\bar{x} = \frac{1}{n}\sum x_i \qquad (1)$$

Where, a *is a unit vector and* $\bar{R} \geq 0$, $\bar{R} = \|\bar{x}\| \qquad \bar{x}_0 = \|\bar{x}\|^{-1}\bar{x}$

The mean resultant length has the following minimizing property, where *S(a)* is the arithmetic mean of the squared Euclidean distance between x_i and $\boldsymbol{a}$.

$$\begin{aligned} S(a) &= \frac{1}{n}\sum \|x_i - a\|^2 \\ &= 2(1-\bar{x}^T a) \\ &= 2(1-\bar{x}_0{}^T a) \qquad (2) \end{aligned}$$

S(a) is minimized subject to $(a^T a=1)$,

$$when\ a = \bar{x}_0$$

$$\min_a S(a) = 2(1-\bar{R}) = \sigma_{sp}{}^2 \qquad (3)$$

The spherical variance is computed for all combinations of orientations of the surrounding conditioning data found within the estimation search. As an illustration, assume we are using 3 nearby samples for estimation (Figure 2) denote the input axial orientation as 0 and the 'flipped' or opposite orientation 1. The combinations that need to be considered are:

u1	u2	u3	Variance
0	0	0	0.60
1	0	0	0.60
0	1	0	0.58
0	0	1	0.01

Table 1 : Angle variance for all combinations of input data

Cleary the sample at location u_3 should be rotated and point in a similar direction as the other data, this is reflected in the variance of the angles. Significantly more combinations are required for considering a larger number of conditioning data. The combination with the minimum variance is selected and traditional estimation of the components of x is implemented. The methodology is implemented on a synthetic 3D axial data (Figure 3) and component-wise decompositions for the domain of interest are shown (Figure 4).

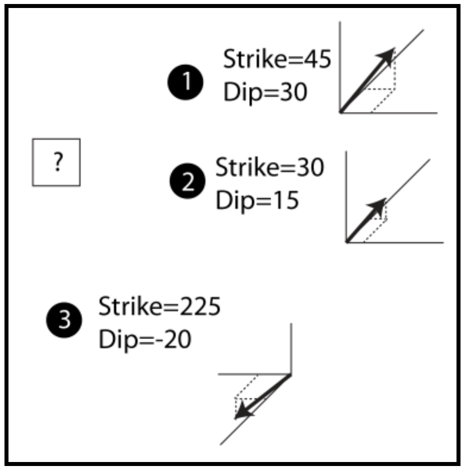

Figure 2 : Estimating at an unknown location (?) with three conditioning data located at u_1,u_2 and u_3.

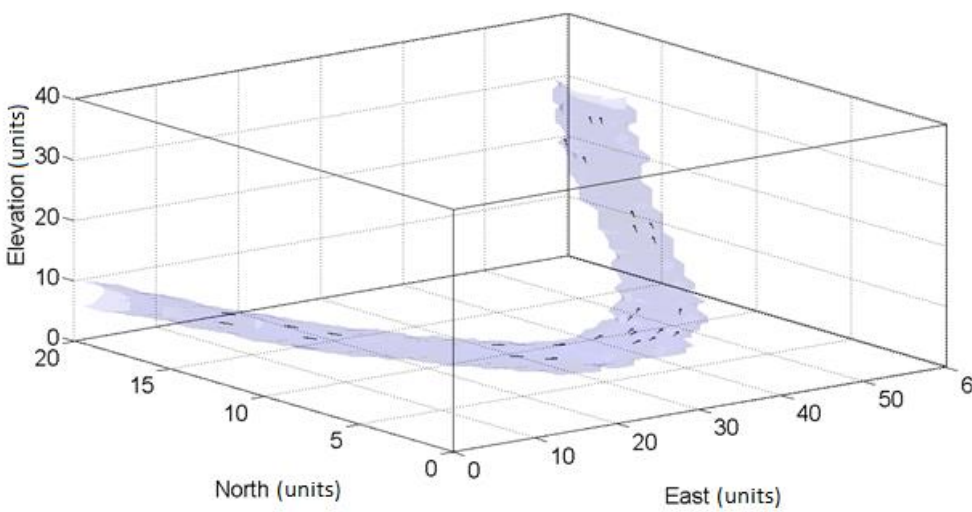

Figure 3 : The shaded represent the shape of the underlying syncline model; at discrete locations the orientations are taken as synthetic angle data and are represented by the arrows.

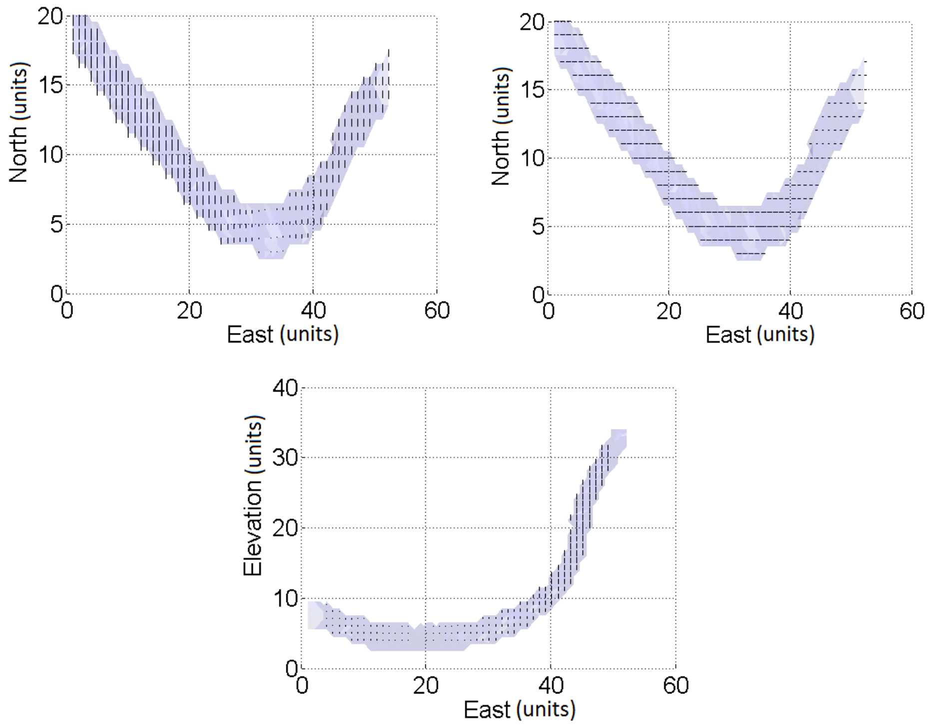

Figure 4 : Component-wise estimation

2. Generating LVA Fields from Centerlines

Skeletons and centerlines are often used to represent objects as they provide a simplified form of 2D and 3D bodies. In this case the geological interpretation/model will be used as the object that we would like to represent. The goal is to reduce the object complexity so that orientation can be easily determined from the skeleton. Topological features and size characteristics are preserved after the transformation (Telea and Wijk, 2002). Sethian's fast marching method (FMM) and its deviant multistencil form presented in Hassouna and Farag (2007) is used to calculate the centerline of a 3D representative geological body. Skeletonization of 3D objects is theoretically meant for compact representation of discrete objects, aiding in visualization, feature tracking and extraction etc.

The skeleton of an object can have several definitions. Blum (1967) defined a skeleton as the locus of the centers of maximal disks contained in the original object (Wang and Basu, 2004). For the calculation of a skeleton from a geological body, the skeleton should have the following properties (1) is a subset of the original dataset (2) has centered geometry within the bounds of the dataset (3) is largely connected and (4) is represented by a minimum number of voxels of the width of one cell (pixel).

Multistencils Fast Marching Method

The Fast Marching Method (FMM) is a technique for tracking a monotonically advancing closed surface proposed by Sethian (1996). The important feature for FMM is that skeleton points are created as the compact fronts evolve and collapse onto each other. Closed surface or interface propagation is identified by its motion; two types of motion are distinguished. A uniformly expanding or contracting, normal to itself, interface can be expressed as the boundary value formulation, whereas an interface advancing erratically is an initial value problem (Telea and Wijk ,2002). In higher dimensions the equation of motion of a propagating front is

$$|\nabla T|F = 1 \ldots \quad (3)$$
$$\textit{where the arrival time } T \textit{ of initial positio is set } 0;$$
$$T(\Gamma_0) = 0$$

The FMM, although highly consistent and accurate in solving the Eikonal equation (3), suffer from numerical errors along diagonal neighborhoods. Hassouna and Farag (2007) incorporate exhaustive diagonal information in FMM by using several stencils centered at each grid point covering all possible neighbours. The method now solves the Eikonal equations at several stencils and picks the solutions satisfying the FMM causality condition, i.e. the arrival time *T(x)* depends on immediate neighbors that have smaller values.

Note that the skeleton is a fairly reasonable representation of the major features of the body (Figure 5); however, in very thick areas the skeleton often invents features that are otherwise absent (i.e. North-West portion of Figure 5) and should be checked manually before using the polylines to interpret an LVA field. In Figure 6 these polylines are used to generate the LVA field by kriging the orientations of the polylines using the methodology presented for generating LVA fields from axial 3D data in Section 1.

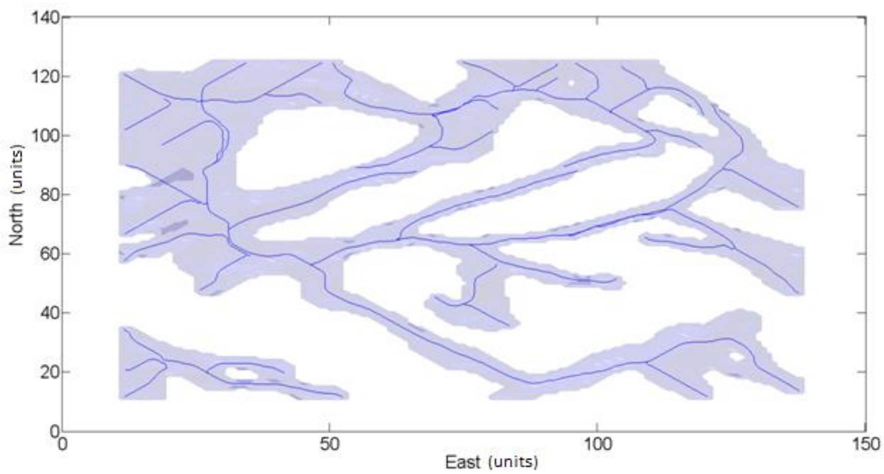

Figure 5 : Centerline created using multistencils FMM

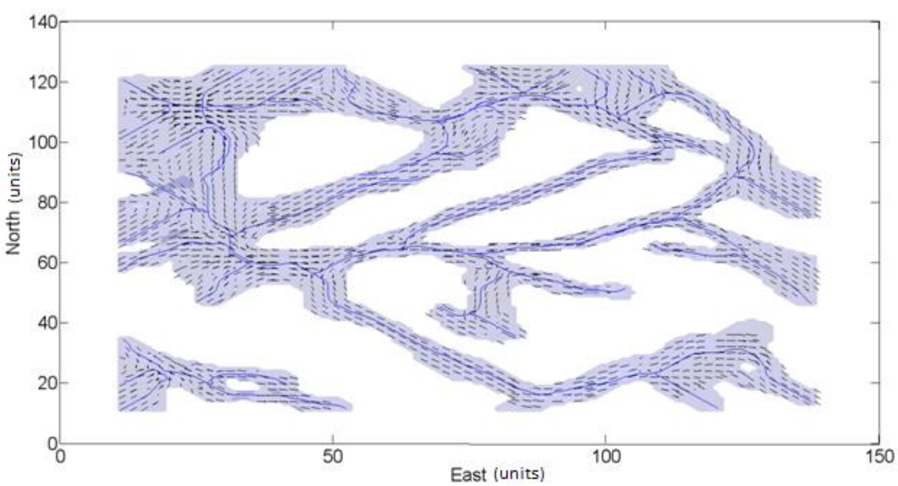

Figure 6 : Plan view of LVA map populated using standard interpolation techniques

3. Adaptive window for Local Orientation Extraction from exhaustive data

When an exhaustive secondary variable is available or mapped with minimal uncertainty, such as geophysical survey or outcrops, they can be used to infer the LVA field of related variables that may not be as densely sampled but are strongly related to the secondary variable. Data from remote sensing techniques provide exhaustive low-resolution information on secondary variables and often the continuity of related primary data is assumed to be similar. Two methodologies for generating orientation from exhaustive data are presented in Lillah and Boisvert (2012); the first obtained orientation through structure tensors and the second methodology (Feng, 2003), uses image and texture analysis techniques. Similar orientations can also be generated using moment of inertia tensor that is derived for the covariance map of the domain at each grid point (MHassanpour, 2007) The basis for these techniques are to calculate the local orientation of continuity based on a window around the location of interest (Figure 8). The definition of the local window is critical and should be sized such that local features of interest are captured inside the window. When the size of the local features varies in the domain, the window should be adapted to match the feature size.

The goal is to determine the optimal search area that best defines local features. In this paper the term 'window' or search neighbourhood is defined by an ellipsoid (an ellipse in 2D). One way to generate the LVA field in this adaptive window approach is to consider all possible combinations of the three parameters, major and minor axis of the ellipsoid search area and a rotation angle (Ө), defining the search for each grid location and choose the set with the maximum *reliability*. Reliability is a quantitative measure of confidence on the estimated orientations (Jeulin and Moreaud, 2008) calculated from the ratio:

$$r = \frac{\lambda_1 - \lambda_2}{\lambda_1 + \lambda_2}, for\, \lambda_1 > \lambda_2 \qquad (4)$$

where, λ_i, i=1,2 are the eigenvalues of a 2D tensor matrix.

The need for such an adaptive local neighborhood is motivated in the following on Figure 7. A synthetic outcrop image is considered that at once captures features of different scales. It is seen that selecting the proposed adaptive neighbourhood framework delineates the LVA of the large and smaller features fairly accurately than choosing either a large or small moving window for the search criteria. The key aspect to generate LVA field in this adaptive window approach is to go through all possible combinations of the three parameters (ranges in major, minor directions and rotation angles) for each grid point and choose the one from the best measure of reliability. The input ranges maybe large and is computationally heavy to calculate orientations for all the possible combinations of the desired parameters. So, a partial-optimization (skip some parameter combinations) process can be done that approximates the results very well (Figure 9).

The steps involved are:

1. At the initial grid location calculate local orientation considering the exhaustive set of parameter combinations
2. Select the combination that gives the highest reliability measure, and store the parameter values as the base case
3. In the next pixel, calculate orientations only for the base case and parameter values that deviate only slightly from the base case. Update the base case values and proceed sequentially

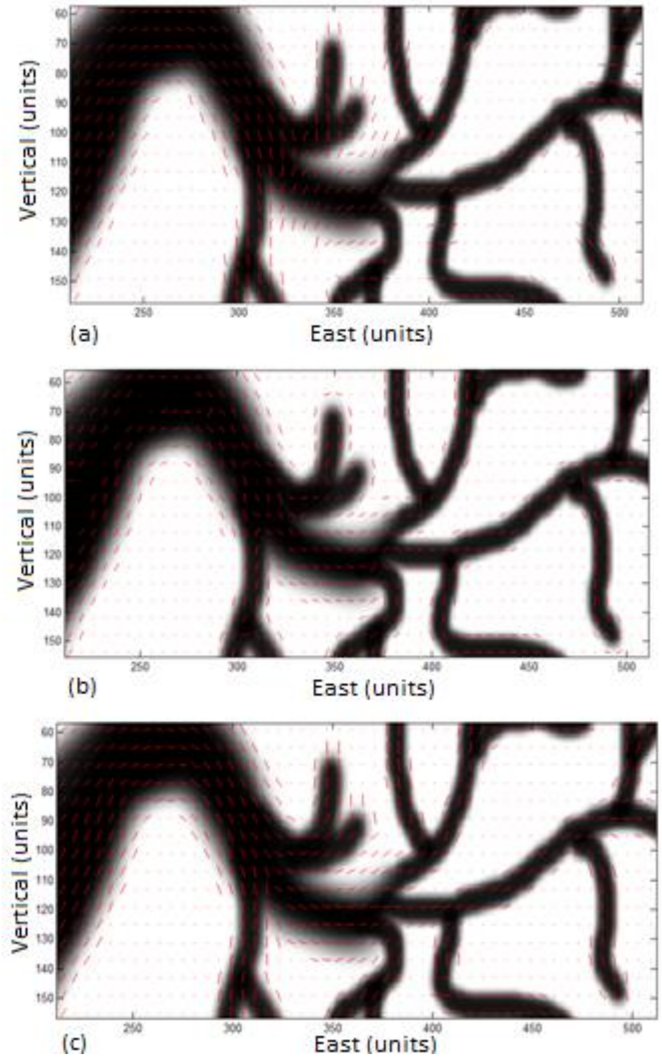

Figure 7: (a) and (b) show the LVA field generated using a large (20×20) and a smaller window. A more accurate LVA representation is found using the adaptive scheme (c).

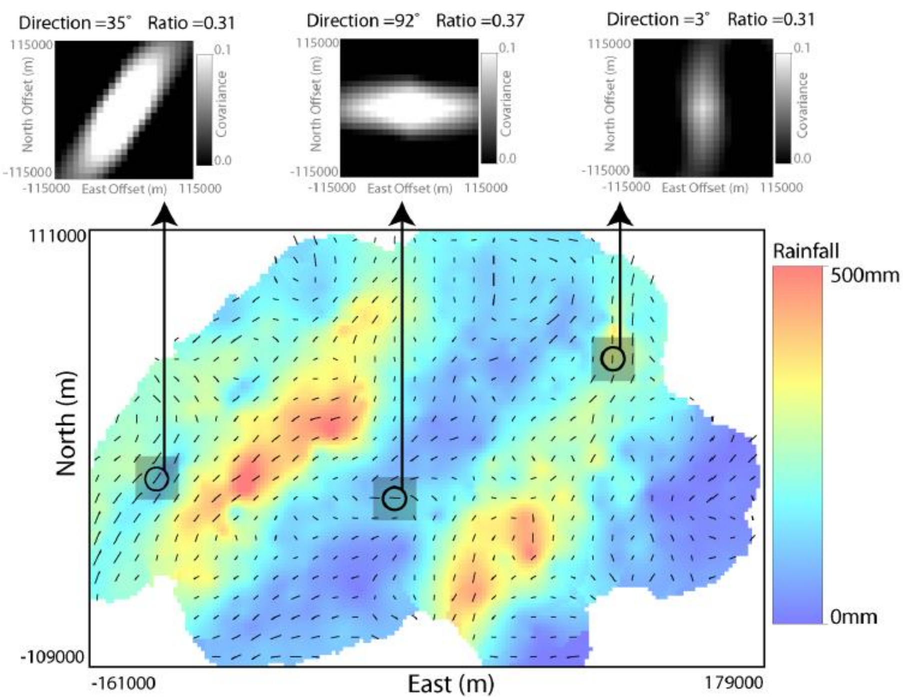

Figure 8: The idea of a moving window is illustrated here. The LVA field is generated using a square window of (10×10) pixels. Sizes of the windows at different locations appear as shaded squares on the image. (Boisvert,2010)

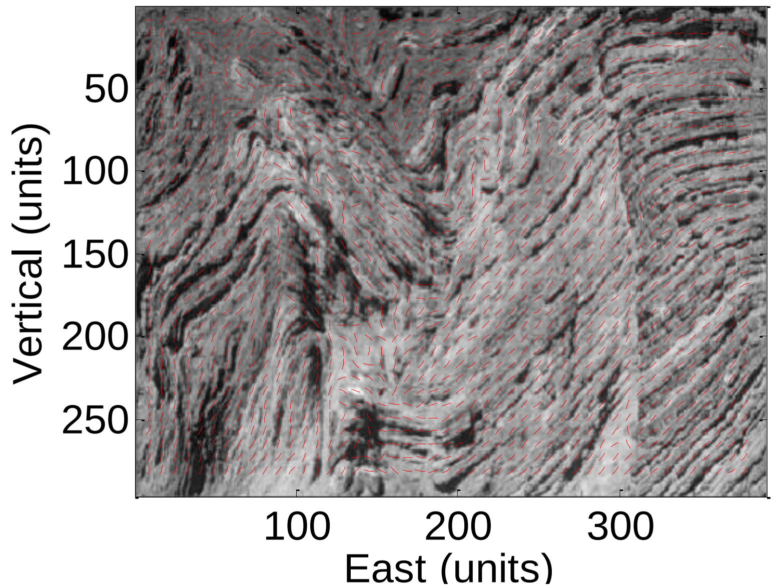

Figure 9: The partial optimization approach to finding the LVA map using structure tensors

The LVA follows the underlying features quite closely. However, here the method is illustrated in a fairly small model. Considering all the combinations with three different parameters can be computationally infeasible for larger and more complex models. Additionally, in 3D there are six parameters to optimize. Instead the partial optimization scheme is implemented (Figure 7). In most cases the continuity of the formation varies smoothly in space and is similar for adjoining locations (cells). As a result, performing the *full optimization* at the first point and using this optimized result as the starting point for the adjacent location reduces CPU time considerably with similar results (Figure 10).

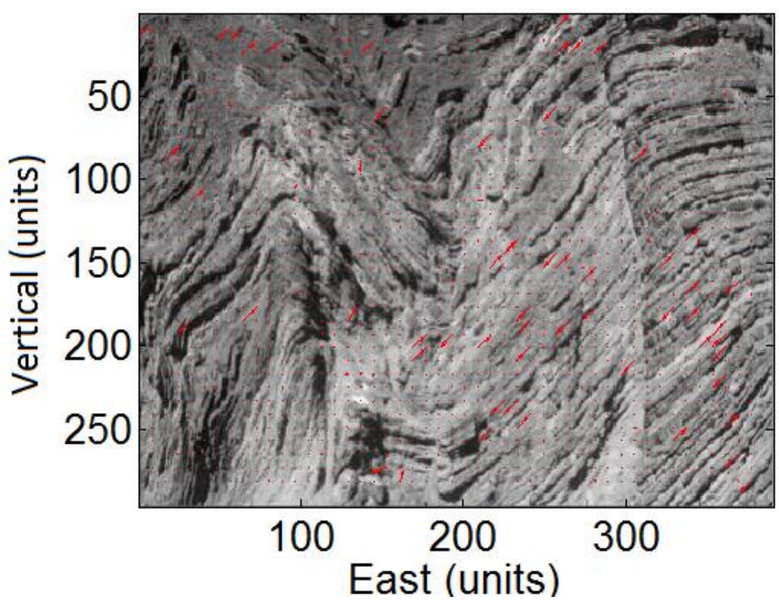

Figure 10: Delta plot of LVA field showing the difference between full and partial optimization.

Conclusions

Generating LVA fields from several data sources are presented and they are fairly accurate representations of underlying deposit. Exhaustive data, such as outcrop images, information on secondary variables (seismic surveys), are best assessed with a moving window framework using any of the tensor based methodologies discussed. The difficulty in incorporating point axial data is examined and their implementation in both 2D and 3D show satisfactory results. If the case presented is based on geologic interpretation, skeletonization of complex formations would yield a compact shape representation before the locally varying anisotropy conditions are deduced. Therefore, the nature of the data is the dictating factor in selecting an LVA generation technique.

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References

Blum,H.,1967,A transformation for extracting new descriptors of shape: Models for the Perception of Speech and Visual Form, IT Press,362-380.

Boisvert, J.,and Deutsch,C.,2011,Programs for kriging and sequential Gaussian simulation with locally varying anisotropy using non-Euclidean distances: Computers & Geosciences, 37(4), 495-510.

Boisvert,J.,2010,Geostatistics with Locally Varying Anisotropy: PhD thesis, Univeristy of Alberta,55-63.

Deutsch, C., and Lewis, R.,1992, Advances in the practical implementation of indicator geostatistics: In Proceedings of the 23rd APCOM Symposium Tucson, AZ, SME/AIME,169-179.

Feng, X., and Milanfar, P.,2002, Multiscale Principal Components Analysis for Image Local Orientation Estimation: Signals, Systems and Computers,Conference Record of the Thirty-Sixth Asilomar Conference,1,478-482.

Hassouna, S. M.,and Farag, A. A.,2007,Multistencils Fast Marching Methods: A highly accurate solution to the Eikonal equation on Cartesian domains: IEEE Transactions on Pattern Analysis and Machine Intelligence, 29(9),1563-1574.

Jeulin,D., and Moreaud,M.,2008,Segmentation of 2D and 3D textures from estimates of the local orientation: Image Analysis and Stereology,27(3),183-192.

Lillah,M.,and Boisvert,J.,2012, Inference of 2D and 3D locally varying anisotropy fields for complex geological formations:Ninth International Geostatistics Congress,Oslo,Norway, Extended Abstract.

Mardia,K., and Jupp, P.,2000,Directional Statistics: John Wiley and Sons, 430.

Mohammadhassanpour,R.,2007,Tools for multivariate modelling of permeability tensors and geometric parameters for unstructured grids: Master’s thesis, University of Alberta,61.

Sethian,J.,1999,Level Sets Methods and Fast Marching Methods: Cambridge University Press.

Stroet, C., and Snepvangers, J., 2005, Mapping curvilinear structures with local anisotropy kriging: Mathematical Geology, 37(6):635-649.

Telea,A.,and Wijk,van J.,2002,An augmented Fast Marching Method for computing skeletons and centerlines: In Proceedings of the symposium on Data Visualization,Barcelona,Eurographics,251-259.

Wang,T., and Basu,A.,2007,A note on ‘A fully parallel 3D thinning algorithm and its applications’: Pattern Recognition Letters,28,501-506.

Xu, W.,1996, Conditional curvilinear stochastic simulation using pixel-based algorithms: Mathematical Geology, 28(7):937-949.