

## A comparison of 5D reconstruction methods

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### Summary

A comparison is made between three 5D reconstruction methods— Projection Onto Convex Sets (POCS), Tensor Completion (TCOM), and Minimum Weighted Norm Interpolation (MWNI). A method to measure of the quality of synthetic data reconstructions is defined and applied under various scenarios. Two different measures of performance in the case of real data reconstructions are also provided and applied to a real data example taken from a land dataset acquired in the Western Canadian Sedimentary Basin. We find that TCOM and POCS are better able to reconstruct data in the presence of low *SNR*. We also find that TCOM provides superior results in most synthetic data scenarios, but in the case of real data reconstruction all three methods have similar performance, with POCS giving slightly better preservation of amplitudes.

### Introduction

5D Reconstruction aims to provide a fully sampled noise free estimate of the data. The majority of processing steps benefit from a fully sampled dataset such as 3DSRME (Dragoset et al., 2010) and AVA imaging (Sacchi and Liu, 2005). There are various strategies to tackle the reconstruction problem. Algorithms fall under the following two categories: those based on wave equation principles (for example Ronen (1987)), and those based on signal processing principles. Methods based on signal processing are by far the most commonly used for interpolation and denoising of seismic data because they do not require velocity information to reconstruct the data. Such methods include prediction error filter techniques (Spitz, 1991; Naghizadeh and Sacchi, 2007), transform based methods (Abma and Kabir, 2006; Liu and Sacchi, 2004), and rank reduction based methods (Trickett et al., 2010; Kreimer and Sacchi, 2011).

This paper considers three methods. Two of the methods, POCS and MWNI, are transform based methods. They reconstruct the data one frequency at a time in the frequency-wavenumber domain. The other method, TCOM, is a rank reduction based method that also reconstructs the data one frequency at a time, but in the frequency-space domain. The following sections summarize the three reconstruction methods under study.

**Projection Onto Convex Sets (POCS)** works by iteratively thresholding the amplitudes of the wavenumber spectrum (Abma and Kabir, 2006).

**Tensor Completion (TCOM)** uses the Higher-Order Singular Value Decomposition and its truncated version to reduce the rank of the core-tensor of the data stored in a tensorial form. It is an iterative procedure, where a reinsertion operation is carried out after the rank reduction (Kreimer and Sacchi, 2011).

Both POCS and TCOM use a weighted reinsertion to simultaneously denoise the original data at each iteration. This can be written  $D^k = \alpha D_{obs} + (1 - \alpha) S D_{est}^k$ ,  $k = 1, \dots, N$ , where  $D^k$  is the reconstructed data in the  $F - X$  domain at the  $k^{th}$  iteration,  $D_{obs}$  is the observed data,  $\alpha$  is a weighting parameter used to

control the level of denoising,  $S$  is the sampling operator, and  $D_{est}^k$  is the spectral estimate of the data at the  $k^{th}$  iteration. In the case of POCS  $D_{est}^k$  is generated by iteratively thresholding the data in the wavenumber domain, and in the case of TCOM  $D_{est}^k$  is generated by rank reduction of the core-tensor.

**Minimum Weighted Norm Interpolation (MWNI)** minimizes an objective function that is the sum of the  $\ell_2$  norm of the misfit and a norm that constrains the solution to be sparse in the wavenumber domain. In its current implementation, we used a conjugate gradients based solver to solve this inverse problem (Liu and Sacchi, 2004).

## Evaluation of results

In the case of synthetic data where we know the true noise-free fully sampled data  $d_{true}$  we can define a measure of the quality as

$$Quality = 10 \cdot \log \frac{\|d_{true}\|_2^2}{\|d - d_{true}\|_2^2}, (dB). \quad (1)$$

where  $\|x\|_2^2$  denotes the squared  $\ell_2$  norm of  $x$ , and  $d$  are the reconstructed data. In the case of real data where the noise-free fully sampled data is unknown we need a different measure of the quality of the reconstruction. Using the input data as a reference we can define the fit of the data by

$$Fit = 10 \cdot \log \frac{\|S \circ d_{input}\|_2^2}{\|S \circ (d - d_{input})\|_2^2}, (dB). \quad (2)$$

where  $S$  is the 5D sampling operator which has a value of 1 at sampled locations, and a value of 0 at unsampled locations. In this notation  $\circ$  indicates the Hadamard product. Although the fit only considers data at existing trace locations and does not consider the noise level of  $d_{input}$ , it is still a valuable tool in judging the performance of a reconstruction. A visually smooth result with a low measure of fit could be underfitting the data, while a visually noisy result with a high measure of fit could be overfitting the data.

Another method of judging the performance of reconstruction utilizes stacked data. Stacking consists of averaging all offset and azimuth traces within a single midpoint location. In the presence of random noise stacking provides a factor of  $\sqrt{fold}$  improvement to the amplitude  $SNR$  (defined as the variance of the signal divided by the variance of the noise). This is useful for gauging the results of reconstruction because it provides an estimate of the true noise free data at all midpoint locations. The quality can then be computed using equation 1.

## Examples

### Synthetic data examples

To test the quality of synthetic data reconstructions a variety of 5D synthetic datasets were generated to fall under various categories. For each category the data have dimensions of 150 time samples, and 16 samples in all four spatial dimensions. The data consist of three events with differing dip and/or curvature depending on the scenario. In all cases except when curvature was being evaluated linear events were used. In all cases when  $SNR$  was not being evaluated a value of  $SNR = 5$  was used. All of these scenarios and their respective qualities are listed in Table 1. Each of the three reconstruction methods are individually parameterized for each scenario. In the case of high  $SNR$  it is clear that all methods provide a very high quality result (the difference between  $Q = 12.85$  dB and  $Q = 39.5$  dB is

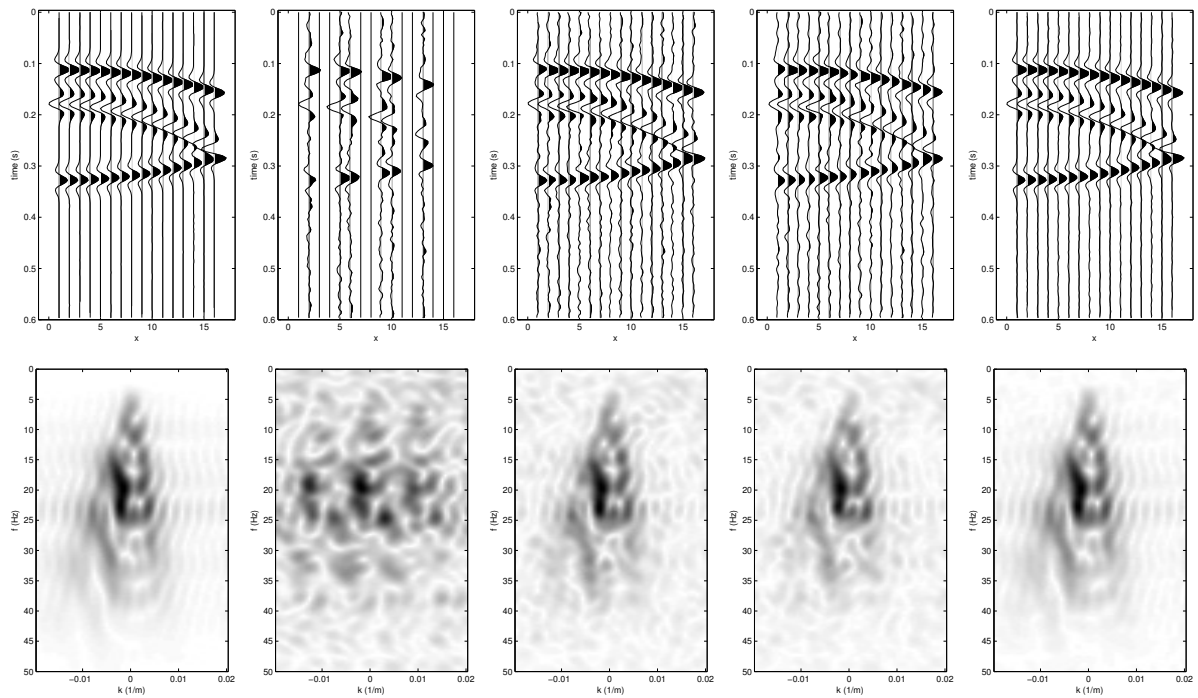


Figure 1: Slices of 5D data shown in  $X - T$  with a low level of curvature (the 5<sup>th</sup> scenario of table ??) in all four spatial directions,  $SNR = 5$ , and 60% of traces decimated. (a) noise free fully sampled data, (b) noisy decimated data, (c) POCS reconstruction, (d) MWNI reconstruction, (e) TCOM reconstruction. (f) - (j) show the corresponding  $F - K$  plots.

difficult to discern visually). In this case TCOM provides the highest quality reconstruction followed by POCS and MWNI. In the case of low  $SNR$  TCOM and POCS do a much better job than MWNI. This is likely because of the power of the weighted reinsertion step used by POCS and TCOM compared to the IRLS approach used by MWNI. In the case of sparse sampling all three methods perform similarly, while in the case of highly dipping events MWNI has the best performance. In the case of low curvature (shown in Figure 1) POCS and MWNI have a similar performance of reasonable quality, while TCOM greatly outperforms these methods. In the case of high curvature both POCS and MWNI fail while TCOM reconstructs the data to a high quality. The reason for this is that POCS and MWNI reconstruct the data by imposing sparsity in the 4D wavenumber domain for each frequency. Linear or gently curved events can be reasonably reconstructed by a sparse approximation in the wavenumber domain, but this approximation can fail in the presence of strongly dipping events. For this reason Fourier reconstruction methods are often applied on small spatial windows of data where a linear approximation can be made. Conversely, TCOM is able to reconstruct highly curved events as the reconstruction is performed in the 4D spatial domain for each frequency. Curved events do not greatly increase the rank of the core tensor allowing for their accurate reconstruction.

Next the methods are applied to a dataset that has been decimated to varying degrees. The quality of the reconstruction is shown in Figure 2 for each of the three methods. The input data are composed of three linear events of moderate dip in all directions with a  $SNR = 5$ , dimensions of 150 time samples, and 16 samples in all four spatial dimensions. All three methods provide high quality reconstruction results to decimation levels approaching 80%, although it is clear that TCOM is providing a higher quality result than both POCS and MWNI. It is curious that the quality of MWNI improves gradually from a decimation level of 0% to approximately 70%. This is likely due to the difficulty the method has attenuating noise on existing traces (the method lacks a weighted reinsertion step as is used for both POCS and TCOM). As the number of input traces are reduced the ability of MWNI to estimate the noise free fully sampled data is improved (until the input data becomes too sparse).

Scenario	POCS	MWNI	TCOM
High <i>SNR</i>	26.38	12.85	39.51
Low <i>SNR</i>	7.28	2.54	10.40
Sparse Sampling	9.09	8.17	11.39
Highly Dipping	9.97	18.11	14.77
Low Curvature	8.21	7.62	21.74
High Curvature	1.02	1.70	15.06

Table 1: Quality of reconstruction results (measured in dB) for 5D synthetic data under various scenarios.

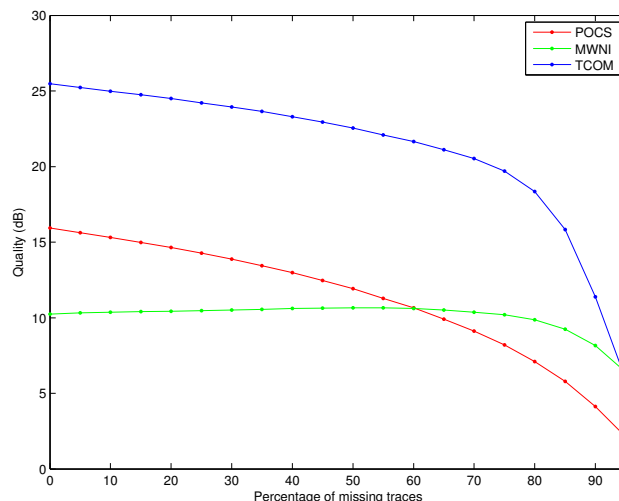


Figure 2: Quality of the reconstruction vs. the level of decimation of the input synthetic data. The data have  $SNR = 5$  and spatial dimensions of  $16 \times 16 \times 16 \times 16$ . They are comprised of three linear events with moderate dip in all directions.

## Real data example

The real data example considers the reconstruction of a land dataset acquired over a heavy oil target in Northern Alberta, Canada. The vertical component of the data is binned into 10m CMP bins, 100m offsets, and 36 azimuth sectors. A single CMP "snail gather" is shown in Figure 4. The reconstruction results for all three methods are quite similar but have some noticeable differences. The POCS result (b) is quite smooth with the exception of several noisy traces which are present on the input data. MWNI (c) appears to have fit these traces with the interpolated data giving an overall noisier result, while TCOM (d) appears to have smoothed out the high amplitudes to the surrounding traces. The TCOM result also appears to have dimming of amplitudes at near offsets, where few traces were present on input. The measurement of fit given in equation 2 confirm these observations. POCS is found to have a fit of 6.10 dB, TCOM a fit of 1.45 dB, and MWNI a fit of 7.73 dB. This confirms that TCOM is likely underfitting the data, while MWNI is likely overfitting the data. The smooth character of the POCS result combined with its high fit suggests that it is doing a reasonable job of attenuating the noise while fitting the existing observations. Calculating the quality of the reconstructions using the stacked data we find that all three methods have a similar reconstruction quality but the POCS reconstruction has the highest quality with a value of of 8.16 dB, MWNI a quality of 7.80 dB, and TCOM a quality of 6.75 dB. The stacked data after application of a 100Hz high cut filter is shown in Figure 5. A likely reason for TCOM obtaining a lower quality compared to POCS and MWNI is the dimming of amplitudes of some of the major events, such as the event with a peak frequency of  $\sim 40$ Hz with a two-way-time of  $\sim 525$  ms. This is also apparent when viewing the power spectra of the reconstructions shown in Figure 3. What is also apparent in the power spectra is the reconstruction of low frequencies. All three

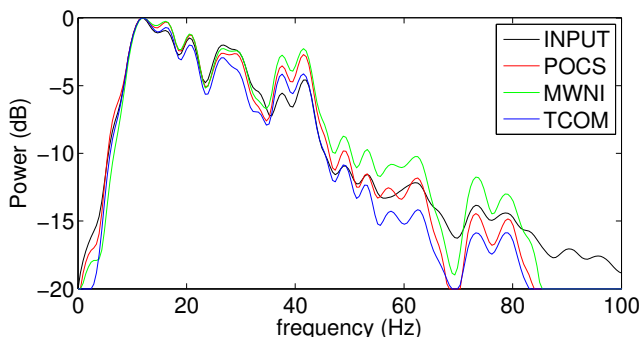


Figure 3: Power spectra of the stacked data.

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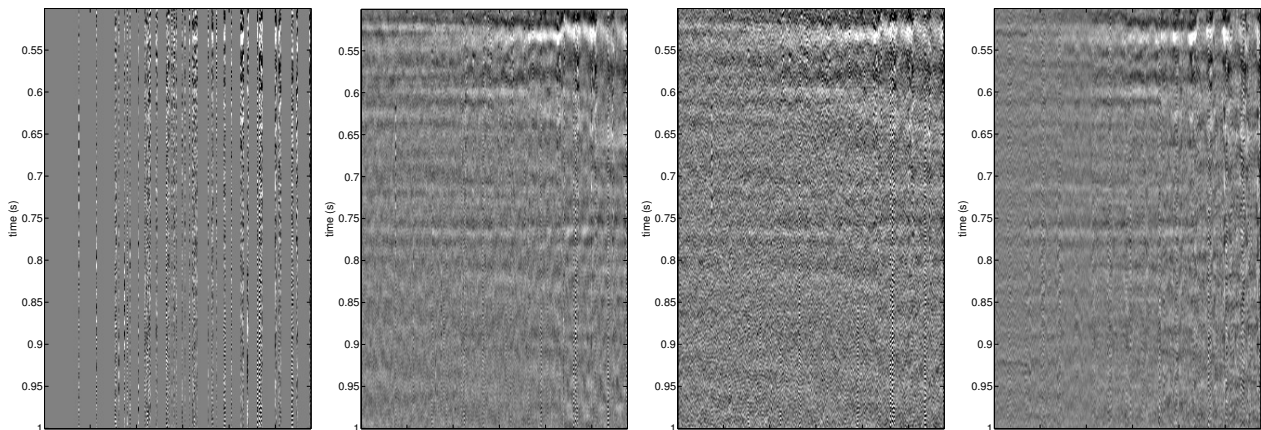


Figure 4: 5D reconstruction of a land dataset. A single midpoint location (1,14) sorted by offset (1-8) and azimuth sector (1-36) "snail gather" is shown. The input data with missing traces (a). Reconstruction using POCS (b). Reconstruction using MWNI (c). Reconstruction using TCOM (d).

methods are reducing the power of some low frequency energy, but POCS appears to be maintaining this energy the most followed by MWNI then TCOM. In the case of high frequencies MWNI does the best job of maintaining these amplitudes, though it is likely that much of this energy is noise. As observed on the prestack gathers MWNI is giving the noisiest reconstruction result, followed by POCS and TCOM. Based on these analyses POCS appears to be giving the best reconstruction overall.

## Conclusions

We find that in the case of synthetic data we can more easily quantify the quality of the reconstructions. In the case of real data this becomes more difficult as we lack the true data at all spatial locations necessary to measure the quality. We can define a measure of fit that compares the data at existing trace locations. Used in conjunction with a visual inspection of the pre-stack data we can use this to judge whether the reconstruction is underfitting or overfitting the input data. We also find that the stack of the input data provides an estimate of the noise free, fully sampled data at all midpoint locations which can be used to measure the quality of the reconstruction results. We find that in the case of synthetic data reconstructions TCOM provides superior results to both POCS and MWNI under most scenarios. In the case of low *SNR* both TCOM and POCS outperform MWNI. This is likely due to the success of the weighted reinsertion strategy that both of these methods employ. In the reconstruction of highly curved events we find that TCOM greatly outperforms the other two methods. In the reconstruction of real data we find that all three methods provide comparable results, but MWNI appears to be less proficient at denoising the data during reconstruction, and POCS is best able to preserve amplitudes during reconstruction.

## Acknowledgements

This abstract is presented upon request after being first presented at the 2012 SEG Convention in Las Vegas Nevada. The authors would like to thank the sponsors of the Signal Analysis and Imaging Group (SAIG) at the University of Alberta.

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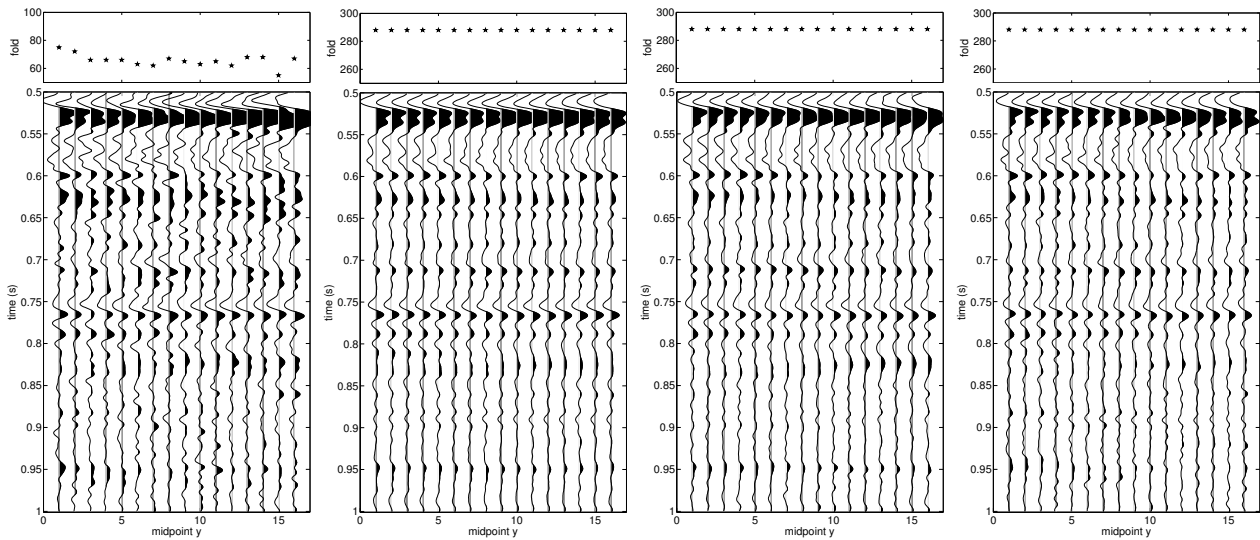


Figure 5: 5D reconstruction of a land dataset. A stacked inline is shown after application of a 100Hz high cut filter. The input data with irregular fold (a). Reconstruction using POCS (b). Reconstruction using MWNI (c). Reconstruction using TCOM (d).

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