On sampling optimization based on mutual coherency criterion

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Summary

Acquisition design plays a very significant role in seismic exploration and data processing. An optimized seismic acquisition design will require less resources and therefore, it can reduce the total cost of seismic exploration. Nevertheless, finding the optimal locations of sources and receivers in a seismic survey is a long-standing problem which has received less attraction in last few decades. It has been well known that higher bandwidth seismic data can be recovered from random sampling of a fixed number of sensors than uniform or regular sampling. However, controlling the maximum gap between sensors and satisfying other logistic constrains (e.g., land obstacle, preferred surface topographic regions) are not possible to ensure in random sampling. Therefore, in this paper, we have proposed a continuous non-uniform sampling (CNUS) technique to determine the optimal locations of receivers for seismic survey design while satisfying the maximum gap and logistic constraints. The proposed sampling method adopts the concept from the field of compressive sensing (CS). Our main goal is to reduce the mutual coherency between sampling scheme and sparsifying transform. However, the design of optimal receiver pattern is a non-linear optimization problem and hence, we have implemented global optimization method to solve this problem. Numerical experiments on optimal sampling technique show good performance.

Introduction

In conventional land data acquisition design, the field geometry is generally assumed dense and orthogonal not only to avoid spatio-temporal aliasing artifacts but also to obtain high-fidelity and high-resolution seismic data (Kerekes, 1998). Nevertheless, this classical acquisition technique requires resources that drastically increase the total cost of the survey and impose adverse impact on the environment (Cordsen et al., 2000). Hence, it is essential to make an optimal survey design with a limited number of sources and receivers. A number of methodologies have already been developed which implement computationally expensive cost functions such as wave equation based illumination analysis, full forward modelling of seismic wave-fields and imaging (Zhu and Cao, 2012), and the distribution of fold and azimuths (Hansruedi Maurer and Boerner, 2010).

Recently, a new mathematical tool named Compressive Sensing (CS) has been developed which permits sub-Nyquist sampling for data reconstruction (Donoho, 2006). Based on CS theory, a compressible signal can be recovered exactly from a set of measurements that are far fewer than the classical Nyquist sampling rate. A fundamental requirement to implement this technique in geophysics is to having a sparse representation of seismic data in some domain (Candes and Walkin., 2008). Fortunately, plenty of algorithms (e.g., Zwartjes and Sacchi (2007); Sacchi (2009); Hennenfent and Herrmann (2008)) have been developed that utilize the sparsity characteristics of seismic data. However, regular decimation of sampling is the main barrier to successfully implement the sparsity-promoting inversion methods as it yields a periodic aliases of the original signal. In contrast, non-uniform or random sampling can distribute the energy in frequency domain in such a way so that the actual spectrum can be distinguished from the noise like incoherent signal. However, random sampling is not possible to implement in real acquisition scenario as it makes large gaps between receivers and does not satisfy the logistic constraints. Large gap makes a big problem for local interpolation method (Trad, 2009). To overcome this problem, Hennenfent and Herrmann (Herrmann and Hennenfent, 2008) proposed discrete jitter under-sampling technique that have the advantages of random sampling and, at the same time, can control the maximum gap size. Nevertheless, gridding/binning used in the former method assigns a seismic trace to its nearest bin center which brings in a significant positioning error (Hindriks and Duijndam, 2000). That's why, it is efficient to consider the actual locations of non-uniform samples. In this paper, we have chosen non-uniform discrete Fourier transformation (NDFT) as a sparsifying domain which considers the exact locations of receivers without any binning. Based on recent result in CS theory (Elad, 2007), a projection scheme can be optimally designed with respect to a transform dictionary whereby the accuracy of reconstruction can be improved even using reduced number of samples. Several authors (e.g., Vahid Abolghasemi and Sanei (2010); Elad (2007)) defined mutual coherency between transformation and measurement matrix as a criterion for the accuracy of reconstruction. In this paper, we have adopted mutual coherency criteria to optimize sampling which favours sparsity-promoting reconstruction. We have tackled this non-linear optimization problem using global optimization method.

The article is organized as follows. First we review the principles of CS method and mutual coherency criteria. Next, we introduce the proposed optimization method NDFT. Eventually, we implemented numerical experiments to manifest the effectiveness of the proposed optimization technique.

Theory

Compressive sensing (CS) is an efficient technique which applies a non-adaptive measurement matrix to keep the structure of the signal. It can acquire signals with much smaller sampling rate than the Nyquist rate (Baraniuk, 2008). It is a mathematical tool which provides a joint sampling and compression process for signals (Elad, 2007). Using a sampling matrix $\mathbf{S} \in R^{p \times n}$ where $p \ll n$, based on CS theory, $\mathbf{x}$ can be reconstructed from sampled data $\mathbf{y}$. In the following equation, ε is the measurement noise.

$$\mathbf{y} = \mathbf{S}\mathbf{x} + \varepsilon \tag{1}$$

However, two conditions such as sparsity and incoherence, require to be satisfied to reconstruct data using CS theory. If $\mathbf{x}$ is a signal and $\mathbf{D}$ is a dictionary such as Fourier or wavelets, then $\mathbf{x}$ can be represented as $\mathbf{x} = \mathbf{D}\alpha$. Here, α is the vector of coefficients which represents $\mathbf{x}$ as a linear combination of $\mathbf{D}$. A signal is said to be sparse in dictionary **D** if a very few coefficients of α is non-zero. Fortunately, seismic data is proved to be sparse in Fourier and other domain (Sacchi (2009); Hennenfent and Herrmann (2008)). Second condition in CS theory is that the coherence between sampling matrix (**S**) and transformation dictionary (**D**) should be as small as possible. Mutual coherency is one way to verify this constraint.

The mutual-coherence is defined as the largest absolute and normalized inner product between different columns in $\tilde{\mathbf{D}}$ ($\tilde{\mathbf{D}} = \mathbf{S}\mathbf{D}$). It can be represented by the following equation.

$$\mu(\tilde{\mathbf{D}}) = \max_{1 \le i,j \le k \text{ and } i \ne j} \frac{|\tilde{\mathbf{d}}_i^H \tilde{\mathbf{d}}_j|}{||\tilde{\mathbf{d}}_i|| \cdot ||\tilde{\mathbf{d}}_j||} \tag{2}$$

A different way to represent the mutual coherency is by considering the Gramian matrix of $\hat{\mathbf{D}}$ where $\hat{\mathbf{D}}$ is computed by normalizing each column of $\tilde{\mathbf{D}}$.

$$\mathbf{G} = \hat{\mathbf{D}}^H \hat{\mathbf{D}}. \tag{3}$$

Therefore, we have to deduce an optimized sampling so that the mutual coherency becomes very small. If the above two conditions (e.g., sparsity and mutual coherency) are met, then we can reconstruct the seismic data by solving the following non-convex optimization problem.

$$\begin{aligned} &\underset{\alpha}{\text{minimize}} \quad ||\alpha||_1 \\ &\text{subject to} \quad \mathbf{y} \cong \mathbf{S}\mathbf{D}\alpha. \end{aligned} \tag{4}$$

In this paper, we have assumed non-uniform discrete Fourier transformation (NDFT) as a sparsifying domain and hence the seismic data can be represented as follows:

$$\mathbf{y} = \mathbf{D}\alpha, \tag{5}$$

where, $\mathbf{D}_{n\times m} = exp^{-i2\pi k_m x_n}$, and x_n is the exact locations of receivers. The adjoint of NDFT operator can be represented by the following equation.

$$\hat{\alpha} = \mathbf{D^H y}, \tag{6}$$

$$= \mathbf{D^H D}\alpha. \tag{7}$$

Here, $\mathbf{D^H D}$ is a circulant or symmetric matrix. Artifacts in NDFT coefficients (equation [7]) can be reduced by approaching $\mathbf{D^H D}$ as an identity matrix. It is similar to minimizing the mutual coherency of $\mathbf{D}$ as represented by Gramian matrix. Therefore, the following optimization problem needs to solved to find the optimum sampling locations (i.e., x_n).

$$\underset{x\in X}{\text{argmin}}\ \mu(\mathbf{D}) \tag{8}$$

Here, we have implemented simulated annealing (SA) to solve this non-linear optimization problem.

Examples

To investigate the performance of the proposed method in 1D spatial scenario, numerical experiments have been accomplished with different percentages of decimation. To proceed, we have initiated with a regularly under sampled pattern of receivers. Subsequently, we assigned the maximum allowable perturbation of each receiver. In fact, this will regulate the maximum gap in the optimized receiver distribution. In the first example, 40% receivers were decimated uniformly out of 100 and subsequently, SA has been implemented to minimize the mutual coherency by satisfying the maximum gap constraints. The optimized pattern of receivers is portrayed in Figure 1a which shows that receivers are almost evenly distributed. In contrast, random sampling pattern is shown in Figure 1b where receivers' pattern has large discrepency and clusteredness. Convergence of the algorithm is shown in Figure 2. The impulse responses of optimized and random sampling are shown in Figure 3. To rigorously examine the behaviour of optimized sampling under different scenarios, we compute the mutual coherency and maximum gap as a function of percentages of measurements. In each undersampling rate, we generated 100 optimized and random sampling pattern and computed their corresponding mutual coherency and max gap while keeping the number of iteration of global optimization method constant. The mean and standard deviation of mutual coherency and maximum gap are shown in Figures 4a, and 4b respectively. The figures demonstrated that optimized sampling always outperforms random sampling both in terms of mutual coherency and maximum gap.

We have extended 1D spatial scenario to two spatial dimensions. Like 1D case, we have initiated with a regular under sampled pattern of receivers in 2D case too. To proceed, we have generated classical Cartesian acquisition grid with 75% regular undersampling. It is shown in Figure 5a. Subsequently, optimized sampling distribution, depicted in Figure 5b, has been generated by applying optimization technique. Random sampling is shown in Figure 5c. The random sampling has higher discrepancy and larger gap than optimized sampling. The wavenumber spectra of random and optimized sampling have been shown in Figure 6. The mutual coherency of optimized and random sampling are 0.432 and 0.569 respectively which refers optimized sampling has less artifacts than random sampling.

Conclusions

In this paper we have optimized the locations of sparse receivers both in 1-D and 2-D spatial cases. We have considered mutual coherency as a criterion to optimize sampling which favours sparsity-promoting reconstruction method. The regularly decimated receivers' pattern has been perturbed using a constraint of maximum perturbation to control the maximum gap between sensors. The similar technique can be implemented to meet other logistic constraints (e.g., land obstacle, preferred surface topographic regions etc.). Numerical experiments for different percentages of measurements have been implemented to verify this method. Monte Carlo simulation has been applied to make comparisons between optimized and random sampling. The results show that optimized sampling outperforms random sampling both in terms of mutual coherency and maximum gap.

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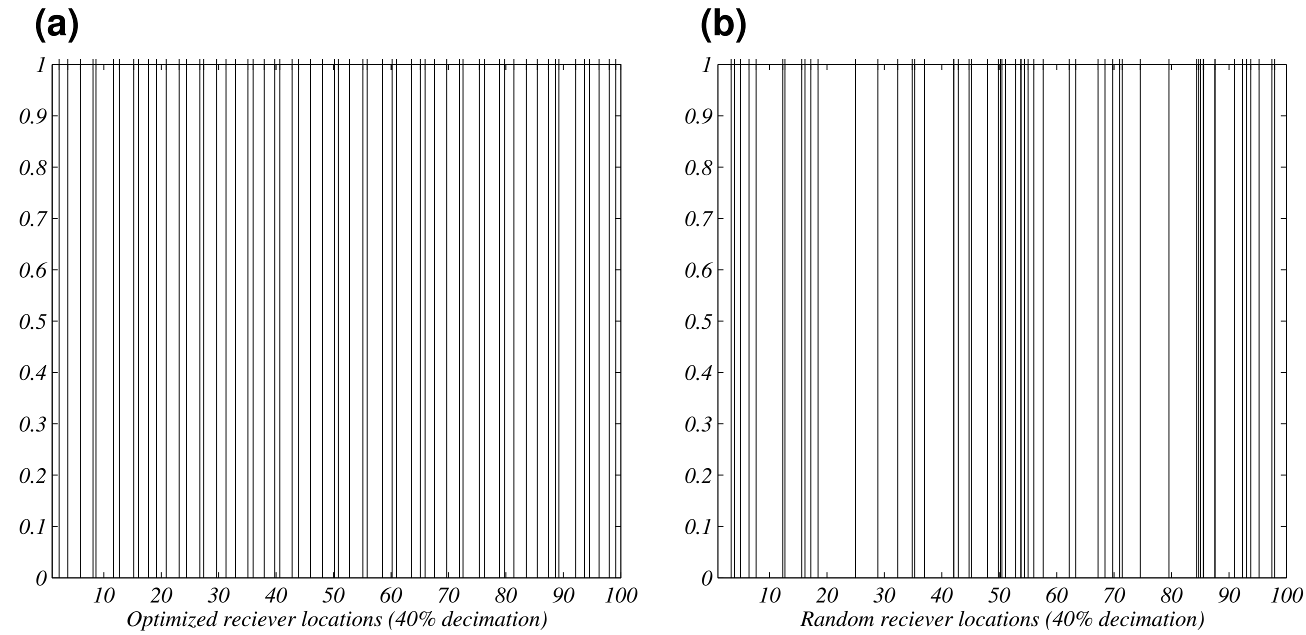

Figure 1 *Receiver distribution. (a) Optimal receiver locations (with 40% decimation). (b) Random receiver locations (with 40% decimation).*

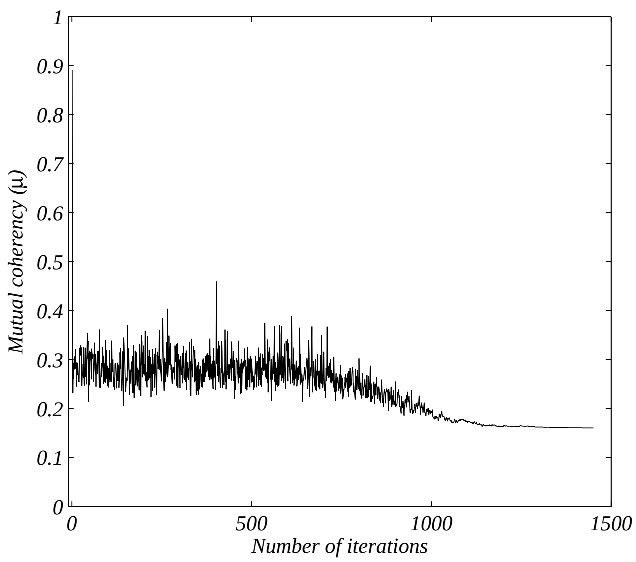

Figure 2 *Convergence of the Simulated Annealing algorithm.*

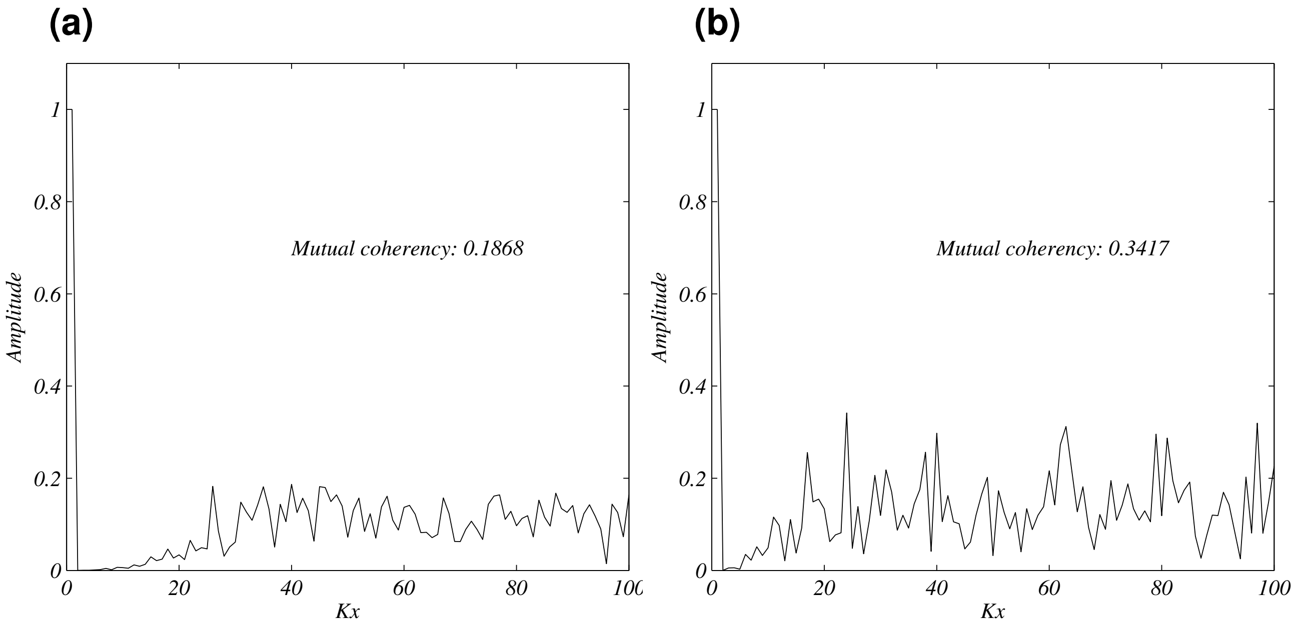

Figure 3 *Impulse response (with 40% decimated receiver). (a) Impulse response of optimized sampling distribution. (b) Impulse response of random sampling distribution.*

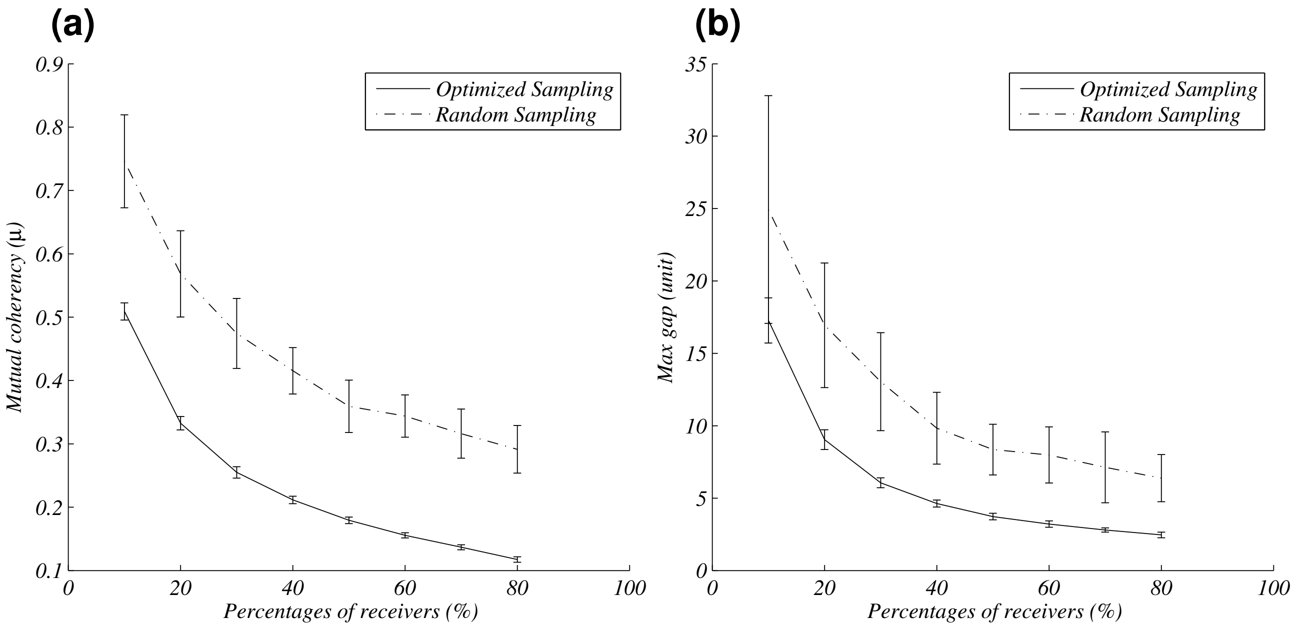

Figure 4 *(a) Mutual coherency and (b) maximum gap distribution as a function of percentages of measurements, with random sampling and optimized sampling.*

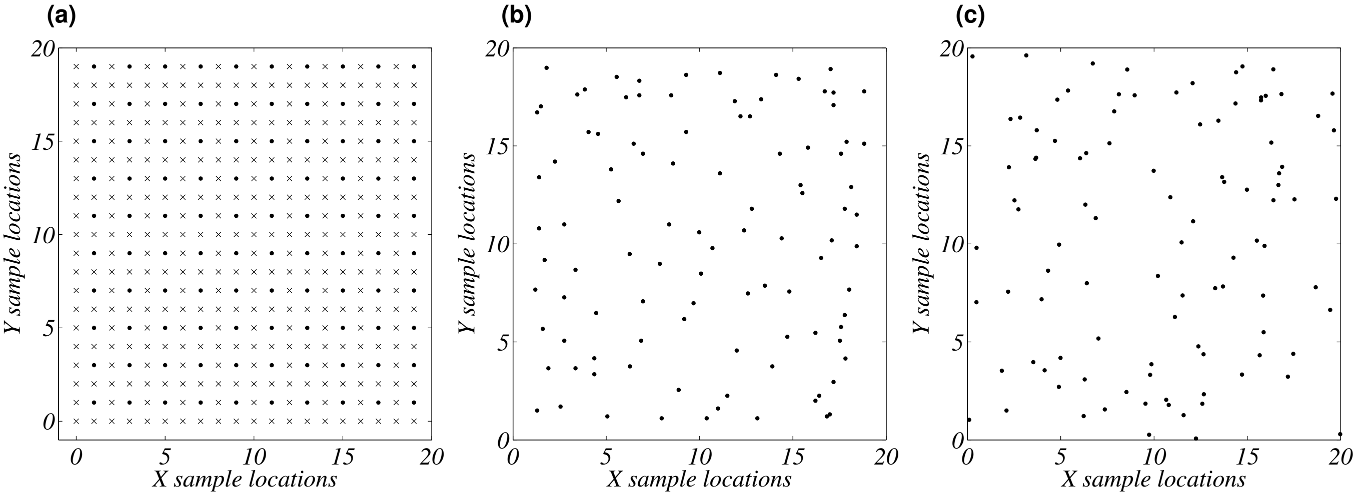

Figure 5 *2-D Receiver distribution. (a) Initial receiver locations (with 75% decimation). (b) Optimal receiver locations (with 75% decimation). (b) Random receiver locations (with 75% decimation).*

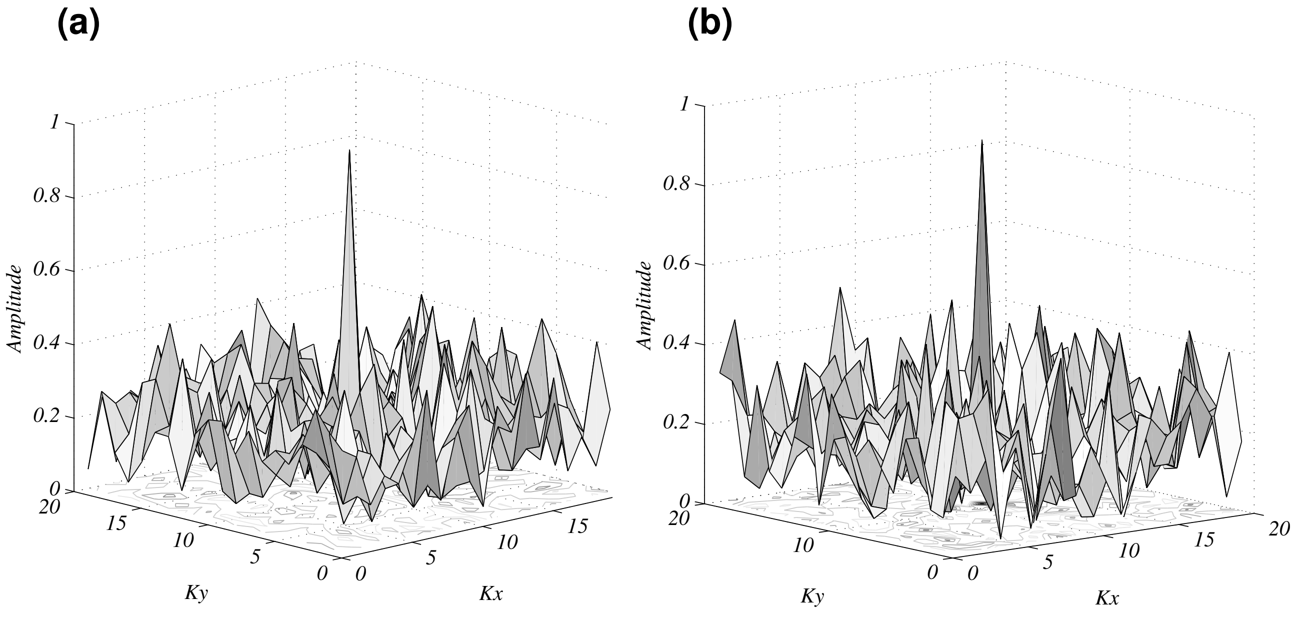

Figure 6 *2-D Impulse response. (a) Impulse response of optimized sampling distribution ($\mu = 0.432$). (b) Impulse response of random sampling distribution ($\mu = 0.569$).*