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Calculation of Amplitude and Velocity in General Anisotropic Media

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Summary

Seismic exploration in anisotropic media requires special processing, imaging and interpretation work flows. Some of the cornerstones of seismic data analysis for isotropic media are of limited use when anisotropy is introduced. In this paper, a unified solution for slowness and amplitude in general anisotropic medium is presented and the results are compared with direct lab measurments. For given direction and polarization of incident plane wave, solutions for slowness of reflected and transmitted waves are derived from eigenvalues and eigenvectors of the Christoffel-Kelvin equations. Continuity of traction and displacements are the two boundary conditions that are solved to give the amplitudes of reflected and transmitted waves. Here we present the governing equations and solutions for slowness, reflectivity and transmissibility for all phases with examples from triclinic, monoclinic and orthorhombic anisotropic symmetries. Comparing reflectivity from analytical solution and direct lab measurements from Phenlic CE leads credence to the accuracy of solution.

Introduction

The study of crystals, particularly for use in early electronic oscillators, motivated some of the earliest studies of elastic wave anisotropy. has its main contribution in early elastic wave analysis concerning crystal oscillators(Rokhlin 1974). Today, however, the study of elastic wave propagation in anisotropic media has had the largest impact in applied geophysics as it greatly influences seismic data processing, imaging and interpretation. Dynamic corrections, AVO attributes, and migration techniques are only few of many seismic tools for which consideration of anisotropy improves the utility of the data. Many researchers from various background in seismic studied the effect of anisotropy on elastic wave propagation. Hearmon's (1960) research involved the application of elasticity on wave propagation through various anisotropic media. Bakus (1962) illustrated how elastic anisotropy is created from horizontal layering to produce transversely isotropic media. Musgrave (1970) and Auld(1973) solved for slowness and polarization for given ray path of plane wave propagating in general anisotropic media. Thomsen (1986) introduced weak elastic anisotropy to minimize number of stiffness parameters.

With a three and two term approximations of the Zeoppritz equation by Shuey (1985), Ruger (1997) and Tsvankin (1997), tried to mimic the isotropic AVO equations for maximum orthorhombic anisotropy with weak anisotropy and weak boundary assumptions. Details about weak anisotropy and weak interface can be found at Tsvankin (1997). Cheadle et al. (1991) provided physical modeling for orthorhombic material through ultrasonic lab at University of Calgary.

Chen et al (2001) created a three term AVO cross-plot in anisotropic media. With all these advances in theoretical solutions, Bouzidi and Schmitt (2012) and Oritz-Osornio and Schmitt (2013) used ultrasonic goniometry to monitor and model exact reflectivity and transmissibility of anisotropic objects in water tank. Their elastic and visco-elastic wave modeling have motivated authors to expand it to general anisotropic scale.

In this paper, we employ the unified solution of Musgrave (1970) for slowness and amplitudes for all of

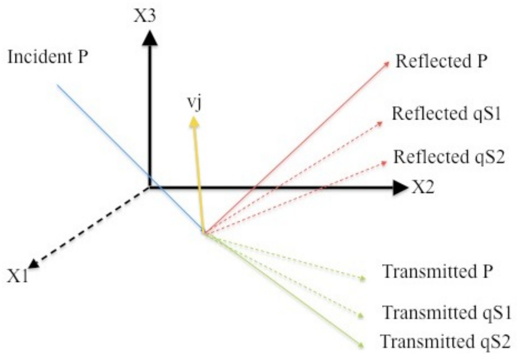


Figure 1 *Geometry of problem, v_j is normal to the boundary interface.*

the wave modes generated upon reflection form an arbitrary interface. The algorithm so developed is then tested using a variety of different anisotropic symmetries of increasing complexity from isotropy through to triclinic. Finally, the experimental results of the direct ultrasonic reflectivity measurements from a water-phenolic CE interface obtained by Oritz-Osornio and Schmitt (2013) will be compared with an existing analytical solution.

Theory

In order to find out anisotropic media effects on elastic wave propagation, lets start with general wave equation 1 and plane wave equation 2.

$$\rho \ddot{u}_i = \frac{1}{2} c_{ijkl} \frac{\partial}{\partial x_j} \left(\frac{\partial u_k}{\partial x_l} + \frac{\partial u_l}{\partial x_k} \right) \tag{1}$$

$$u_k = A^0 P_k e^{(-iK(n.r-Vt))} \tag{2}$$

where u_k is component of displacement vector, A^0 is incident wave amplitude, $\mathbf{P}_k$ is displacement vector, K is wavenumber, V is wave velocity, n is wave normal, $\mathbf{r}$ is position vector, $\mathbf{c}_{ijkl}$ is stiffness matrix and ρ is density of medium. Solution of velocities and displacements are derived from substituting 2 into the 1. This raises system of equation known as Christoffel-Kelvin equation 3, (Musgrave, 2002).

$$(\mathbf{T}_{ik} - \rho V^2 \delta_{ik})\mathbf{P}_k = 0 \tag{3}$$

or

$$\begin{bmatrix} \mathbf{T}_{11} - \rho V^2 & \mathbf{T}_{12} & \mathbf{T}_{13} \\ \mathbf{T}_{12} V^2 & \mathbf{T}_{22} - \rho V^2 & \mathbf{T}_{23} \\ \mathbf{T}_{13} V^2 & \mathbf{T}_{23} & \mathbf{T}_{33} - \rho V^2 \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix} = 0$$

where $\mathbf{T}_{ik} = \mathbf{C}_{ijkl}\mathbf{n}_j\mathbf{n}_l$. Nontrivial solution comes from non-zero displacement vector, so $|T_{ik} - \rho V^2 \delta_{ik}| = 0$ releases phase velocities which are eigenvalues of 3. Displacement vector is also calculated from eigenvalues and eigenvectors of 3.

To determine the properties of reflected and refracted waves , without loosing generality of this approach, consider an incident plane wave arriving from above to a plane boundary between two anisotropic media, figure 1. General Snell's law for anisotropic media can be formulated as (1) all generated reflected and refracted waves are in the same plane perpendicular to boundary interface as the incident ray . (2) All projected horizontal slownesses ($\mathbf{S}$) from incident($\mathbf{S}^I$), reflected($\mathbf{S}^R$), and transmitted($\mathbf{S}^T$) rays on the boundary interface must be equal to one another.

For typical horizontal boundaries, all non-vertical slowness elements must be equal, $\mathbf{S}_H^I = \mathbf{S}_H^R = \mathbf{S}_H^T$, (Musgrave, 1970). With these conditions in mind, finding the vertical components of the reflected and the transmitted waves requires use of the Christoffel-Kelvin equation 3 $\mathbf{S}_3^{Ri}$ and $\mathbf{S}_3^{Ti}$ separately, knowing **S** = **n**/V. One can follow the steps developed by Rokhlin et al. (1985) to find a solution for the slownesses. For each reflected and refracted ray, six solutions exist of which only three are physically valid. The geometry of the problem enables us to pick those proper three solutions. The reflected and transmitted angles and the velocities of the 6 resulting elastic wave modes are calculated at this stage.

Reflectivity and Transmissibility Calculation

With these slownesses the particle displacement vectors for all six of the generated elastic waves ($i = 1,2,3$ for reflected waves and $4,5,6$ for transmitted waves in order of $qP, qS1, qS2$) may then also be caluculated. To maintain the rigidity of the boundary interface, two principle boundary conditions must be satisfied, the continuity of the displacement vector and the continuity of the traction force on the interface, 4.

$$\begin{aligned} \mathbf{u}_i^I + \Sigma \mathbf{u}_i^R &= \Sigma \mathbf{u}_i^T, \\ (\sigma_{ik}^I \upsilon_k) + \Sigma(\sigma_{ik}^R \upsilon_k)^R &= \Sigma(\sigma_{ik}^T \upsilon_k)^T \end{aligned} \tag{4}$$

where υ is normal to the boundary interface. We can rewrite 4 in the following six algebraic equations (5) where the amplitudes are only unknowns.

$$\begin{aligned} A^{(1)}P_i^{(1)} + A^{(2)}P_i^{(2)} + A^{(3)}P_i^{(3)} + A^{(4)}P_i^{(4)} + A^{(5)}P_i^{(5)} + A^{(6)}P_i^{(6)} = A^{(0)}P_i^{(0)}, i = 1,2,3 \\ A^{(1)}\mathbf{C}_{ijkl}^R \upsilon_j \mathbf{S}_k^{(1)} \mathbf{P}_i^{(1)} + A^{(2)}\mathbf{C}_{ijkl}^R \upsilon_j \mathbf{S}_k^{(2)} \mathbf{P}_i^{(2)} + A^{(3)}\mathbf{C}_{ijkl}^R \upsilon_j \mathbf{S}_k^{(3)} \mathbf{P}_i^{(3)} \\ + A^{(4)}\mathbf{C}_{ijkl}^R \upsilon_j \mathbf{S}_k^{(4)} \mathbf{P}_i^{(4)} + A^{(5)}\mathbf{C}_{ijkl}^T \upsilon_j \mathbf{S}_k^{(5)} \mathbf{P}_i^{(5)} + A^{(6)}\mathbf{C}_{ijkl}^T \upsilon_j \mathbf{S}_k^{(6)} \mathbf{P}_i^{(6)} = A^{(0)}\mathbf{C}_{ijkl}^T \upsilon_j \mathbf{S}_k^{(0)} \mathbf{P}_i^{(0)} \end{aligned} \tag{5}$$

Reflectivity and transmissibility for all generated phase are calculated from amplitude ratio, 6.

$$\begin{aligned} R^{(\alpha)} &= \frac{A^\alpha}{A^0}, \alpha = 1(qP), 2(qS1), 3(qS2) \\ T^{(\alpha)} &= \frac{A^\alpha}{A^0}, \alpha = 4(qP), 5(qS1), 6(qS2) \end{aligned} \tag{6}$$

must be valid for all solutions.

Examples

In this section, calculated phase velocities from major anisotropic symmetries (Bass, 2013) are displayed in figure 2

The PP reflection coefficient of Phenolic CE sample was measured at Experimental Geophysics Group (EGG) lab, details are in (Ortiz-Osornio and Schmitt, 2013) and (Bouzidi and Schmitt, 2012). Figure 3 illustrates calculated and observed reflectivity for given phenolic CE elastic properties for various incident and azimuth angle.

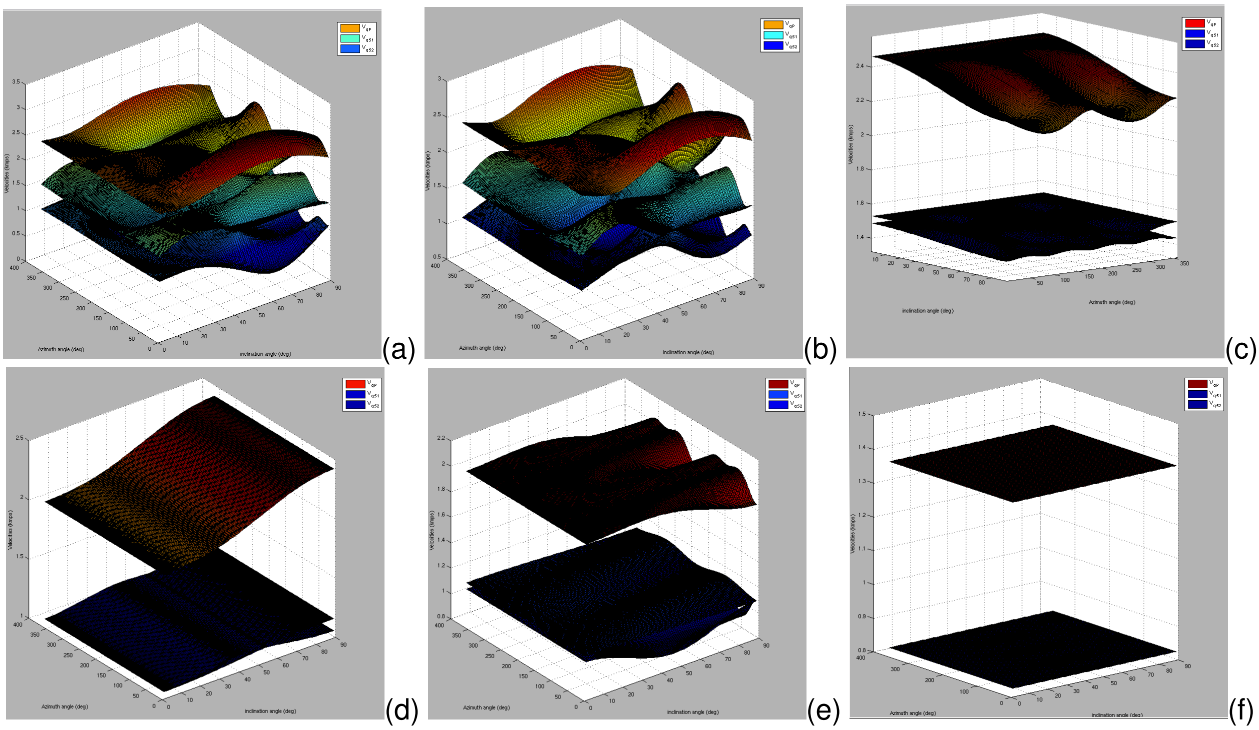
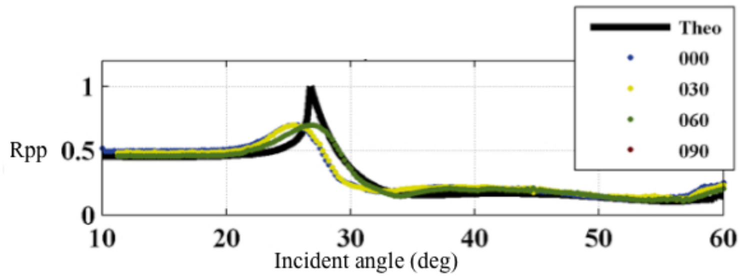
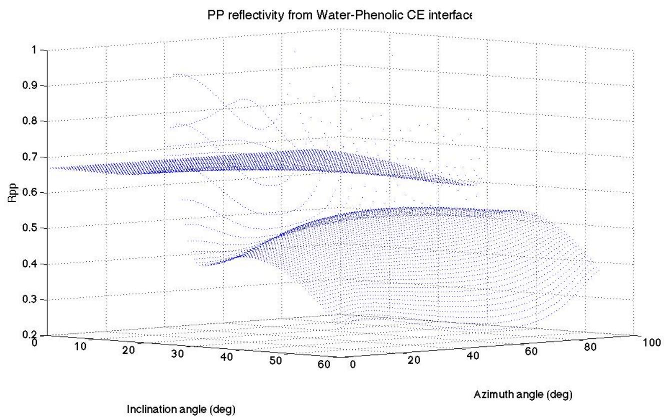


Figure 2 *Calculated phase velocity in common anisotropic symmetries, (a) Triclinic (b) Monoclinic (c) Orthorhombic (d) VTI, (e) HTI and (f)Isotropic. Calculated velocity (vertical axis) versus azimuth (left axis) and inclination angle (right axis).*



(a)



(b)

Figure 3 *P-wave reflection coefficient from water-phenolic CE interface (a) Observed (b) Calculated.*

Comparing results from PP reflectivity, direct measurements and theory have the same trend before and after the critical angle, however measured data R_{pp} are smoother than theory, this is due to the effects of the transducer beam, which cannot be a plane wave, in the vicinity of critical angles. This is discussed in detail by Bouzidi and Schmitt (2012)

Conclusions

In this paper we developed an algorithm to calculate the elastic wave behavior in general anisotropic media from incident plane wave. An important part of this is that we can now determine the reflectivity from the interface between two arbitrary anisotropic half-spaces. The algorithm includes the solution for slownesses, amplitudes, and polarizations of the generated qP, qS1, qS2 reflected and refracted waves. As a test, the PP reflectivity of phenolic CE is calculated analytically and compared to the solution. Analytical solutions are highly correlated with the actual lab measurements. Future works will be dedicated to application of this solution in inversion of surface seismic data.

Acknowledgements

The authors thank NSERC and the Canada Research Chairs program for support of this project of Experimental Geophysics Group (EGG) at the University of Alberta.

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