# A framework for linear and nonlinear Converted wave time-lapse difference AVO

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## Summary

Multicomponent time-lapse amplitude-variation-with-offset (AVO) may improve approximating time-lapse difference data. The difference data during the change in a reservoir from the baseline survey relative to the monitor survey are expressed for converted wave. We define a framework for the difference data in order of both baseline interface contrast and time-lapse changes. The higher order terms represent corrections appropriate for large contrasts. We conclude that in many plausible time-lapse scenarios the increase in accuracy associated with higher order corrections is non-negligible for converted wave as well as P-wave. Furthermore coupling between baseline and time-lapse quantities is non-negligible when contrasts are large.

## Introduction

Production or employment of enhanced oil recovery techniques will affect the reservoir properties such as fluid flow and pressure. 4D Time-lapse monitors these changes with acquiring a baseline survey prior to the production and one or more monitor surveys over an interval of time during which the geological/geophysical characteristics of a reservoir change (Lumley, 2001; Landro, 2001).

Multicomponent surveying has developed rapidly in both land and marine acquisition with many applications in seismology including reservoir monitoring. The elastic properties of a rock change when the pressure and fluid flow is altered in a reservoir due to production. This raises the necessity of multicomponent 4D time-lapse analysis in a reservoir (Stewart et al., 2003).

A framework has been formulated to model linear and nonlinear elastic time-lapse difference for P-P sections (Jabbari and Innanen, 2013). The study described here focuses on applying the perturbation theory in a time-lapse amplitude variation with offset (Time-lapse AVO) methods to derive a framework to model the difference data for the converted wave.

## Theory and Methods

Time-lapse AVO connotes the analysis of changes to the offset or angle dependence of reflection coefficients from the baseline to the monitor survey. Consider an incident P or S wave striking the boundary between two elastic media which are incidence medium and reservoir with rock properties $V_{P0}$, $V_{S0}$, $\rho_0$ (above) and $V_{PBL}$, $V_{SBL}$, $\rho_{BL}$ (below). The reservoir properties change to $V_{PM}$, $V_{SM}$, $\rho_M$ in the monitor survey.

Reflection coefficients for the baseline and monitor survey are calculated by solving the Zoeppritz equations, once with the target properties [BASELINE] and once with the target properties [MONITOR]. Difference reflection coefficients are determined by subtracting the baseline reflection data from the monitor reflection data.

To model this difference in a physically interpretable way, we introduce two groups of perturbation parameters; perturbations “b” representing the change from the incidence medium to the target medium in the baseline survey, and perturbations “a” representing target medium changes from the baseline to monitor survey (Innanen, 2013; Solt and Weglein, 2012):

$$b_{VP} = 1 - \frac{V_{P_0}^2}{V_{P_{BL}}^2}, \quad b_{VS} = 1 - \frac{V_{S_0}^2}{V_{S_{BL}}^2}, \quad b_{\rho} = 1 - \frac{\rho_0}{\rho_{BL}},$$

$$a_{VP} = 1 - \frac{V_{P_{BL}}^2}{V_{P_M}^2}, \quad a_{VS} = 1 - \frac{V_{S_{BL}}^2}{V_{S_M}^2}, \quad a_{\rho} = 1 - \frac{\rho_{BL}}{\rho_M},$$

Substituting these perturbations into the two requisite instances of the Zoeppritz equations (modeling the baseline and the monitoring reflection amplitudes), we derive a series expansion for the difference data reflection as follows (Aki and Richards, 2002).

$$P_{BL} \begin{bmatrix} R_{PP} \\ R_{PS} \\ T_{PP} \\ T_{PS} \end{bmatrix} = b_{BL} \quad R_{PS}^{BL}(\theta) = \frac{\det(P_P)}{\det(P)} \qquad P_M \begin{bmatrix} R_{PP} \\ R_{PS} \\ T_{PP} \\ T_{PS} \end{bmatrix} = b_M \quad R_{PS}^{M}(\theta) = \frac{\det(P_P)}{\det(P)}$$

$$S_{BL} \begin{bmatrix} R_{SS} \\ R_{SP} \\ T_{SS} \\ T_{SP} \end{bmatrix} = c_{BL} \quad R_{SP}^{BL}(\varphi) = \frac{\det(S_S)}{\det(S)} \qquad S_M \begin{bmatrix} R_{SS} \\ R_{SP} \\ T_{SS} \\ T_{SP} \end{bmatrix} = c_M \quad R_{SP}^{M}(\varphi) = \frac{\det(S_S)}{\det(S)}$$

Where $P_{BL}$ $P_M$, $S_{BL}$, and $S_M$ are Zeoppritz metrics for the baseline and monitor survey for incident P and S wave. $P_P$, $S_S$ are the P and S metrics with the second column replaced by the vector b and c respectively. R’s and T’s are reflection and transmission coefficients for PS and SP waves.

$$\Delta R_{PS}(\theta) = R_{PS}^{(M)}(\theta) - R_{PS}^{(BL)}(\theta) \qquad \Delta R_{SP}(\varphi) = R_{SP}^{(M)}(\varphi) - R_{SP}^{(BL)}(\varphi)$$

$$\Delta R_{PS}(\theta) = \Delta R_{PS}^{(1)}(\theta) + \Delta R_{PS}^{(2)}(\theta) + \Delta R_{PS}^{(3)}(\theta) + \dots \qquad \Delta R_{SP}(\varphi) = \Delta R_{SP}^{(1)}(\varphi) + \Delta R_{SP}^{(2)}(\varphi) + \Delta R_{SP}^{(3)}(\varphi) + \dots$$

where θ and φ are P and S waves incident angles on the interface between the cap rock and reservoir.
Linear and higher order terms for PS and SP are:

$$\Delta R_{PS}^{(1)}(\theta) = k_1^1 a_{VS} + k_2^1 a_\rho \qquad \Delta R_{SP}^{(1)}(\varphi) = l_1^1 a_{VS} + l_2^1 a_\rho$$

$$\Delta R_{PS}^{(2)}(\theta) = k_1^2 a_{VS}^2 + k_2^2 a_\rho^2 + k_3^2 a_\rho b_\rho + k_4^2 a_{VS} b_{VS} + k_5^2 \left(a_{VP} a_{VS} + a_{VP} b_{VS} + b_{VP} a_{VS}\right)$$
$$+ k_6^2 \left(a_{VP} a_\rho + a_{VP} b_\rho + b_{VP} a_\rho + a_{VS} a_\rho + b_{VS} a_\rho + a_{VS} b_\rho\right)$$

$$\Delta R_{SP}^{(2)}(\varphi) = l_1^2 a_{VS}^2 + l_2^2 a_\rho^2 + l_3^2 a_\rho b_\rho + l_4^2 a_{VS} b_{VS} + l_5^2 \left(a_{VP} a_{VS} + a_{VP} b_{VS} + b_{VP} a_{VS}\right)$$
$$+ l_6^2 \left(a_{VP} a_\rho + a_{VP} b_\rho + b_{VP} a_\rho + a_{VS} a_\rho + b_{VS} a_\rho + a_{VS} b_\rho\right)$$

where all coefficients are functions of $V_{P0}$, $V_{S0}$, sinθ and sinφ. These coefficients plus third order terms are presented in details in the reference (Jabbari and Innanen, 2013).

## Numerical behaviour of truncations of the difference AVO series

We will complete this initial discussion with an examination of the numerical influence of the low order nonlinear terms of the series. We conduct a numerical test **to** examine the derived linear and non linear difference time lapse AVO terms for the PS and SP waves**.** We plot the exact difference reflection coefficient associated with such a change (black curve). This curve is compared with the linear (blue curve) and higher order approximations (red and green curves) embodied in the equations above (Figure 1). The second and third order approximations are in a good agreement with the exact difference in the pre-critical regime. Since we truncate our approximations beyond first order in $\sin^2\theta$, this is expected and serves to define the domain of their applicability (higher order terms in the incidence angle can be used if so desired. We conclude that the nonlinearity of the relationship between the difference reflection coefficient and perturbations in both the baseline medium and the time-lapse changes may be significant and non-negligible in geophysically plausible scenarios.

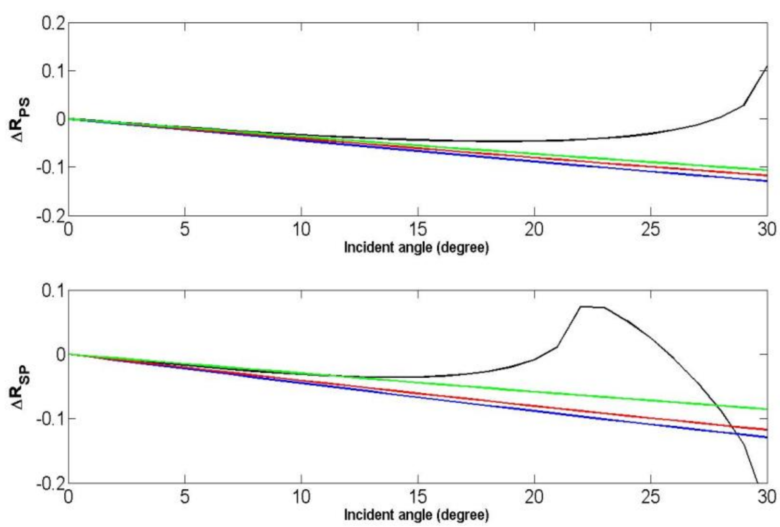


Figure 1: $\Delta R_{PS}$ and $\Delta R_{SP}$ for the exact, linear, second order, and third order approximation. Elastic incidence parameters: $V_{P0}$ =2000 m/s, $V_{S0}$ = 1500 m/s and $\rho_0$ = 2.0 g/cc ; Baseline parameters: $V_{PBL}$ = 3000 m/s, $V_{SBL}$ =1700 m/s and $\rho_{BL}$ = 2.1 g/cc ; Monitor parameters: $V_{PM}$ = 4000 m/s, $V_{SM}$ = 1900 m/s and $\rho_M$ =2.3 g/cc.

## Conclusions

Perturbation theory can be used to pose time-lapse seismic monitoring problems in a quantitative and interpretable and easily analyzable way. Forms for elastic difference reflection coefficients that closely resemble standard linearized AVO equations are derivable, with nonlinear corrections that include coupling terms between baseline and time-lapse changes.

Changes in the fluid saturation and pressure will have an impact in elastic parameters of subsurface, such as P and S wave velocities and density. Jabbari and Innanen (2013) have already investigated P-wave time-lapse AVO and showed that adding the higher order terms to the linear approximation for difference time-lapse data increases the accuracy of $\Delta R_{PP}$ and corrects the error due to linearizing $\Delta R_{PP}$. In the current research, we extended this work by formulating a framework for the difference reflection data for converted wave. The results showed that, including higher order terms in $\Delta R_{PS}$, and $\Delta R_{SP}$ improves the accuracy of approximating time lapse difference reflection data, particularly for large contrast cases. Also $\Delta R_{PS}$ and $\Delta R_{SP}$ are different when the higher order terms are included in the approximating the difference data, which is the case for the exact difference data.

Numerical studies indicate that in geophysically plausible (though reasonably large-contrast) scenarios these nonlinear terms can have significant impact in pre-critical regimes.

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