CORRELATION TECHNIQUES — A REVIEW*

By

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ABSTRACT

Correlation techniques are in the process of passing from the research laboratory to the field. This paper seeks to aid the transition. The three main sections of the paper have these objects:

1. To state in words the basic principles on which correlation techniques are based, and the connection between these principles and more familiar concepts which are already among the tools of the exploration geophysicist.

2. To review briefly the published applications of correlation techniques in science generally.

3. To discuss some of the additional applications which are now emerging in exploration geophysics.

The treatment is graphical and illustrative, rather than rigorous. The paper is addressed to the practising exploration geoscientist and to the newcomer to correlation techniques. A list of about 175 references is given.

INTRODUCTION

Some things, like two peas in a pod, are similar; others, like chalk and cheese, are not.

Throughout science, however, we find instances where the situation is not as clear as this, and where it is desirable to establish a *measure* of the similarity between two quantities. For such applications we require the techniques of correlation.

The mathematical treatment of these techniques is given excellently in the existing literature (for example, by Lee, 1960), and the use of these techniques for research purposes has been established for more than a decade. Until recently, however, the actual operation of correlation has been a slow one, so that correlation techniques could hardly be considered for routine use. Today, correlation in *real time* is entirely practical, and there is little doubt that correlation techniques will soon take their place in the day-to-day operations of the practising exploration geophysicist. Consequently it seems appropriate, at this time, to present the basic propositions of correlation theory in physical or graphical terms, to review briefly the published applications of correlation techniques throughout the broad field of science, and to discuss the emergent applications in practical geophysics.

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The Basic Propositions

A good method of measuring the similarity between two waveforms is to multiply them together, ordinate by ordinate, and to add the products over the duration of the waveforms. Thus to assess the similarity between waveforms a and b of figure 1, we multiply ordinate p_1 by ordin-

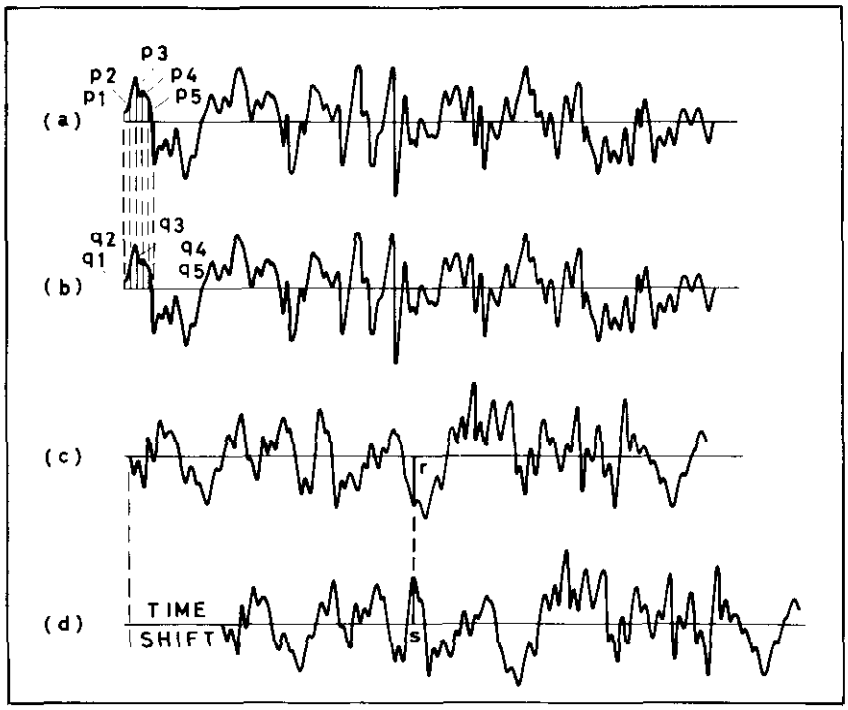

FIG 1.—Waveforms a and b are identical in shape and time. Waveforms c and d are identical in shape but d is shifted in time. The similarity between a and b is large, while that between b and c or between c and d is small.

ate q_1, ordinate p_2 by q_2, p_3 by q_3, and so on, and finally we add these products to obtain a single number which is a measure of the similarity. In this example, waveforms a and b are identical, so that every ordinate, positive or negative, contributes a positive term to the sum; the sum is therefore large, and the waveforms are pronounced similar. If, however, we perform the same process on waveforms b and c, we find that each positive product is offset by another negative product, and the sum is small; the waveforms are pronounced dissimilar.

Now let us consider waveforms c and d. They are identical in shape, but one is displaced in time. If we perform the process of multiplying ordinates (of which r and s are typical), we find again that every positive product tends to be offset by a negative product, and that the sum is small. Thus if we were to plot the similarity between a waveform of the type of figure 1c and a time-shifted version of itself, we should expect

the resulting curve to assume small values for large time shifts, and to rise to a large positive maximum when the time shift is zero. This curve is called the auto-correlation function.

The auto-correlation function of a waveform is a graph of the similarity between the waveform and a time-shifted version of itself, as a function of this time-shift.

We may obtain some feeling for the nature and merit of the auto-correlation function by considering the two examples of a sine wave and a sample of broad-band noise. The sine wave (figure 2) becomes exactly

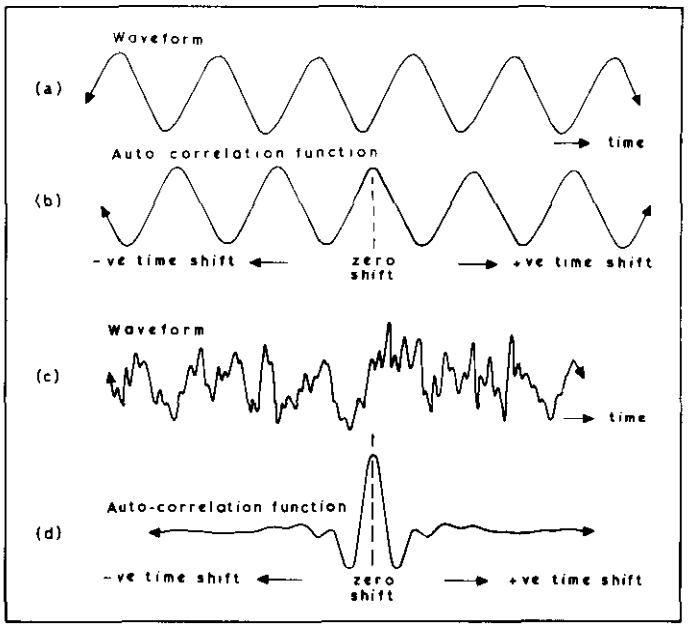

FIG. 2.—Waveform *a* is a section of a sine wave. Waveform *c* is a section of continuous broad-band noise. Waveforms *b* and *d* are the corresponding auto-correlation functions. Note that the x-axis of an auto-correlation function represents time *shift*, not time.

similar to itself whenever the time shift is an integral number of periods, and so the auto-correlation function must itself be periodic. In fact, the auto-correlation function of a sine wave is also sinusoidal, having the same frequency, and being symmetrical about the point which represents zero time shift (figure 2b). The broad-band noise of figure 2c is quite different; a very small time shift is sufficient to destroy the similarity, and the similarity never recurs. The auto-correlation function (figure 2d) is therefore a sharp impulse, decaying quickly from the central maximum to very low values at large time shifts. Moreover, the same argument would apply if we took any other sample of noise having the same frequency content; the auto-correlation function would be the same. Evi-

dently the auto-correlation function of a waveform does not depend on the actual waveform itself, but on its frequency content.

This is worth considering in Fourier terms. We know* that the frequency content of a waveform is defined by the amplitude spectrum, and that a variety of waveforms can be synthesized by associating a particular amplitude spectrum with a variety of phase spectra. For example, if (as in figure 3) we consider cosinusoidal Fourier components and set them all to zero phase at the time origin, then the waveform represented by the sum of these components must have a maximum at the time origin (where all the components reinforce) and must be symmetrical about the time origin. The adoption of any other phase spectrum must decrease, in general, the degree of reinforcement at the time origin and the completeness of the cancellation away from the time origin; thus other phase spectra usually produce waveforms which have lesser peak amplitude and greater duration—which are *dispersed* over a longer time. We know also that we may derive the power spectrum of a waveform by squaring the absolute amplitude of the Fourier components; thus the power spectrum accents any departures from "flatness" in the amplitude spectrum, and discards completely the information in the phase spectrum. If now we associate these well-known facts with our previous conclusion that the auto-correlation function depends not on the actual waveform but only on its frequency content, we find it easy to accept this proposition:

> *The auto-correlation function contains the same information as the power spectrum of a waveform. However, this information is presented in the form of a function of time, rather than frequency. The function may be visualized by imagining that the Fourier components of the waveform are squared in amplitude, set into phase at the origin of a new time scale, and added together.*

Thus a *waveform* is synthesized by combining Fourier components with the amplitudes given by the amplitude spectrum and the phases given by the phase spectrum; the *auto-correlation function of a waveform* is synthesized by combining Fourier components with the amplitudes given by the power spectrum of the waveform, and with zero phase.

Now we know that if we inject an input waveform (having known amplitude and phase spectra) into a system (having known amplitude-frequency and phase-frequency responses) we obtain an output whose amplitude spectrumi is given by multiplying the input amplitude spectrum by the system amplitude-frequency response and whose phase spectrum is given by adding the input phase spectrum to the system phase-frequency response. For any waveform, therefore, we may define a system whose amplitude-frequency response has the same form as the input amplitude spectrum and whose phase-frequency response is exactly opposite to the input phase spectrum (that is, it offsets each phase lead with an equal lag, and each lag with a lead); then the amplitude spec-

*For an elementary discussion of this background material, see Anstey (1965).

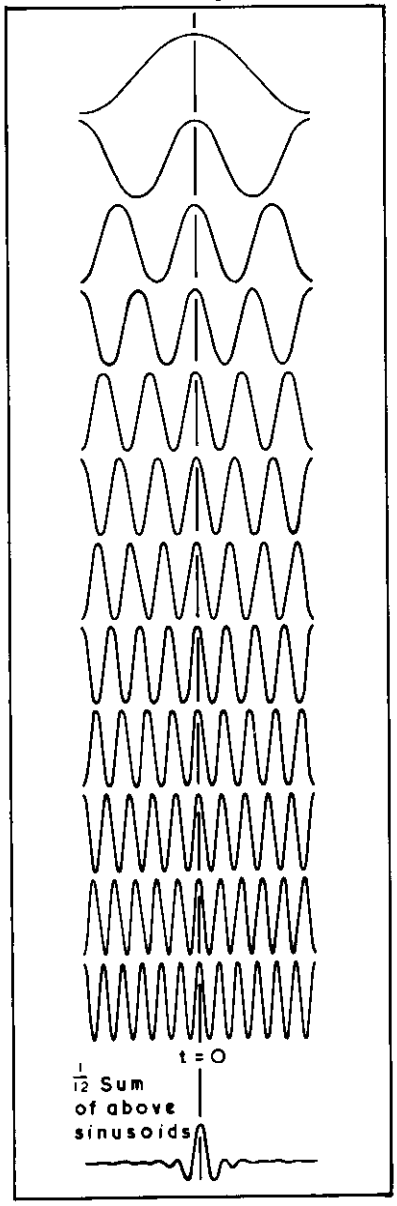

FIG. 3.--Approximation to the synthesis of the waveform corresponding to a rectangular amplitude spectrum and a zero phase spectrum.

trum of the output is equal to the power spectrum of the input, and the phase spectrum of the output is zero. Such a system is said to be a *matched filter* for its particular waveform. Thus we are led to this proposition:

The process of auto-correlation of a waveform is equivalent to passage of the waveform through its matched filter.

At this stage we have established two alternative approaches to the auto-correlation function: the approach through the concept of similarity as a function of time shift, and the approach through the power spectrum and the matched filter. In figures 4 and 5 we illustrate these two approaches, using as an example a waveform which has proved to be of particular utility in correlation techniques—the linearly swept-frequency waveform.

In figure 4 we construct the auto-correlation function by the process of plotting the similarity between the waveform and a time-shifted ver-

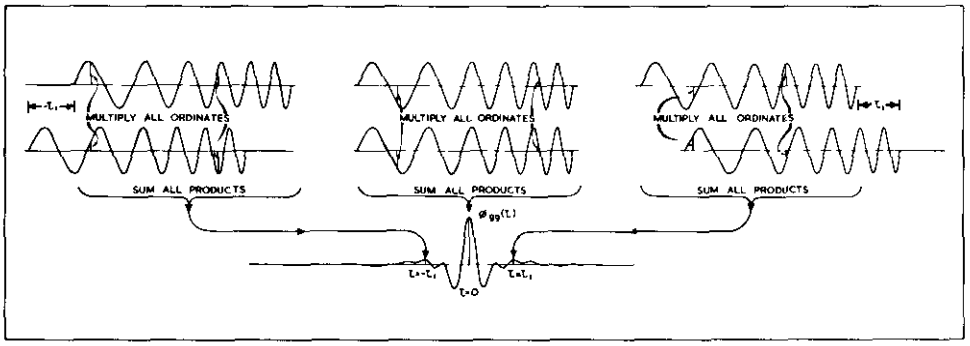

FIG. 4.—Illustration of the process of constructing the auto-correlation function of a swept-frequency waveform.

sion of itself. The centre section of the diagram illustrates the "overlay" position; for this the time shift is zero, the ordinate products are all positive and the sum is large. The left and right sections of the diagram illustrate the situation at a time τ_1 before and after the overlay position; some of the ordinate products are positive and some are negative, and their sum is small. Then our earlier propositions tell us that the variation of this product sum, as a function of the time shift, defines the form of the auto-correlation function shown on the lowest trace.

In figure 5 we illustrate the alternative approach from the power spectrum. Our intuition tells us, correctly, that if the sweep rate of the swept-frequency waveform is constant and small, then all frequencies are approximately equally represented; the amplitude spectrum is flat between the end frequencies of the sweep, and substantially zero beyond them. Consequently the power spectrum also has this rectangular form. If we now take a suitable set of Fourier components representing this power spectrum, draw them all in phase at the origin of a new time scale, and add them together (just as we did in figure 3), we obtain the auto-correlation function. For the particular case of a slow sweep, and for illustrative purposes only, we may visualize the swept-frequency wave-

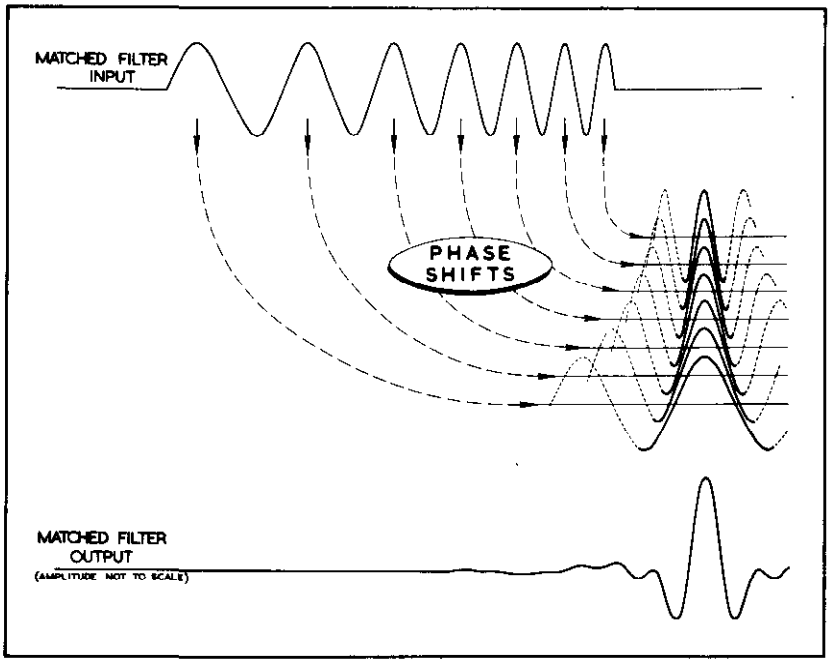

FIG. 5.—Alternative approach to the process of constructing the auto-correlation function of a swept-frequency waveform.

form as the *sequential* arrangement of those parts of the Fourier components which make a positive contribution to its shape; loosely we may say that "the frequencies are strung out in line." Then the construction of the auto-correlation function (or the action of the matched filter, which is the same thing) may be viewed in terms of the phase shifts illustrated in figure 5.

We note that in the approach of figure 4 the central maximum of the auto-correlation function occurs because the similarity between the two waveforms is large when there is zero shift between them, and the parts of the auto-correlation function which correspond to large shifts are small because then the similarity between the waveforms is small. In the Fourier approach of figure 5, the maximum of the auto-correlation function occurs because all components are in phase at this point; away from the maximum the components interfere destructively to produce low amplitudes. The two approaches are, of course, interchangeable and equivalent. The merit of the Fourier approach is that it allows us to think of the auto-correlation function in terms which are very familiar to us, so that to the extent that we may look at a waveform and estimate its spectrum, we may also estimate its auto-correlation function. The merit of the similarity approach is that it remains valid for waveforms (for example, continuous noise-like waveforms) to which Fourier concepts may not be applied.

The auto-correlation function, then, is concerned with the similarity between a waveform and a time-shifted version of itself. As was implicit in our discussion of figure 1, we may apply the same measure of similarity to the case of two waveforms which are not identical. In figure 6 we visualize one waveform (the upper one) as stationary and the other as sliding bodily past it from right to left; then we view the two waveforms

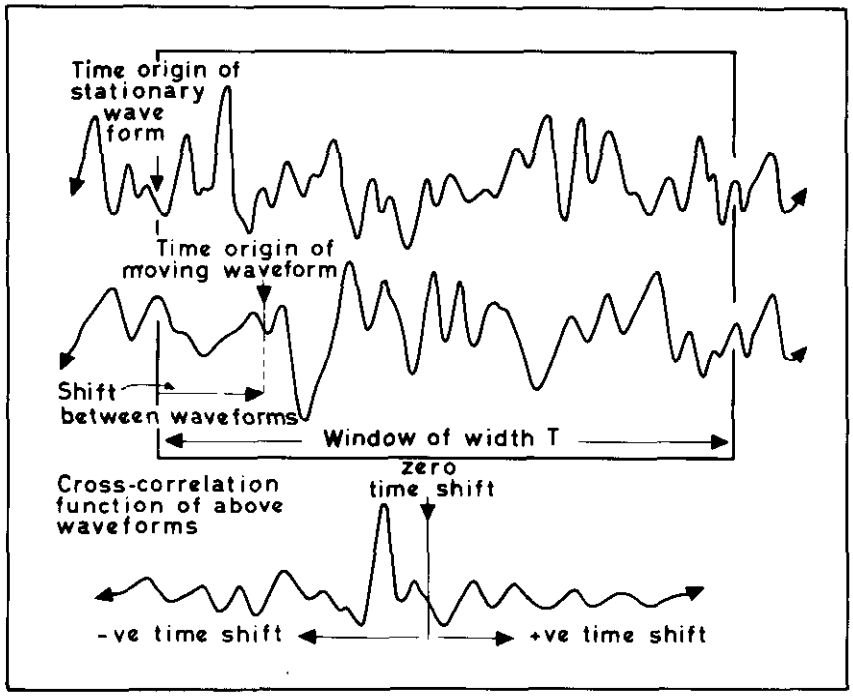

FIG. 6.—The upper two traces represent the waveforms being cross-correlated. The bottom trace is the cross-correlation function; the maximum in it shows that for the indicated value of time shift there is a marked similarity between the waveforms, even though this is barely visible to the eye.

through a "window" of width T, and we assess the similarity of the two waveforms within this interval by our previous method of multiplying ordinates and summing products. The bottom trace plots this similarity as a function of the time shift between the wave-forms.

The cross-correlation function of two waveforms is a graph of the similarity between the two waveforms as a function of the time shift between them.

The first example we should consider represents the basic problem of all communications and echo-ranging systems: a signal of known waveform is transmitted into a medium, and is received again unchanged in form but immersed in noise—what is the best way of detecting the sig-

nal? If we cross-correlate the received signal against the transmitted signal, the cross-correlation function must include two parts: a first part representing the auto-correlation function of the transmitted signal (which is common to both waveforms), and a second part representing the cross-correlation of the transmitted signal with the received noise. The form of the first part we know from the spectrum of the transmitted signal. The second part, since it represents the cross-correlation of dissimilar waveforms, can contain only low amplitudes, and so it is reasonable to expect that the signal-to-noise ratio should be improved by the correlation process. We might say that during correlation the transmitted signal is *searching to find itself* in the received signal. The degree to which it can do so is indicated by figure 7, where the first trace represents the transmitted signal and the second trace the same signal immersed in noise to the extent that it cannot be positively identified by eye. The third trace represents the cross-correlation of the first two traces; the signal (in the form of its auto-correlation function) is now elevated far above the noise.

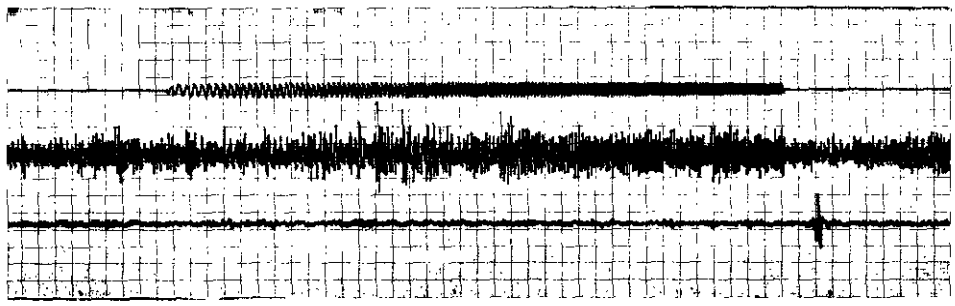

FIG. 7.—The top trace represents the transmitted waveform. The centre trace shows this waveform buried in noise. The bottom trace is the cross-correlation of the other two; the signal (in the form of its auto-correlation function) is elevated well above the noise.

We have already seen that the auto-correlation function of a waveform can contain only those frequency components present in the waveform itself. By extension of this, we find it easy to accept this proposition:

The cross-correlation function of two waveforms contains only those frequencies common to both waveforms.

This suggests that it should be possible to use the correlation process as a general filter. For example, we may correlate a complicated waveform against a sample of a sine wave, and the output contains only those components of the complicated waveform which are very close to the frequency of the sine wave. Thus we have a sharply peaked filter, and it is not difficult to see that the sharpness of the peak can be increased without limit by taking longer and longer samples of the sine wave. This illustrates a general point:

Correlation techniques are most powerful when the waveforms being correlated consist of many cycles.

Correlation filters are not limited to sharply peaked responses. For example, we have said that the swept-frequency signal of the top trace of figure 7 has a spectrum which is flat within the sweep limits, and substantially zero beyond them. This means that the correlation process, in producing the third trace, does not see any of the noise frequencies of the second trace which are outside the sweep limits; the process of correlation against a signal having a rectangular spectrum is, as far as the noise is concerned, a filter having a near-ideal rectangular pass-band.

We may be even more general. For example, let us cross-correlate a signal—any signal—against the pulse-like waveform of the third trace of figure 4. We know from the foregoing material that this waveform represents a rectangular amplitude spectrum and a zero-phase spectrum. The result of the cross-correlation, therefore, will be exactly the same as if we passed the original signal through a filter having a rectangular pass-band and zero phase. In completely general terms:

Cross-correlation against an arbitrary waveform is equivalent to passage through a filter having an amplitude-frequency response of the same form as the amplitude spectrum of the waveform, and a phase-frequency response of the same form as the phase spectrum of the waveform, but reversed in sign (that is, with a lead instead of a lag, a lag instead of a lead).

It is interesting to compare the cross-correlation process, which we see may be viewed as a *completely general linear operator,* with the process of convolution or superposition. This latter process, as we know, furnishes a general method of constructing the response of a system to an arbitrary input waveform when its response to a simple spike impulse is known. The input waveform is divided into a succession of impulses, a proportionally-scaled impulse response is drawn for each one of these impulses, and the output waveform is obtained by adding all these impulse responses. Alternatively the process may be effected by reversing the impulse-response in time, by sliding it along the input waveform, and by multiplying ordinates and summing products in the manner of a cross-correlation. The two processes of correlation and convolution have the same generality and are, in fact, equivalent.

Cross-correlation against a certain waveform is identical to convolution with the same waveform reversed in time.

We have previously stated that for every waveform there is a matched filter, whose output (when the waveform is injected at the input) is the auto-correlation function of the waveform, and we have specified the characteristics of the matched filter in the frequency domain. From the last proposition we may deduce an alternative specification:

The matched filter corresponding to a particular waveform has an impulse response which is the waveform itself, reversed in time.

We can illustrate this very simply in terms of a swept-frequency waveform (Fig. 8). Suppose that we have a "black box" such that the in-

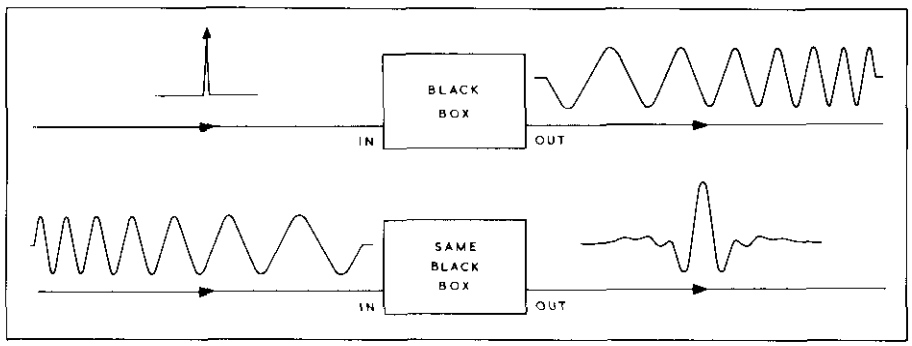

FIG. 8.—A certain black box has an impulse-response of swept-frequency type (first line). The same black box is then the matched filter for this waveform reversed in time.

jection of a spike impulse at the input produces a swept-frequency waveform at the output, and let us say that the sweep starts at the low frequencies and rises to high frequencies. In other words, the black box delays the high frequencies more than the low frequencies. Then let us record this sweep, reverse it in time (for example by replay backwards), and inject it at the input of the black box. The high frequencies enter the box first but they suffer a long delay. The low frequencies come last, but they suffer a lesser delay. Conequently all frequencies emerge in their original relation, and form a simple impulse (which may be viewed as the auto-correlation function of the swept-frequency waveform, or as a band-limited version of the original spike).

We have stated above that cross-correlation is a linear operator. Perhaps we should form an explicit proposition from this:

> *Cross-correlation is a linear operation, so that when it is associated with other linear operations (filtering, attenuation, differentiation, and so on) the order in which these operations are performed does not affect the final result.*

Our last proposition is concerned with the application of correlation techniques to methods of specifying the behaviour of a system. Thus any practical system modifies the waveform applied to it, producing an output which depends on the characteristics of the system. If we accept that the similarity between the output and the input is indicative of the characteristics of the system, and that cross-correlation is a method of measuring the similarity between two waveforms, we are not surprised to find that the cross-correlation function between input and output waveforms specifies the system behaviour over the bandwidth of the input signal. If we make the bandwidth of the input signal greater than the pass-band of the system, we have this useful result:

> *For a broad-band input, the cross-correlation function of the input and output of a system is the impulse response of the system.*

This simplified discussion of the rudiments of correlation theory makes no pretence to be rigorous. More formal treatments may be found in the literature (for example Lytle, 1957; Lee, 1960; Turin, 1960; Barber, 1961). No mention has been made here of the approximation introduced when the waveforms considered are of finite duration; a formal treatment of this is given by Schroeder and Atal (1962).

APPLICATIONS IN SCIENCE GENERALLY

1. *Pulse-Compression Echo-Ranging Systems*

The performance of all echo-ranging systems—radar, sonar, seismic, flaw-detection—is giverned by the energy principle (Woodward, 1953; Stewart and Westerfield, 1959). In simplified terms, this states that, for given noise conditions, the detectability of an echo is prescribed by the signal *energy* (that is, by the product of its average power and its duration). Conventional echo-ranging systems cannot increase the duration of their search signals without sacrificing the resolution between reflectors, and so their approach to better penetration can be only through an increase in the power. Sooner or later, however, there comes a limit: electrical breakdown occurs in a radar waveguide, or cavitation occurs on the face of a sonar transducer, or extreme non-linearity occurs in the earth near a seismic charge. The solution to this difficulty is the transmission of a *long* search signal (so that high energy and long range are assured) and then the *compression* of the long signal to a short pulse (so that good resolution is preserved). This principle is known as pulse-compression.

The method of compressing the received signal is, as we might guess from all the foregoing material, the cross-correlation of the received signal against the transmitted signal (or—what is the same thing—the passage of the received signal through the filter matched to the transmitted signal). This process, under the conditions normally prevailing in echo-ranging systems, represents the theoretically optimum method of detecting an echo; as our above propositions show, it provides the virtual elimination of all noise frequencies outside the signal spectrum, it provides the shortest wavelet compatible with the spectrum employed, and it provides the highest peak wavelet amplitude compatible with the signal energy. Figure 7 may be taken as indicative of the advantages of a pulse-compression system.

One family of pulse-compression radar systems uses a swept-frequency search signal, and has come to be known as "chirp" radar (from the catch-phrase "Not with a bang, but with a chirp"). Relevant references include Hüttman (1940, 1961), Dicke (1945), Darlington (1949), Cauer (1950), Krönert (1957), Chin and Cook (1959), Cook (1960, 1961), Klauder *et al.* (1960), Klauder (1960), May (1960, 1962), Meeker (1960), Meitzler (1960, 1961), Ohman (1960), O'Meara (1960), Fitch (1961a, b), Hoover (1961), Kagawa and Ishizaki (1961), Lange (1961), Lerner (1961), Merrihew (1961), Ramp and Wingrove (1961a, b), Tiberio (1961), Lohrman (1962), Skolnik (1962), Adams (1963), Bernfeld (1963), Ja-

cobus (1963), Mims (1963), Yost (1963), Benjamin (1964), Bernfeld *et al.* (1964), Milne (1964), Adamsbaum (1965), Logotch and Cook (1965), Lancaster (1965), Marchese (1965) and Mortley (1965).

A second family of pulse-compression radar systems makes use of the fact that it is the spectrum of the search signal, rather than its actual waveform, which defines the form of the final compressed pulse. Thus various random and noise-like waveforms may be employed as search signals. Discussions relevant to these random-signal radars have been given by Guanella (1939), Lee and Wiesner (1950), Bussgang (1951), Faran and Hills (1952), Lytle (1957), Horton (1959), Turin (1960), Welti (1960), Fishbein and Rittenbach (1961), Katzman and Frost (1961) Miller (1961), Spilker and Magill (1961), Huffman (1962), Golomb (1963), Sakamoto *et al.* (1963), Spilker (1963), Zadeh *et al.* (1963), Band (1964), and Lockheed (1964). A remarkable early demonstration of a radar of this type was the registration of radar echos from Venus, a distance of 28 million miles (Price *et al.*, 1959). Pulse-compression is not peanuts.

When pulse-compression techniques are applied to sonar, the use of a noise-like search signal allows the detection of a submarine or surface vessel without there being any direct means for the latter to know that its presence has been detected. Pulse-compression sonar systems have been discussed by Sproule and Hughes (1944), Stewart and Westerfield (1959), Parks and Downing (1960), Clay (1961), Clay and Liang (1962), Federici (1962), Jones (1962), Middleton (1962), Rowlands (1962), Stewart and Allen (1962), Remley (1963), Allen and Westerfield (1964), Clay *et al.* (1964), Parvelescu and Clay (1964), Persons *et al.* (1964), Eisenmenger (1965), Rowlands (1965), Russo and Bartberger (1965) and Weston (1965).

A pulse-compressive method of seismic exploration, known commercially as the "VIBROSEIS"* system, is by now well known to members of this Society, having been described by Doty and Crawford (1953), Doty and Lee (1954), Crawford and Doty (1955), Crawford and Clynch (1955), Doty (1959), Crawford, Doty and Lee (1960, 1961) and Goupillaud and Lee (1963). This system, like its pulse-compressive counterparts in radar and sonar, promises to replace the conventional method for most applications.

A general summary of the theory and practice of pulse-compression systems has been given by Anstey (1963).

2. *Communications*

The problems of communication in a noisy medium have much in common with the problems of echo-ranging. The application of pulse-compression principles to the transmission of teletype and voice-freqnency information can yield four major advantages: an improvement in signal-

*A trademark of Continental Oil Co.

to-noise ratio, sophisticated measures to minimize fading, the provision of "secure" communications unintelligible to anyone without the appropriate matched filter, and conservation of radio bandwidth. Relevant references include Price and Green (1958), Sussman (1960), Wozencraft (1961), Winkler (1962), Aein and Hancock (1963), Anon (1964) and Chiles and Lafuse (1964).

3. *The Detection of Periodic Signals in Noise*

Two methods of recovering a periodic signal from noise emerge from our basic propositions above.

The first is applicable if the frequency of the periodic signal is known, and consists of cross-correlation of the noisy signal against a sinusoid. This is an extremely powerful method limited only by the purity and duration of the reference sinusoid; it is comparatively simple to recover a periodic signal immersed in 100 times its own amplitude of noise.

The second method is applicable if the frequency of the periodic signal is not known, and consists of the auto-correlation of the noisy signal. This is not as powerful as cross-correlation against a sinusoid—in effect we are correlating a dirty signal against a dirty signal instead of a dirty signal against a clean signal—but it is still capable of very large improvements in the signal-to-noise ratio. The auto-correlation function is the superposition of figures 2b and 2d, so that, provided the waveform sample considered has sufficient duration, the periodic signal is evident in the outlying parts of the auto-correlation function. These methods and their applications are discussed by Lee, Cheatham and Wiesner (1950), Lee and Wiesner (1950), Jones and Kelly (1952), Rudwick (1953), Brazier and Barlow (1956), Mercer (1958), Barlow (1959), Raemer and Reich (1959), Smith and Lambert (1960), the American Gas Association (1962), Bakewell (1963), Bass and Men' (1963) and Krauss (1963).

4. *Determination of Spectra*

Clearly the method of cross-correlating against samples of sinusoids may be used to establish the amplitude spectra of periodic signals (in the manner of a set of narrow-band filters), and the resolution with which the spectra may be established is limited only by the purity and duration of the reference sinusoids.

For non-periodic noise-like signals of the type found most commonly in nature, the best method for determining spectra is the construction of the auto-correlation function and the subsequent manipulation of this, by mathematical methods, to the power spectrum. A rapid approximate method of deriving the spectrum from the auto-correlation function is the cross-correlation of the latter against a slowly swept-freqnency waveform, followed by rectification and smoothing.

A comparison of the determination of spectra by digital narrow-band filtering and by auto-correlation is given by Cartwright, Tucker and

Catton (1961) in an application to oceanography. A modified correlation method is used by Akamatu (1956) for the determination of the spectra of microtremors; also relevant to this reference are Tomoda (1956), Melton and Karr (1957) and Ekre (1963). It begins to emerge that the ear, long thought to operate largely on a spectral basis, may include auto-correlation as one of its functions (Kraft, 1950; Stevens, 1950; Cherry, 1961).

5. *General Filtering Operations*

From our basic propositions, we may deduce that cross-correlation may be used as a completely general filter, with entirely independent control of amplitude and phase. Furthermore, a change from one filter characteristic to another involves only a change in the reference waveform against which the input is cross-correlated; no modification of the equipment itself is required. Thus cross-correlation represents an extremely flexible method of filtering (Jones and Morrison, 1954; Smith, 1958).

Since cross-correlation filtering is not an intuitively obvious process, an example may be helpful. Let us correlate an input waveform against a slowly and linearly swept-frequency signal. The output is small, because the waveforms are in general dissimilar; it is of a complicated nature, and longer than the input signal. Now let us correlate this output against the same swept-frequency signal reversed in time. The final output represents the input signal filtered through a near-ideal filter having a rectangular pass-band and zero phase. The same output would be obtained if, instead of the swept-frequency signal, we used *any* waveform of the same power spectrum (even a band-limited noise-like waveform). However, three advantages accrue from the use of a swept-frequency signal. First, the phase-compensation of any part of the main or ancillary equipment is a simple matter, achieved by shifting the phase of appropriate parts of the second swept-frequency waveform. Second, pass-bands of arbitrary shape may be obtained either by suitable amplitude modulation of the swept-frequency waveform or — more interestingly — by variations in the sweep rate. Third, after the first correlation the frequency components of the original signal may be regarded as separated in *time,* and this allows highly sophisticated data enhancement techniques in some applications.

6. *Determination of Impulse-Response*

One of our basic propositions showed that we could determine the impulse-response of a system by cross-correlating the output with a suitable broad-band input. But why should we wish to do so, when we can determine it much more easily by the simple injection of a spike?

The answer lies in figure 7. For an echo-ranging system may be said to determine the impulse-response of the transmission medium—a seismic reflection record is the impulse-response of the reflective earth — and so we may deduce that an impulse-response determined by cross-correlation is insensitive to the presence of noise or other signals which may be

injected into the system between its input and its output. Thus the cross-correlation method is desirable whenever the determination must be made in the presence of a high noise level, or in the presence of other signals (such as the system's normal throughput), or on a system having a small dynamic range. A compelling illustration is in industrial process-control systems, where it is desirable to measure the characteristics of the system without stopping the plant (Goodman, 1955; Reswick, 1955; Balchen and Blandhol, 1960; Buland, Cooper and Margolis, 1960). Other examples occur in the testing of aircraft, buildings and acoustical systems (Lee and Wiesner, 1950; Bussgang, 1951; Goff, 1955; Mercer, 1958; Zverev and Kalachev, 1960; Gayford, 1961; Isobe and Idogawa, 1964); further examples must surely emerge in neurophysiology.

7. *The Identification or Location of Signal Sources*

In our discussion of pulse-compressive echo-ranging systems, we visualized the transmission of a suitable known waveform, the identification of reflected versions of this signal by cross-correlation of the received and transmitted signals, and the consequent location of reflectors by study of the reflection travel times. For some purposes it is not necesary to generate a transmitted signal; if we observe a suitable broadband signal occurring naturally at a first point, and if we correlate this against a signal observed at a second point, then we identify those disturbances which propagate between the two points, and we determine their apparent velocities.

Iyer (1958), Rykunov (1961) and Töksoz (1964) have applied this method to determine the velocities and directions of microseisms. Butterfield, Bryant and Dowsing (1960) have applied it to the measurement of the speed of moving steel stip in a rolling mill. Goff (1955) has applied it to the measurement of the acoustic absorption of panels. Burd (1964) has applied it to the identification of the path by which sound "leaks" out of a studio. Other applications include the determination of the contributions of each of many machines to the noise level in a control room, the determination of the relative contributions of inner and outer aircraft engines to the noise level in the cabin, the location of noise sources in mechanical, missile and sonar equipment, and the measurement of cross-talk in multichannel communication systems subject to intermodulation distortion (Lee and Wiesner, 1950; Goff, 1955; Gilford and Greenway, 1956; Gilbrech and Binder, 1958; Briggs, 1960; Bull, 1960; Keast and Kamperman, 1960; Dickey and Craig, 1964; Lund and Urick, 1964; Cron *et al,* 1965). An inverted application is described by Denham (1963), who is concerned with determining the minimum spacing of the elements of an array which will ensure that the noise at these elements is substantially incoherent.

Clearly, many of the examples we have considered in this section may be taken as illustrating that during correlation each one of the waveforms is searching to find itself in the other. Thus correlation forms the basis of all systems of *pattern recognition.* All current attempts at the automatic recognition of speech, script and print represent some form

of correlation technique. The same principles govern the matching of fingerprints, the identification of the after-shocks of an earthquake and the matching of maps and pictures (Diamantides, 1964).

EMERGENT APPLICATIONS IN EXPLORATION GEOPHYSICS

The most widespread and commercially important application of correlation techniques today is in the "VIBROSEIS" method of seismic reflection and refraction work. Indeed it was the commercial necessity of high routine production which furnished the incentive for the development, by S.S.L., of the real-time correlator now extensively used in "VIBROSEIS" exploration.

In this discussion, however, we are concerned with the whole field of exploration geophysics. We may expect a potential application of correlation techniques wherever some geophysical measurement may be expressed as a waveform, or as its two-dimensional equivalent—a pattern. Examples of waveforms are gravity and magnetic profiles, well logs and, of course, seismic traces. An example of a pattern is a multi-trace seismic reflection. Let us consider under what circumstances correlation techniques may usefully be applied to these examples.

First, we must reaffirm that correlation gives its clearest answers when the waveforms considered consist of many cycles.

Second, we must note that correlation gives an *average* measure of the similarity between two waveforms. Thus it is *quite unsuited* to the detection of one small pip on one of two otherwise-identical waveforms (Fig. 9). It is *perfectly suited* to the detection of an extended section

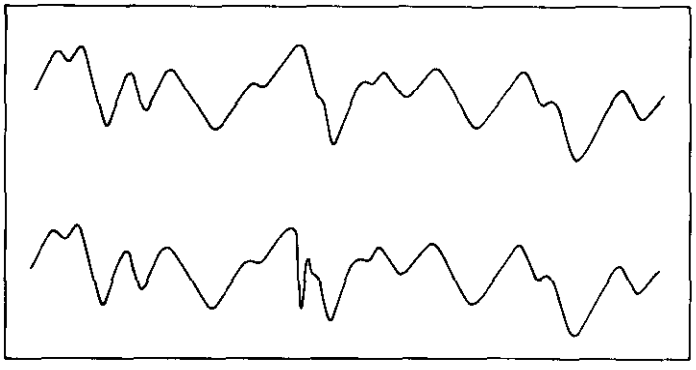

FIG. 9. Cross-correlation is not the correct process for distinguishing between these waveforms.

of common content in two waveforms, particularly when this common content is obscured by noise or other irrelevant signals (Fig. 7).

Third, we must note that correlation is comparatively insentitive to local differences in the *amplitudes* of the two waveforms. Thus in figure

10 there would be only a minor difference between the auto-correlation function of waveform *a* and the cross-correlation function of waveforms *a* and *b*.

Fourth, we must note that correlation is extremely sensitive to the "stretching" of one of the waveforms. Thus the correlation process looks at waveforms *a* and *c* of figure 10, and pronounces them identical; it looks at waveforms *c* and *d*, and can see little similarity between them.

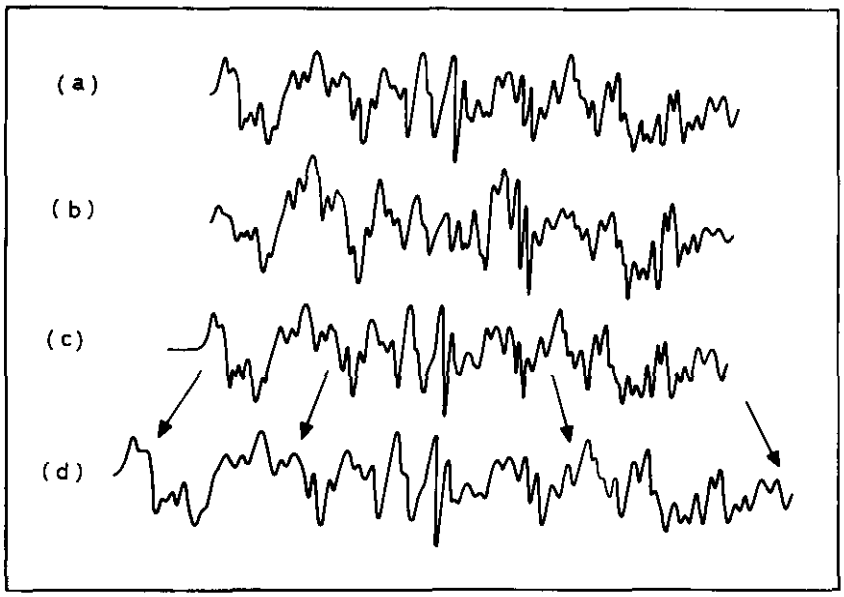

FIG. 10.—Traces *a* and *b* differ primarily in their local amplitudes; correlation is relatively insensitive to this type of difference. Trace *d* is a stretched version of trace *c*; correlation is very sensitive to this type of difference.

Let us now draw the conclusions relevant to the correlation of well logs. Suppose that special interest attaches to a particular segment of log in one hole, and that the problem is to find and explore this segment in a log from another hole. We cross-correlate the segment against the entire second log (or any lesser portion decreed by geological possibility), If a correlation is clearly indicated (by a sharp pulse in the cross-correlation function) the appropriate part of the second log may then be studied for significant differences in the balance of "amplitudes" and for significant differences in the detail of the logs. It is hardly necessary to stress that the correlation process can find the appropriate part of the second log even when these differences of amplitude or detail are sufficient to make the correlation invisible to the eye. The resolution with which the depth of the segment is determined is, of course, a function of the "bandwidth" of the waveform; if there is a chance periodicity in the beds of the segment the cross-correlation function is less sharp than if the spacing of the beds is random.

If the cross-correlation process produces several indications instead of one (and if this is geologically implausible), the segment which has been taken is too short. If it produces no indications of correlation (and if this is geologically implausible), the segment which has been taken is either geologically too long, or a thinning of the beds in the segment has occurred between the holes. In this case, we may try different degrees of compression or extension of the depth scale of the reference segment until a correlation is indicated (Haites, 1963). Shaub (1963) gives an altenative approach particularly applicable to pinch-outs.

By a straightforward change of terms, we may apply the same considerations to seismic reflection traces (Jones, 1962). Suppose that we have a seismic cross-section characterized by good quality data on both sides, but, for reasons associated with surface conditions or noise, poor data in the centre. Then if we can establish a reference trace in the good part of the section, we may use cross-correlation against this trace to push the continuity across the poor part of the section, even where this continuity is not visible to the eye.

Some features of this technique are worth exploring. First we note that there is an incompatability between the considerations which define the duration of the segment of reference trace employed. If we take a long segment (for example, 2 or 3 seconds of two-way time) we shall be able to detect a recurrence of its pattern, in another trace, even if it is buried deep in noise. However, this demands that the seismic section be conformable within very narrow limits (perhaps within 20 or 30 milliseconds in 2 or 3 seconds). Therefore, in using this technique, we must first decide how far across the section the continuity from a particular reference trace must be extended, and then, for this horizonal distance, we must decide for what duration of reference trace the section can be expected to remain approximately conformable. These decisions then define the maximum noise level in which the continuity may still be detected (Mut, 1959; Faust, 1963).

Clearly the selection of the reference trace is very important. We may use a synthetic trace (Peterson, 1955), or a single good field trace, or a trace compounded from several good ones. Alternatively, we may use a trace which represents the result of stacking many widely-separated reflection paths to a common depth point; in this way, of course, we reduce the contribution of the multiple reflections. Considerations like these suggest a method of field recording in which every device we know for improving record quality is applied just to one trace in every 20 (or 50, or 100), and then the other traces (obtained with a simpler and more economical technique) are used, in combination with a cross-correlation technique, to establish continuity between the special traces.

The above considerations are also very relevant to the whole problem of stacking. We have all seen examples where a number of apparently-useless records have been stacked to yield something usful. We have also seen examples where the stacked record has been no better than one or two of its components. Clearly what is required is a check whether

a particular record will help or hinder the stacking; reinterpreted, this calls for a measure of the similarity between two traces, and therefore is a natural application of correlation techniques. Not only does the cross-correlation function decide whether the record is worth stacking or not; it also indicates the appropriate static correction with a certainty far beyond that obtainable (by the employment of doubtful velocities and an inadequate geometrical model) with a weathering correction. There is no better way of measuring a step-out than with an appropriate correlation technique.

We have spoken of giving the maximum possible enhancement technique to particular selected traces, and one of the techniques which we must consider is inverse filtering of deconvolution. By this we seek to compensate the filtering effect of the earth, off-setting frequency-selective attenuation by appropriate amplification. Correlation techniques are very relevant here. For example, it is well known that the implementation of inverse filtering requires a determination of the initial spectral content of the seismic trace, and that this is best done by correlation techniques (Robinson, 1957; Agard, 1961; Kunetz, 1961; Rice, 1962). It is also well known (Crawford and Doty, 1955) that in the "VIBROSEIS" technique it is feasible to effect inverse filtering by appropriate variation of the sweep *rate,* and that this allows any defined degree of inverse filtering with the minimum expenditure of energy.

Another process related to correlation is also relevant. In conventional work, the process of boosting the frequencies which are absorbed in the earth must clearly degrade the reflection signal relative to any broadband noise which may be present, and thus the conventional inverse filtering process is constrained to exchange signal-to-noise ratio for resolution. If the signal-to-noise ratio is very poor, then we know that the optimum filter is the matched filter, whose amplitude-frequency response has the same form as the amplitude spectrum of the reflection signal. As we consider progressively improving signal-to-noise ratios, we pass through a ratio which allows detection of a reflection without any filtering at all, and then come to ratios which allow progressively increasing degrees of inverse filtering. Inverse filtering is appropriate only to very good records, and the possible degree of resolution improvement is defined rigorously by the signal-to-noise ratio. Therefore it is of the utmost importance to achieve signal-to-noise ratios as high as possible. This requires a modification of view for the exploration seismologist; for most of us a signal-to-noise ratio of unity makes a pleasant change, and we would not ordinarily see any point in increasing the ratio beyond a value of perhaps two or three. The attainment of much greater signal-to-noise ratios by methods which cannot be allowed to decrease the bandwidth of the signal must surely require help from the techniques of correlation.

We have said that inverse filtering affords a means of exchanging signal-to-noise ratio for resolution. Correlation techniques allow certain other exchanges which are of value for particular objectives. Thus multiplicative compounding and the use of "correlation arrays" allow various

exchanges between signal-to-noise ratio, the directivity (or apparent-velocity resolution) of a geophone array, the physical dimensions of an array, and the clarity of indication of the onset of a signal (Dyk, 1956; Melton and Bailey, 1957; Welsby and Tucker, 1959; Linder, 1961; Tucker, 1963; Ksienski, 1964; Ryall, 1964; Shaw and Davies, 1964; Whiteway, 1964).

We have said that correlation is at its best when dealing with long waveforms. Consequently correlation against a waveform of the type of a single seismic wavelet is not a strong method. Correlation against many such wavelets simultaneously, however, can become a very strong method. For example, we may arrange a wavelet on each of many traces, align the wavelets as for a reflection having a particular step-out, and correlate the record against this pattern (Tullos and Cummings, 1961; Baranov and Picou, 1964). Similarly, we may search for the major apparent velocities present on a refraction record (McCamy and Meyer, 1964).

With a greater generality, we might say that two-dimensional correlation is appropriate whenever a geophysical result may be expressed as a number of separate short waveforms forming some sort of pattern. We should note that the correlation technique is very little disturbed by the fact that several such patterns may cross each other. A correlation method is adept at showing whether a waveform or a pattern is composed of several superposed components, and this fact should find application in many branches of exploration geophysics. It may be true that novel geophysical methods have proved unworkable in the past because of confusion from overlapping responses, and that such methods warrant review in the light of modern correlation techniques.

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