THE NATURE OF DIGITAL SEISMIC PROCESSING

By

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INTRODUCTION

Today, most geophysical interpretive and management personnel have had experience with digital processing of seismic data. Results of the processing may be often impressive, but at other times may leave much to be desired. The reasons for this are not always apparent. They must be derived from a knowledge of the processing techniques and theory, which in turn, often involve unfamiliar concepts. The problem is further complicated by rather complex mathematics used to derive the methods and the proofs of the operations.

Under these conditions, management and operating personnel must appraise and evaluate the use of the new techniques. As a result, there is a tremendous surge in the need to learn more about the techniques for the more efficient utilization of the seismic method.

The research geophysicist, the mathematician, and the computer programmer must be familiar with the theory and the proofs, in order to develop the applications and to determine their validity. However, the person who is concerned only in evaluating a given process and determining why certain results are obtained, may sidestep rigorous mathematical procedures in favour of a more pragmatic approach to the problem.

In the following pages, we shall endeavour to develop an explanation of the seismic process in a straightforward manner, using simple concepts and demonstrations. Rigorous proofs will be avoided. The full treatment and proofs may be obtained where desired from the references.

Digital processing of seismic information is based on the methods and procedures of signal theory. The development follows a regular procedure. First, certain assumptions are made and the necessary terms and symbols are defined. Then, a set of theorems is postulated and the corresponding proofs are developed. Practical applications then follow. Thus it is with seismic processing.

BASIC PROBLEM

The first step is to define the problem. A seismic recording consists of a complex signal containing useful signal information mixed together with a great deal of unwanted signal, which is arbitrarily called noise. Much of the so-called noise might be converted to useful information, if it could be properly extracted and classified.

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It is desirable to separate the complex signal into its several components, select those which are useful, and discard those which are not.

In order to sort the signal components into distinctive classes, some key characteristic which can be measured and classified over the set of data must be selected. One of the methods of classifying seismic energy is by frequency. It is well known that useful seismic energy usually falls within a relatively narrow band of frequencies, say 20 to 80 cycles per second. Standard recording practice has been to filter out both the low frequencies, which are often due to such noise as ground roll, and the higher frequencies, which are often wind or shot hole noise. Thus, the seismic trace may be termed a limited set of frequencies. Methods exist for the determination of the frequency components and for selectively modifying the proportions of these components in a seismic trace. Thus, classification by frequencies provides a useful tool for the analysis of seismic signals.

However, classification by frequency will not necessarily uniquely define signal and noise. Both the useful signal and the noise may have similar frequencies. For example, in marine shooting a reverberation may oscillate at roughly 35 cycles, which may be near the center of the band of useful seismic information.

NATURE OF LIGHT

A useful analogy, which is frequently used, may be developed by the examination of a beam of light.

White light is composed of a broad range of frequency components. These components may be separated by passing a beam of white light through a prism, as in Fig. 1. The light frequencies are refracted by the prism at successively greater angles, in such manner that the white light input is transformed into several distinct colours. The output of this process is a continuous spectrum extending from the low frequency, or red end, to the high frequency, or violet end. Although the range is continuous, for most purposes the output is classified by dividing it into a relatively small number of major divisions, e.g., the seven primary colours.

Thus, approximation by sampling at some increment provides a satisfactory representation of the continuous range.

In order to selectively filter or extract particular components of white light, it is possible to pass the light through a prism and then filter the output selectively. A filter could be made by cutting a small slit in an opaque plate. If the plate were positioned in the output, all of the undesired components would be blocked off, and only a selected frequency or component—say red—would be passed through the slit in the plate. This is a filtering process in which the undesired components have been physically removed from the spectrum (Fig. 2).

The unfiltered output of a prism may be passed through a second reverse prism, and the components will again combine to form white

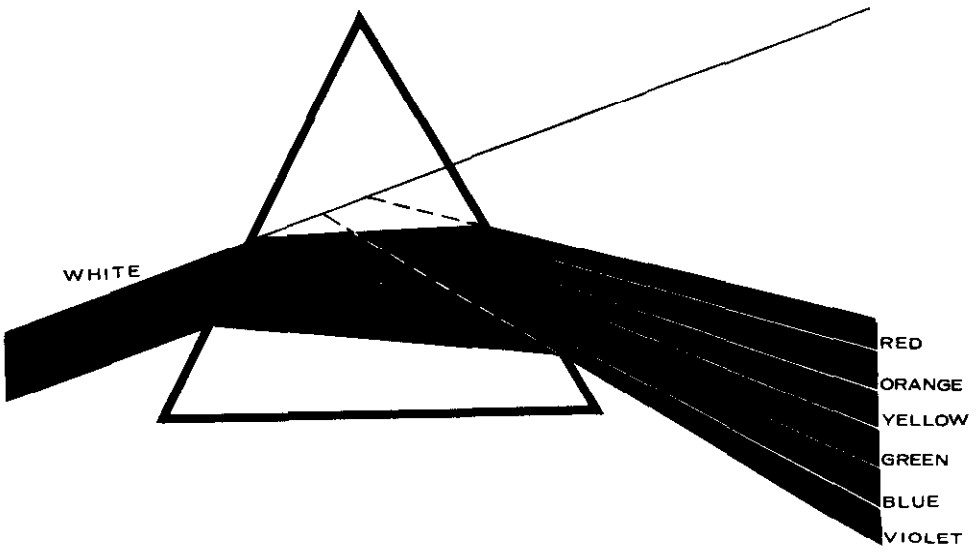


FIG. 1.—Decomposition of white light into a spectrum.

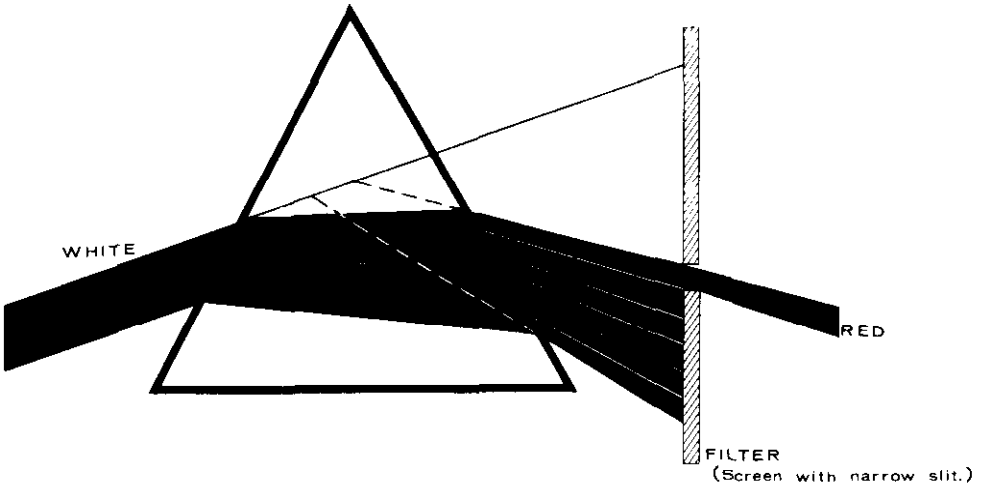


FIG. 2.—A band filter may be used to pass a selected portion of the spectrum.

light. However, if the filtered output is passed through a reversed prism, the output will remain the same colour—say red—in our example. The reason for this is that the other components required to make white light have been physically removed, and it becomes impossible to reproduce the original input of the system.

Another form of filter could be a sheet of coloured glass placed in the beam of the output. The sheet of coloured glass, if it were red, would pass only the red component of the light, and would effectively absorb all others. This filter might be termed a correlation process, in that the colour of the filter is correlated with a desired segment of the spectrum, and thus selectively filters the broad band.

If white light can be transformed into its component colours, or frequencies, the reverse process should also be possible. If individual beams of colour approximating the spectrum are merged into a single beam, the output would then approximate white light. It would be necessary to adjust all of the beams to equal intensity. If the intensity of one of the components was accentuated; say, the intensity or amount of green light was twice that of all the other components, the resulting combined light would be coloured green, i.e., it would contain an excess of that component.

In other words, white light is composed of an equal amount of each of the components which enter into it.

These concepts may be extended into the domain of frequencies which characterize seismic information. For example, white signal and coloured noise. A white signal is nothing more than the sum of equal amounts of each of the frequencies over the range which makes up a given spectrum, whether it be a seismic signal or light. A 'coloured signal' indicates there is an excess of that particular frequency present in the spectrum.

NATURE OF THE SEISMIC SIGNAL

A seismic signal may be defined by a description of frequencies present. This will not provide all of the information required for the basic separation of the desired reflection signal from the noise. The category, which is called noise might actually contain a great deal of useful information if it could be extracted and classified. The problem has always been one of separating the great number of individual components that make up the so-called noise.

Noise may be subdivided into two categories, the periodic and the random elements.

Periodic noise consists primarily of reverberations and/or multiples. These undesirable events will be repeated at regular intervals. The reverberations will be essentially sinusoidal, oscillating at some frequency. The multiples will be repetitive mirror image events, occurring at equally-spaced time intervals.

Ground roll may usually consist of a low frequency wave, although its composition may be more complex than a single frequency oscillation.

In addition, certain oscillatory disturbances which are externally produced may produce periodic noise. These might include 60 cycle induced signal from high-line installations, or from some AC source within the instruments.

With the exception of multiples, each of the periodic noise elements described may be defined in most cases as a single frequency. Under some conditions, multiples may be considered as low frequency events having a period of oscillation twice that of the basic multiple time.

Random noise may consist of wind noise, shot hole noise, and other external disturbances.

Diffractions and refractions are usually treated as noise. These may produce events which occur at essentially random times on the record.

The problem, then, is one of separating, classifying, and refining the material which makes up the recorded seismic signal.

SEISMIC RECORDING PARAMETERS

Fractionating a seismic signal may appear to be a very difficult thing to do, due to the very wide range of components. Examination of a typical trace will show how extensive the analysis must be.

In the first place, the total frequency range of the seismic signal covers only a relatively narrow band. Rarely will a seismic trace contain frequencies over 100 cycles per second. Very low frequencies may be recorded, so the scope of the frequency analysis may be safely limited to a range extending from 1 to 100 cycles per second.

The length of a seismic signal is relatively short. In most areas of the world, rarely is more than 5 seconds of seismic information required to adequately measure the geologic section which can be drilled with current equipment and practices. Over much of Western Canadà, the limit is substantially less, say rarely more than 2 seconds.

If the field recording extends beyod the sedimentary section, it will record merely the background noise and the continuation of any multiples. However, this information can be extremely valuable. Within the limits of the normal section, the recorded signal should contain both noise and reflections, whereas the section below the basement will generally contain only noise and multiples. If the problem is to define noise and multiples without the interference of reflections, the operation is simplified. It may then be possible to separate the noise from the upper part of the record, leaving only the seismic reflections. In practice, techniques for this purpose are not adequately developed, but undoubtedly they will be.

To return to the examination of parameters, it is assumed that in Western Canada a record will be five seconds long. At .002 sampling, each trace may then be defined by 2,500 samples, not a particular large amount of data for a computer.

The third parameter which is used to measure the seismic signal is amplitude. Normally, in digital recording, the maximum amplitude which is likely to be recorded is set at some maximum numerical value, determined by engineering limitations of the equipment. The range between this upper limit and its associated negative value is then div-

ided into an equal number of values. Thus, the signal amplitude will generally be recorded on a digital tape with values which usually will be adequate to fully define the range of the signal. The amplitude range of digital recording values is generally substantially better than analog equipment. Central processing equipment used for digital seismic processing is capable of manipulating numbers much larger than those recorded by field recorders, or those obtained from analog to digital converters.

In some areas the range of present field recorders may be inadequate to fully define the signal with a desired degree of precision. For this reason, new recorders using a technique termed binary gain have been introduced, which record the amplitude values as a base number and a multiplier. This allows a range of recording amplitude which is limited only by the ability of the recording system to deliver distortion-free signal information to the recorder.

In summary, the seismic signal is a combination of a relatively limited number of frequencies of limited amplitude extending over a short finite time duration. The rather narrow limits make the analysis much simpler than might be expected. Unfortunately, the narrow limits may also limit the quality of processing.

SAMPLED TIME SERIES

Digital computers deal with discrete numerical values rather than continuous signals. Therefore, some method must be developed to quantize the seismic trace, which is a continuous signal. A system of coordinates is used to define the seismic trace, in the same manner that X and Y coordinates may be used to define any point on the periphery of a circle.

The coordinates are obtained by sampling the amplitude of the seismic trace at some defined time interval. Thus time becomes the X coordinate and amplitude the Y coordinate. Since the sample interval is constant, only the amplitude values are stored, and the number of values is counted to obtain the time. The coordinate points may be plotted on any suitable scale and the points joined with a continuous line to reconstruct an approximation of the seismic trace (Fig. 3). A better reconstruction can be obtained by use of a special filter, termed a sinc function.

The system of coordinates used to define the seismic trace can at best be only an approximation. The quality of the approximation will be a function of the sample spacing. But processing cost is almost a direct function of the number of samples; hence, it is desirable to obtain the optimum number of samples which adequately define the seismic trace at the minimum processing cost.

SAMPLING THEOREM

Sampling rates may be defined mathematically as a function of the highest frequency which is to be defined on the re-constructed trace.

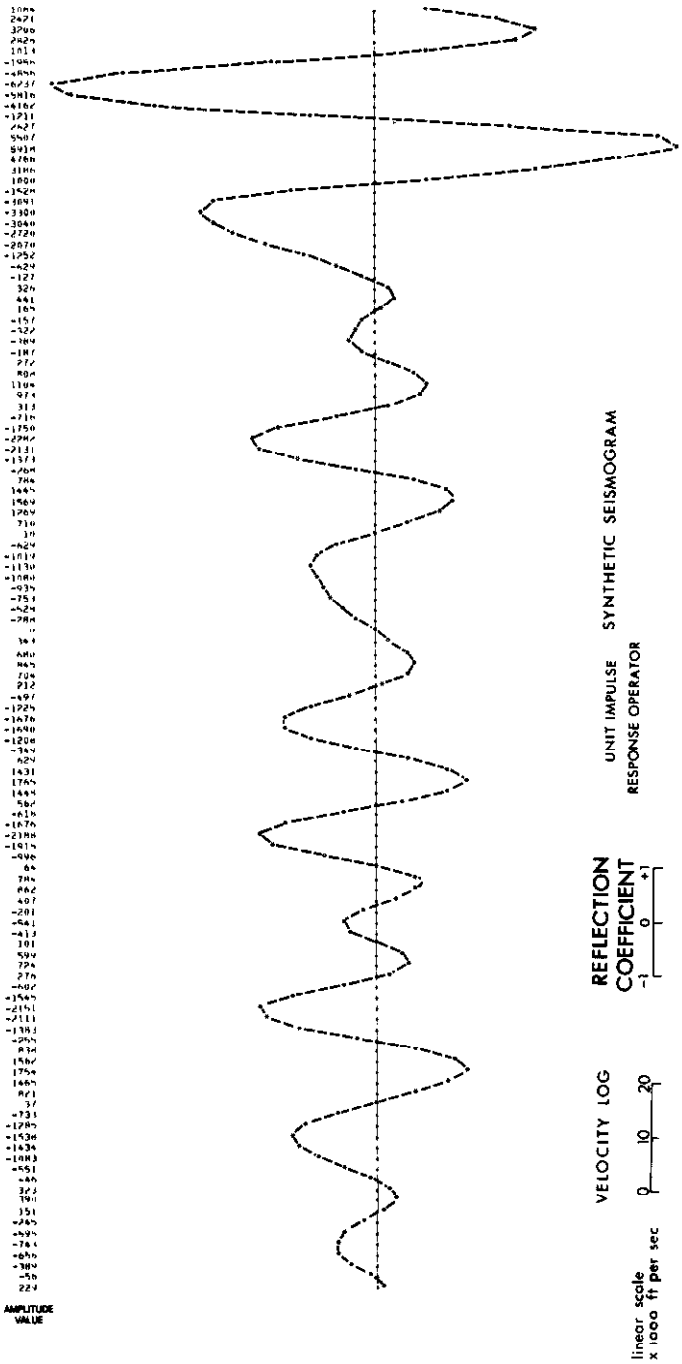


FIG. 3.—Portion of Digitized Seismic Trace.

The basic theorem for uniform sampling rates states that at least two samples must be taken over each cycle of that frequency which is to be adequately defined. In other words, any frequency must be sampled at a rate double that frequency. Conversely, the highest frequency which can be defined by a given sampling rate is half that of the sampling frequency. The highest frequency which can be defined by a given sampling rate is termed the Nyquist, or Aliasing Frequency.

ALIASING

Signal frequencies which are above the Nyquist Frequency will not only be lost during the sampling process, but will produce an undesirable side effect, known as 'aliasing.' Aliasing is a spurious low frequency which appears as a product of sampling of frequencies above the Nyquist Frequency. An example is illustrated in Fig. 4. A given signal, occurring at a frequency of four, is sampled five times over the same interval. When an attempt is made to reconstruct the original signal, the result will represent a signal having an apparent frequency of one cycle. This is the aliased signal.

ALIASING

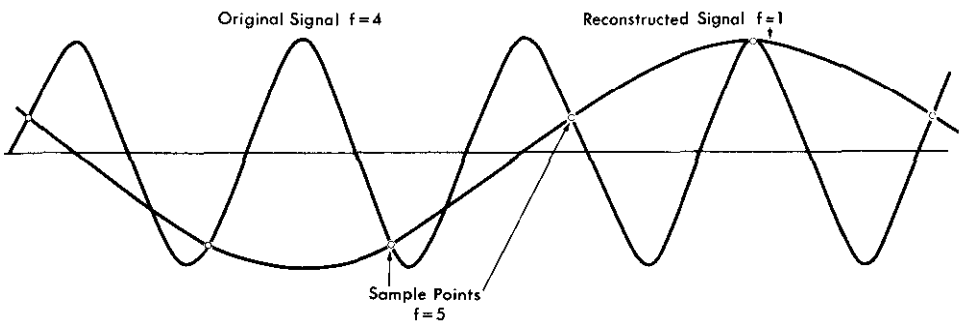


FIG. 4.—Aliasing results from improper sample spacing.

The frequency of the undesirable aliasing may be determined by simply subtracting the original signal frequency from the sampling frequency. The difference will indicate the aliased frequency.

A seismic signal sampled at one millisecond, i.e., 1,000 times per second, will theoretically define all frequencies up to 500 cycles per second, well beyond the normal range of seismic prospecting. Sampling rates of 2 milliseconds, or 500 samples per second, will define 250 cycles per second, and 4 milliseconds will define 125 cycles per second.

In practice other considerations, such as the precision of sampling equipment, noise, and other engineering problems, tend to lower the definition of a sampled signal to something below the theoretical limit.

Common practice is to assume a safe upper frequency limit at one half of the Nyquist Frequency, or one quarter of the sampling frequency. Thus, sampling rates of 1 millisecond are considered adequate to provide definition of 250 cycles per second; 2 millisecond, 125 cycles per second; and 4 millisecconds, 62.5 cycles per second. Commercial equipment for converting analog signals to digital contains sharp high frequency cut-off filters, to ensure that frequencies above the safe limit are removed from the signal.

A sampling rate of 2 milliseconds is considered to provide the optimum combination of adequate frequency definition for seismic work, with the most economical processing cost. Sampling of 1 millisecond is generally considered redundant, and 4 milliseconds will be satisfactory only where the upper limits of the seismic frequency range do not exceed roughly 62.5 cycles per second.

REPRESENTATION OF FREQUENCY

A seismic trace may be described in terms of its frequency components. These in turn must be adequately described in absolute terms or by comparison with some standard.

One method of representation of a given frequency is by a sinusoidal wave sampled in the same manner as a seismic trace. The wave will oscillate about zero. The vertical scale may be measured as amplitude, and the horizontal scale as time (Fig. 5).

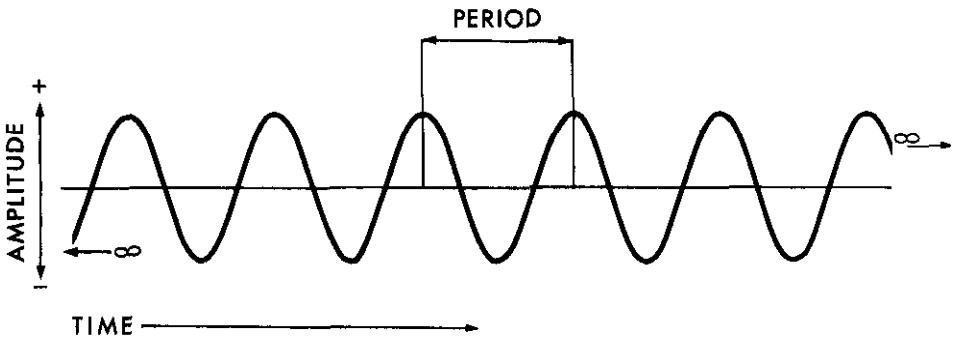


FIG. 5.—Graphic representation of a continuous wave.

Any given frequency may then be identified by the period of a single cycle. If the time scale is in seconds, a period of 100 milliseconds (divided into one second) will indicate a frequency of 10 cycles per second. In this manner, it is possible to transform one system of measurement into another. The frequency is a transform of the period and the period in turn is a transform of the frequency. A transform is nothing more than a means of measuring some quantity in a different way. Thus, the Fourier Transform, which will be examined later, is merely a transforma-

tion of measurement of a seismic trace from terms of time to a definition of that same trace in terms of frequency.

Like the conversion from period to frequency and frequency to period, transforms are reversible. That is, if a transform is possible in one direction, it is also possible to transform in the opposite direction.

The amplitude of a sinusoid may be measured on some arbitrary scale in any convenient unit of measurement. Often, maximum and minimum amplitudes are defined as one and minus one, and a uniform linear scale is applied over the intervening space.

THE COSINE WAVE

A harmonic sinusoidal wave such as that of Fig. 5 may represent the amplitude and period of a given frequency, but does not define the absolute time of any given point. Apparently, the signal started at some arbitrary time in the past and will continue indefinitely into the future. If the wave is to be fully defined, there must be some starting point from which to measure the absolute time.

A cosine wave (Fig. 6) is a fully defined regular harmonic function which starts at time zero with a value of one. It successively oscillates from a value of one through zero to minus one, and back through zero to complete the cycle at a value of one.

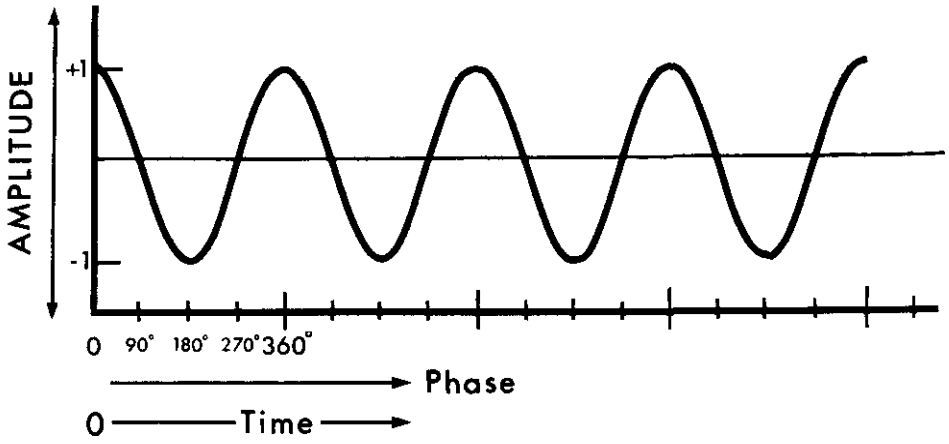


FIG. 6.—Graphic representation of a cosine wave.

GRAPHIC REPRESENTATION OF A COSINE WAVE

A graphic representation of the cosine wave may be obtained from rotation of the radius of a unit circle (Fig. 7). By convention, the radius is extended in the positive direction of the X axis at the initial or zero point of rotation. As the radius is rotated, counter-clockwise, it may

be stopped at any position. A perpendicular is dropped from the intersection of the radius and the circumference to the X axis. The value or amplitude of the cosine is the measured distance from the center of the circle along the X axis to the perpendicular. Thus, at zero degrees, the radius will lie on the X axis and the cosine value will be 1. At 30° of rotation, the measurement will be .866. At 45°, .707 and at 60°, .5. At 90°, the radius will be directly over the center point and the measured value of the cosine would be zero. Beyond 90°, the perpendicular drop from the intersection of the radius with the circumference will intersect the X axis in the negative direction from the center, and the cosine value will be minus.

Successive rotations of the radius will cause the point of intersection of the perpendicular with the X axis to move from plus one through the center point or zero to minus one, and back again, in simple harmonic motion. Thus, a graph of the cosine wave may be represented by plotting amplitude against degrees of rotation (Fig. 6).

The cosine wave may be plotted as a function of degrees of rotation, or phase, rather than time. However, if the radius rotates around the unit circle at some uniform rate of speed, it will complete one full rotation of the circle in some unit of time. Then the scale of phase on the graphic representation of the cosine wave could be replaced by a scale in time.

If the radius is shifted by rotating it a given number of degrees, it will take a certain amount of time to do this, so a phase shift is a time shift. In all cases, a phase shift may be equated to a time shift, and vice versa.

The time shift of a seismic trace can be directly related to a phase shift and may be expressed as such. The phase shift which is inherent in some filters, particularly analog filters, will cause a time shift of the information recorded to which they are applied.

The addition of the phase completes the parameters required to fully describe a given frequency component of a seismic signal. Hence, any frequency component of a seismic signal may be measured by stating its frequency, its amplitude, and its phase.

SYNTHESIS OF A SEISMIC SIGNAL

A basic theorem of signal analysis is that any given periodic signal represents the sum of a number of harmonic frequencies, each of a specified amplitude and phase shift.

This is also the basic assumption used for many of the methods of seismic signal analysis and processing. It is assumed that a seismic signal is the sum of a finite number of frequencies, each with a unique amplitude and phase shift.

To illustrate this, first examine the simplest case. A pure frequency may be represented by a cosine wave, plotted in the form of a graph of amplitude versus time (Fig. 6).

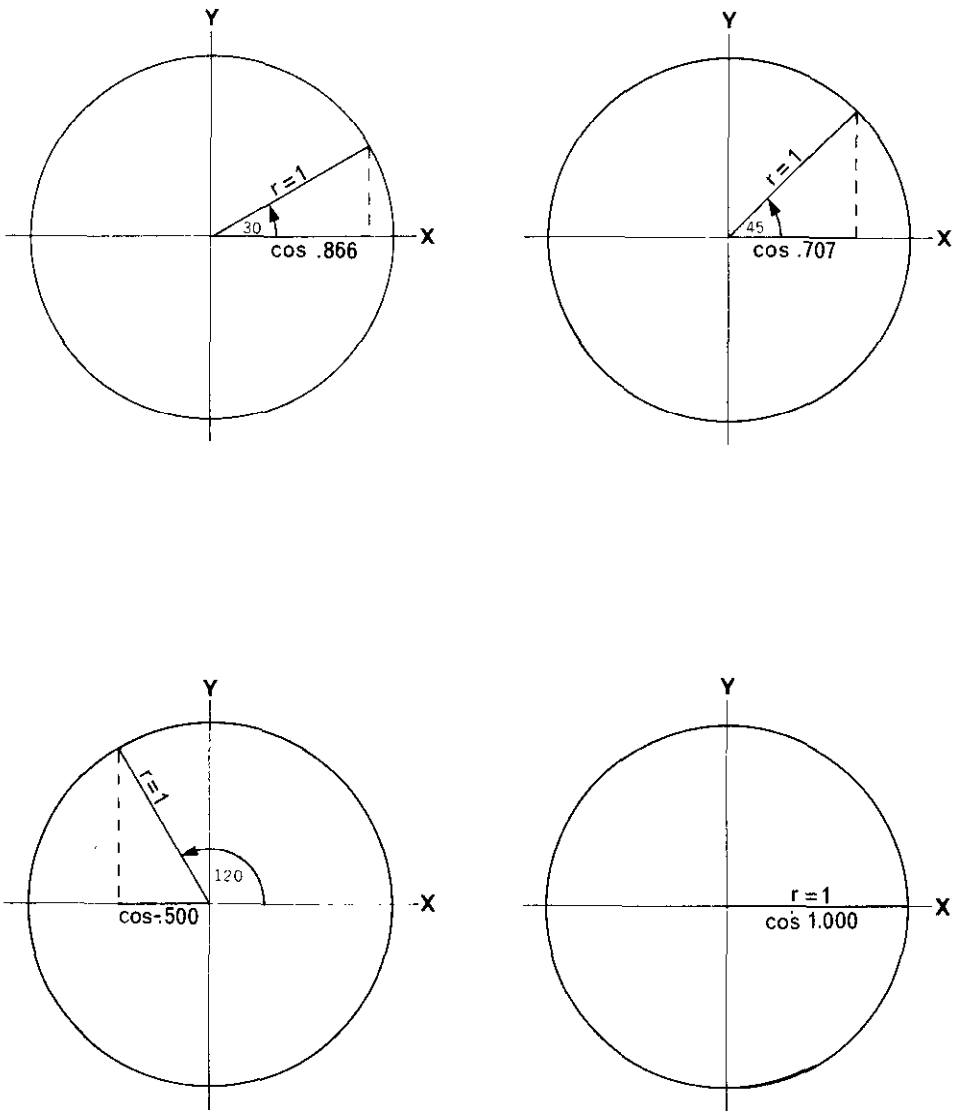


FIG. 7.—Graphic development of the cosine function.

A slightly more complex signal might be represented by the sum of the basic frequency and its first harmonic, i.e., a frequency double that of the base frequency. If the two signals of the same amplitude are summed and then divided by two, to maintain the amplitude ratio, they might appear as in Fig. 8. Changing the amplitude of either of the signals will change the general appearance of the output somewhat.

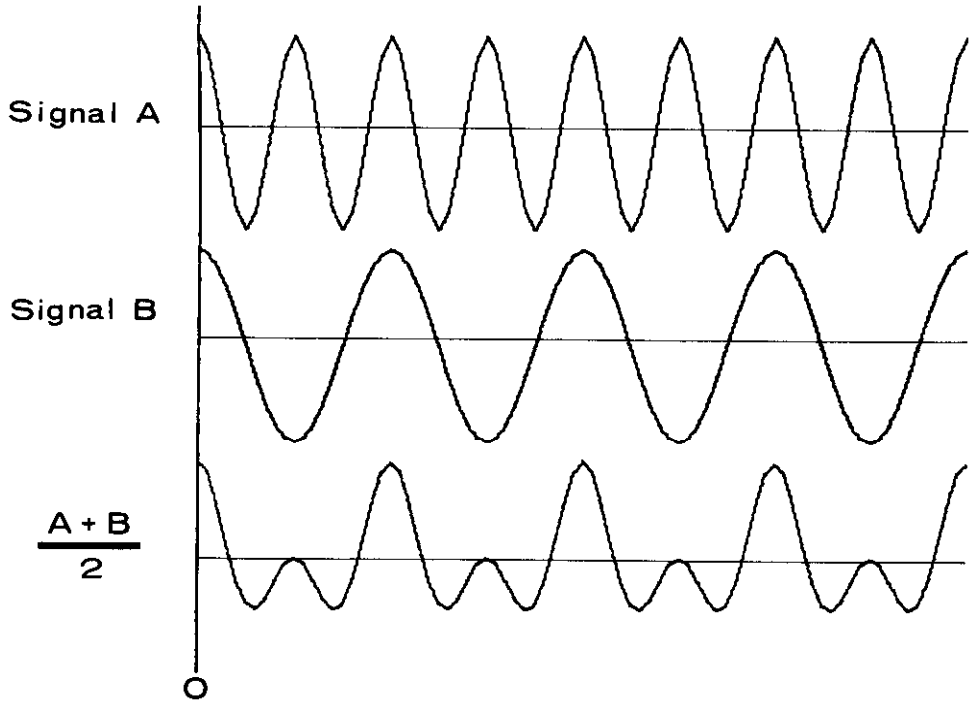


FIG. 8.—The bottom signal represents the sum of the upper two.

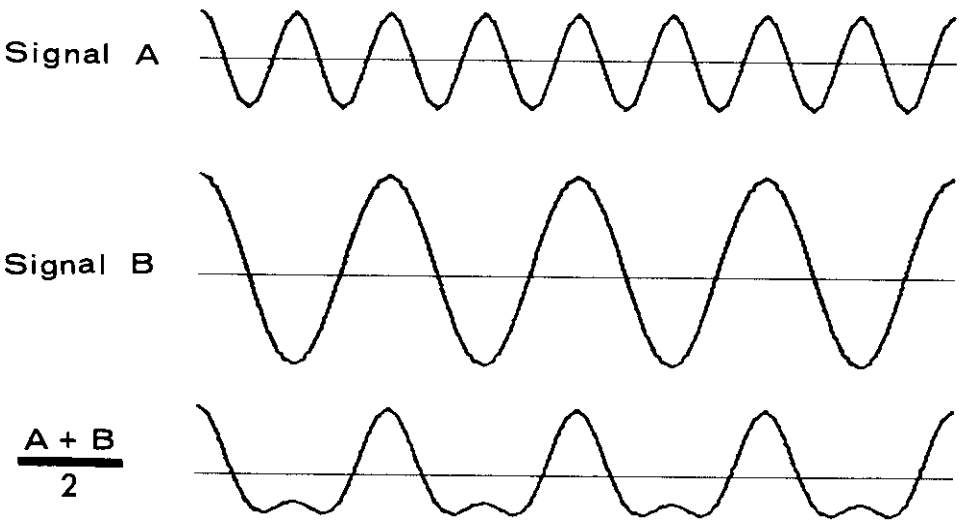


FIG. 9.—Sum of base frequency and first harmonic at two to one amplitude ratio.

For instance, if the harmonic happened to have half the amplitude of the base frequency, the resulting summed signal would appear as in Fig. 9. Thus, varying the amplitude of the combined frequencies will vary the appearance of the resulting signal.

Note also that there is a definite periodicity to the resulting signal. The pattern repeats over each period of the lowest frequency. Generally, the pattern will repeat each time the signals fall into phase. A manner of determining the periodicity of the resulting signal is to find that period into which the period of oscillation of each of the component signals will divide evenly. For instance, if signals of 25 cycles (.040 seconds) and 10 cycles (.100 seconds) are added together, the periodicity of the resulting signal will be .200 seconds, the lowest common numerator of the two periods. If signals of uncommon numerators are combined, the periodicity can become very long.

Summing together several frequencies can produce rather complex signals. For instance, signals of two, four, and eight cycles per second produce the results shown in Fig. 10. Notice that the periodicity of this signal is one half second. This is the period of the 2-cycle per second signal, and is also the length of time that the periods of the 4-cycle and the 8-cycle signals will divide into evenly.

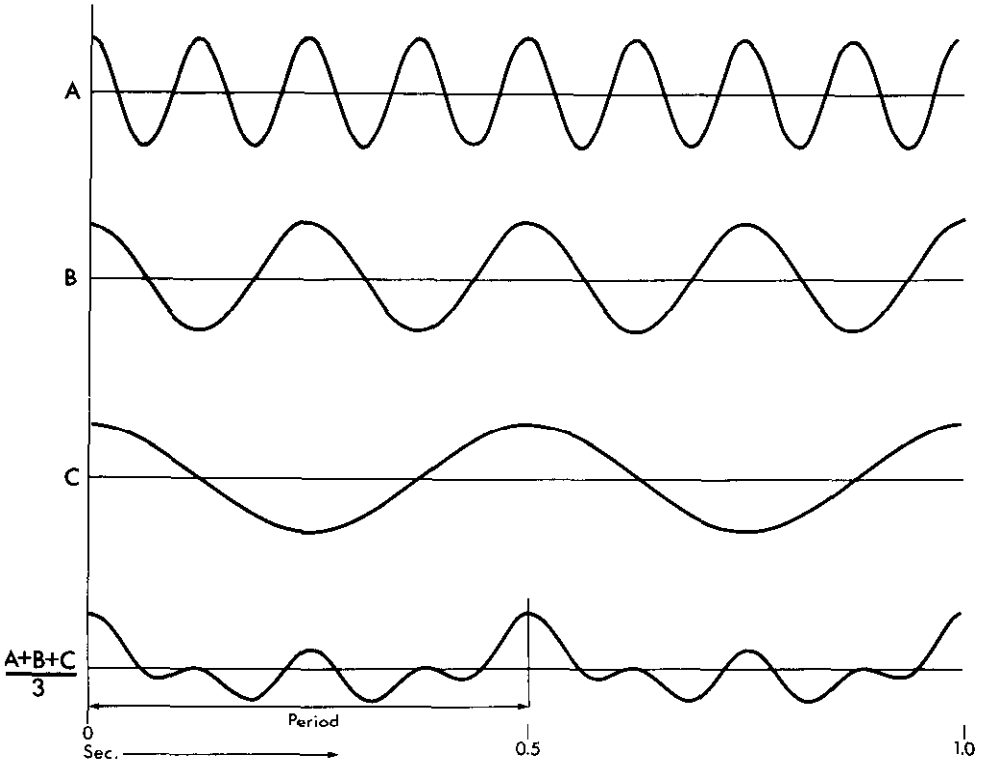


Fig. 10.—Simple signal synthesis.

If three waves summed together happen to be something like 6, 7, and 8 cycles per second, as shown in Fig. 11, the signal becomes more complex and the periodicity increases to one second, which is the smallest period of time common to the period of the three components. Notice that with relatively simple components, in phase, and of the same amplitude, a signal has been produced which is not unlike part of a seismic trace.

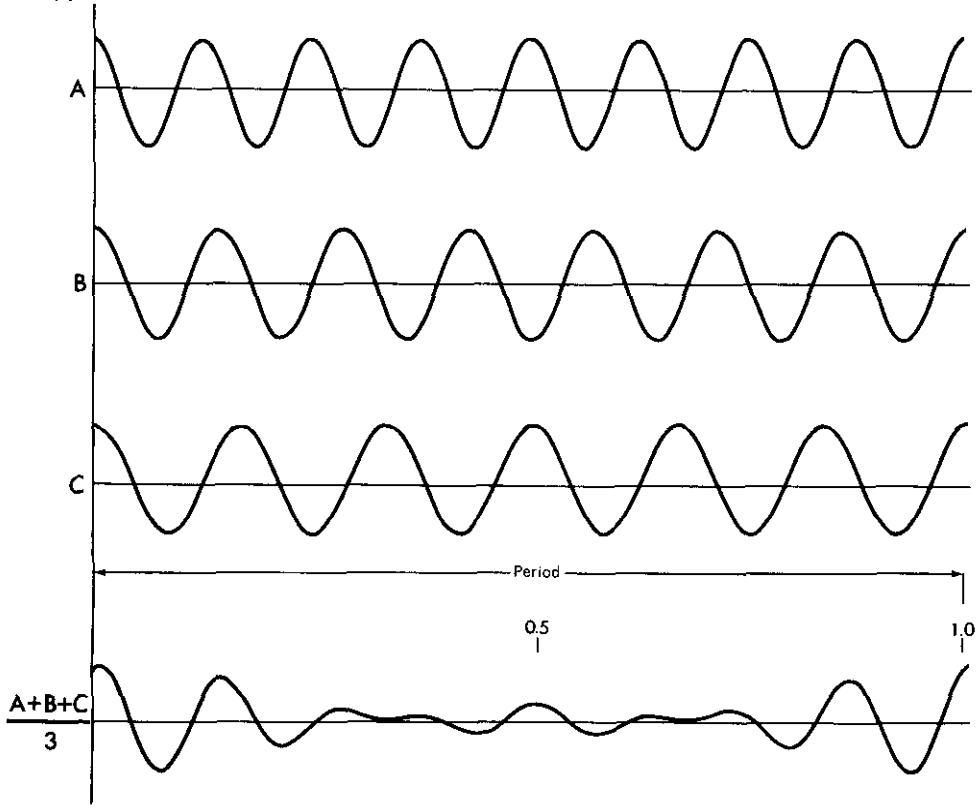


FIG. 11.—Uneven components produce longer period signals.

Thus far, only the number and the frequencies added together have been modified, except for a brief examination of the effects of varying amplitude. The phase may also vary. A composite signal may be changed substantially by a relatively minor change of phase. To illustrate, the simple sum of the signal and its first harmonic is shown in Fig. 12. These two signals are in phase. If the first harmonic is shifted 90° in phase, the appearance of the resulting signal changes from a symmetrical signal to a 'peg leg' type of signal. Note, however, that the periodicity has not changed, merely the appearance or character of the signal within the periodicity has been modified. In such cases, the components will have the same phase relationship at each point of periodicity.

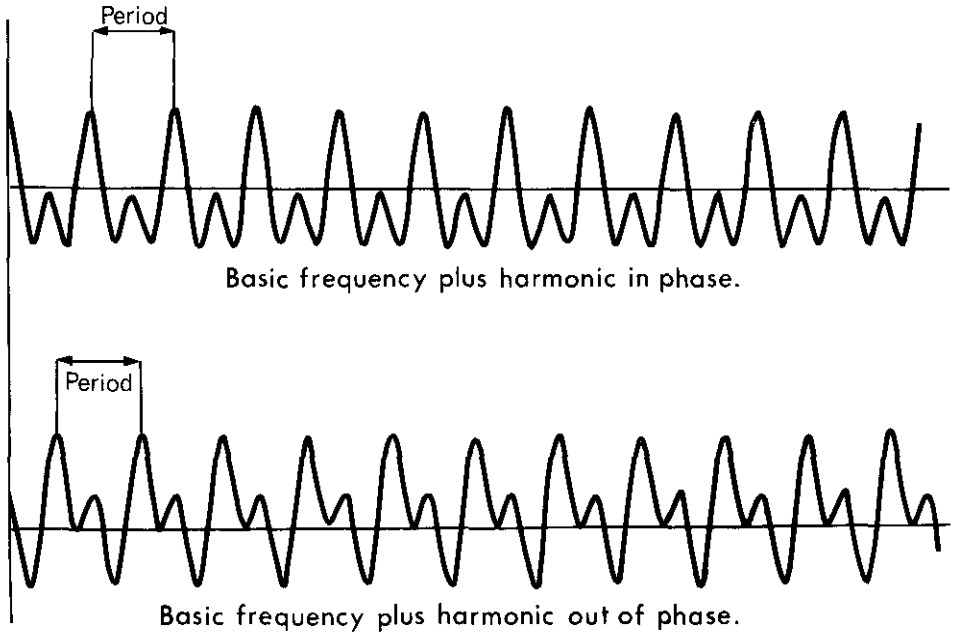


FIG. 12.—Phase shifting components alters the signal character.

Fig. 10 illustrates three traces of frequency, 2, 4, and 8 cycles summed together in phase to produce a periodic signal. In Fig. 13, the same traces have been summed together at slightly different phase shifts. The resulting trace is substantially different from the zero phase example, but the periodicity remains the same.

It is possible in exactly the manner outlined to approximate any seismic trace by summing together a series of harmonic frequencies, each at some specified amplitude and phase shift (Fig. 14).

Obviously, the degree of approximation will depend on the detail to which the components are measured. Component frequencies measured to .5 cycles per second will provide greater detail than multiples of 5 cycles per second.

A band pass filtered signal has a limited frequency range and, therefore, will be limited in the complexity or detail of total signal. Expressed in another way, the band filtered signal can not contain as much information as a broad band trace, i.e., one with a much broader range of frequencies. By the same token, the range of amplitudes which can be recorded will also help determine the amount of information which can be carried in the seismic signal. For these reasons, a broad band field recording is to be preferred ove a narrow band ecording, and a wide amplitude range is to be preferred over a recording of limited amplitude range.

The seismic record in Fig. 14 represents the actual decomposition of a seismic trace into 20 frequency components. These components have

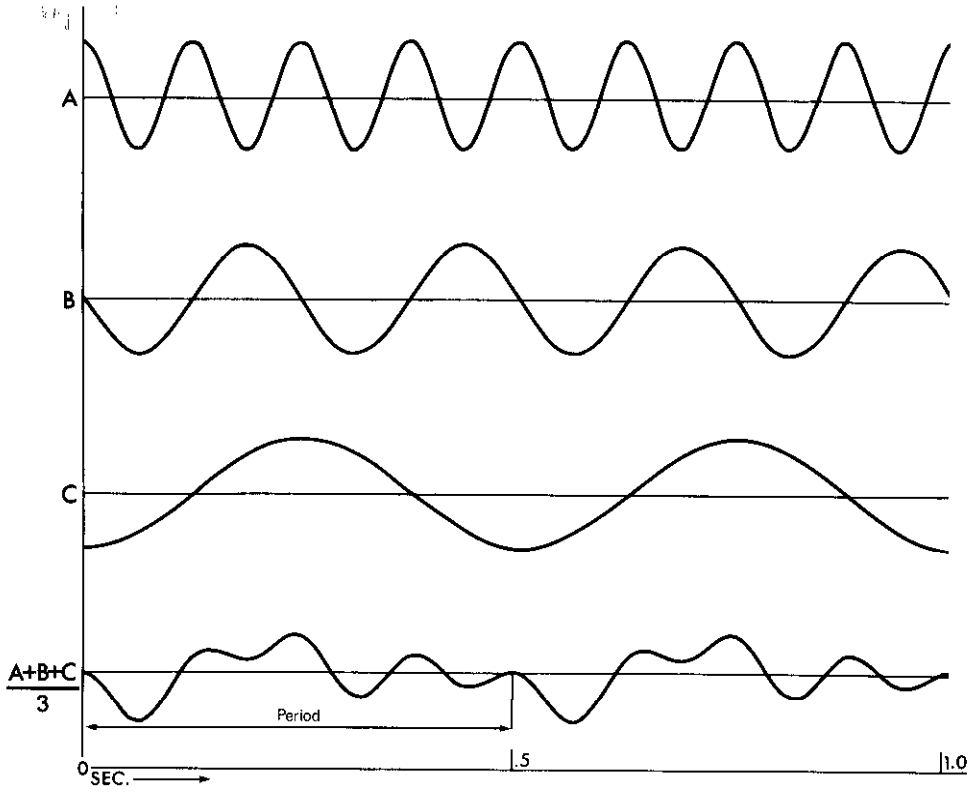


FIG. 13.—Shifting phase alters signal characteristics.

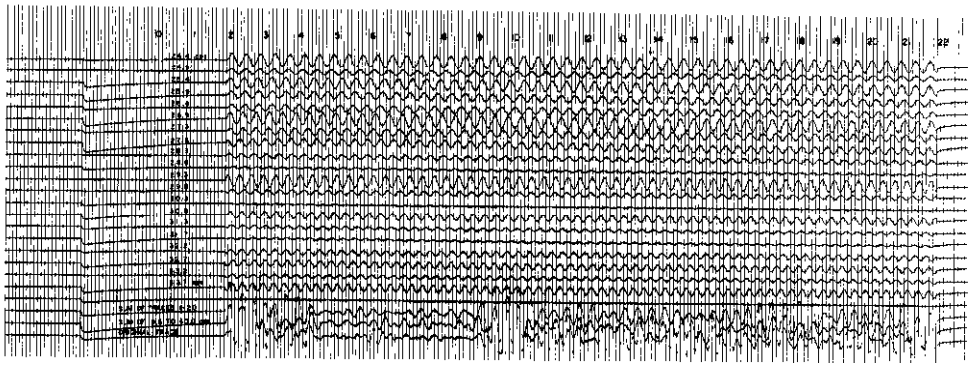


FIG. 14.—Composition of a seismic trace.

then been re-combined to approximate the original trace. The original trace is at the bottom of the figure and the re-constituted trace is above. This is necessarily a rather crude approximation of the trace, in that only 20 frequency steps over a limited range were used. Increasing the number of the frequencies to 40 permits a better approximation to be made. The approximation could be further improved by using a smaller frequency increment. As a general rule, the greater the detail used in sampling and in the trace analysis process, the better will be the result. However, the cost of processing is almost a direct function of the amount of the detail, so some compromise must always be made between the sconomic limits and the quality desired.

Note that in the synthesis of the trace, single values of amplitude and phase shift for each frequency were used. It is conceivable that a rec-

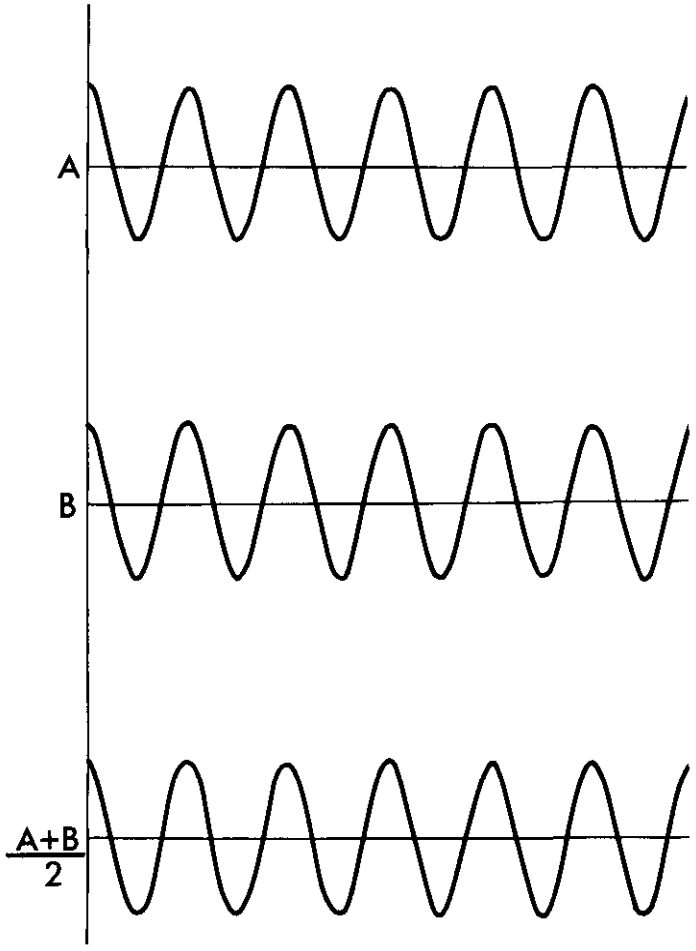


FIG. 15.—The sum of like frequency and phase is similar to the input.

orded signal may include similar frequencies from more than one energy source. For example, wind noise, AC pickup, or motor noise may be sensed by the geophones during the period in which energy is being recorded from the shot.

To examine the effects of duplicate energy sources on the seismic record, assume two identical sources of energy which produce the same frequency and phase. If the two signals are summed and normalized to the same amplitude, the resulting signal will be identical to either of the two energy sources (Fig. 15).

One of the two energy sources may then be shifted in phase with the other by 180°. This could be done by rotating one of the signals 180°, or by lagging the time of one signal one half period with respect to the other. If the two signals are 180° out of phase, the numerical value

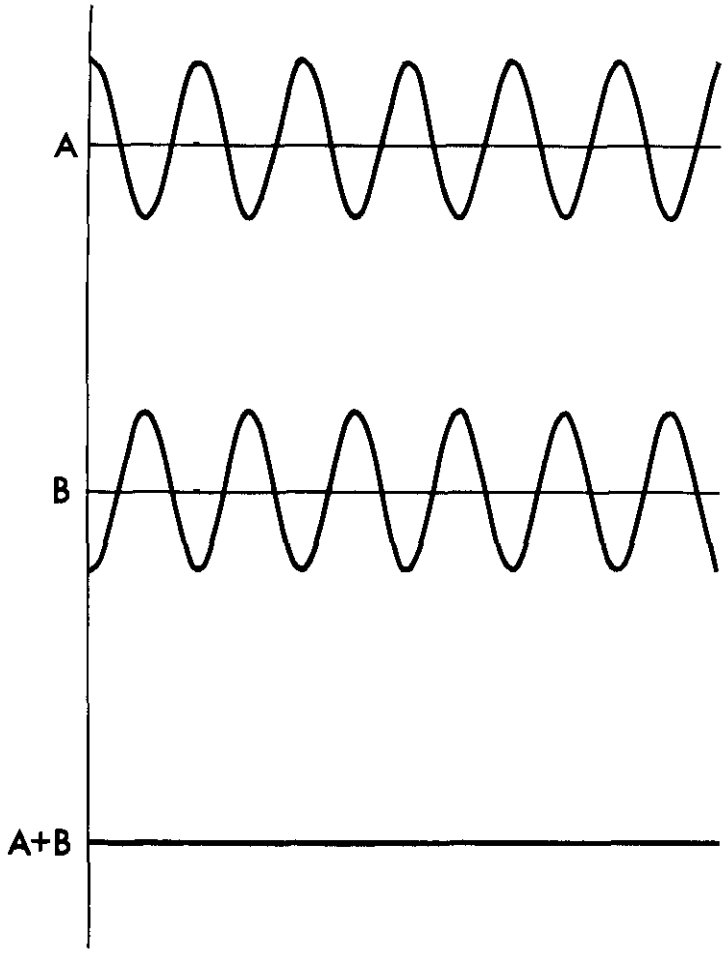


FIG. 16.—The sum of two like frequencies of opposite phase is zero.

of the amplitude of the signals will be equal but opposite in sign. When the two signals are summed, the resulting amplitude will be zero (Fig. 16).

These two examples represent the extremes of phase relationship, i.e., two like frequencies which are first completly in phase, and second completely out of phase. Any other condition must fall somewhere between. If one of the signals is lagged an amount less than 180° with respect to the first, the phase of the resulting sum will be the average of the two, and the amplitude value will be somewhat less than the sum of the same two traces in phase. However, the frequency of the resulting signal remains unchanged (Fig. 17).

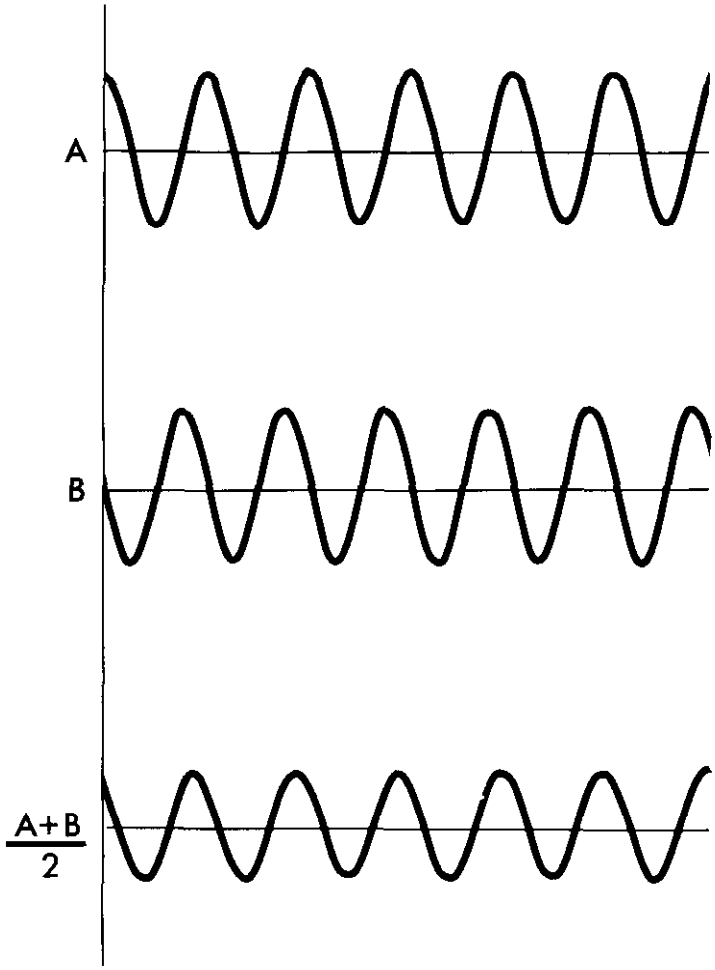


FIG. 17.—Components of the same frequency may be represented by a single signal.

Thus, any two components of equal frequency can be represented by a single component of the same frequency, having the average phase of the two and the algebraic sum of the amplitudes.

Extending this process, the combination of two sources may be combined with a third, and so on, so that any number of signals of the same frequency may be represented in the analysis by a single frequency component of some specified amplitude and phase value.

THE FOURIER TRANSFORM

A seismic trace can be expressed in terms of its frequency, amplitude, and phase. The amplitude of each frequency component will vary, as well as the phase of each frequency. A convenient way of describing the components of the seismic trace is to graph the amplitude and phase as a function of frequency.

The Amplitude Spectrum

Each frequency component which enters into the sum to form the seismic trace will be defined as a constant amplitude over the length of the signal analyzed. Therefore, the amplitude may be graphed as a function of frequency. The example in Fig. 18 is a plot of the amplitude of the frequency components which were used to synthesize the seismic trace in Fig. 14. Frequency is plotted along the abscissa and amplitude on the ordinate.

The Phase Spectrum

The phase component of each frequency is also defined. This may be graphed on the same frequency scale as used for the amplitude spectrum with the phase angle plotted on the ordinate. It is customary to plot the phase within limits of $+180°$ to $-180°$. Any phase shift lying between 180° and 360° may be shifted 180° to 360° to obtain the same phase relationship (Fig. 19).

These two charts represent the Fourier Transform of the seismic signal of Fig. 14. Within the limits of approximation imposed by the sample spacing, the trace may be fully described by the values plotted for parameters of frequency, amplitude, and phase. The same trace in the time domain can be fully defined by a chart of amplitude versus time. The two modes of definition are Fourier Transform pairs.

The Fourier Transform, like all transforms, is reversible. The seismic trace may be reconstituted in the time domain by merely adding together cosine waves at the designated amplitude and phase over the length of time of the signal. The Fourier Transform applies only to periodic functions. Although no periodicity may be evident on the signal, the transform will assume that the signal is periodic over the length of the signal used for computing the transform.

THE UNIT IMPULSE

The source of energy for seismic exploration methods is usually an impulse and, in particular, an impulse developed by explosives.

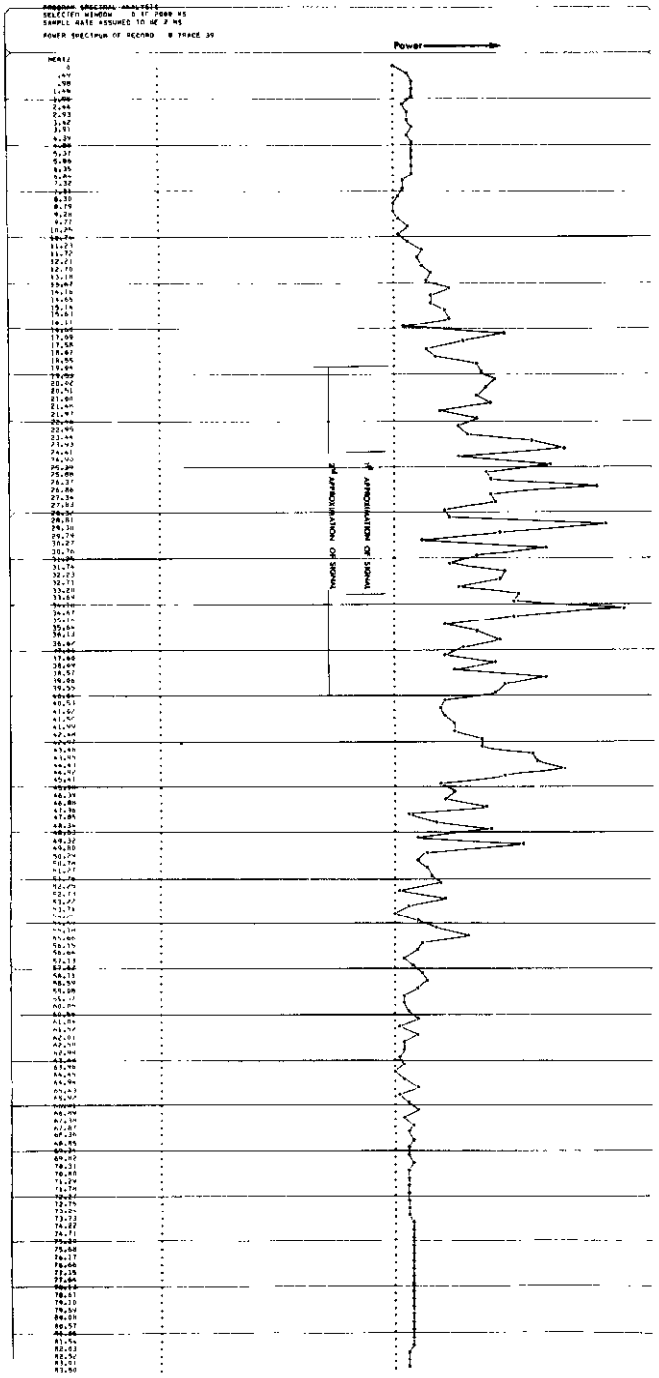


FIG. 18.—Amplitude spectrum of seismic trace of Fig. 14.

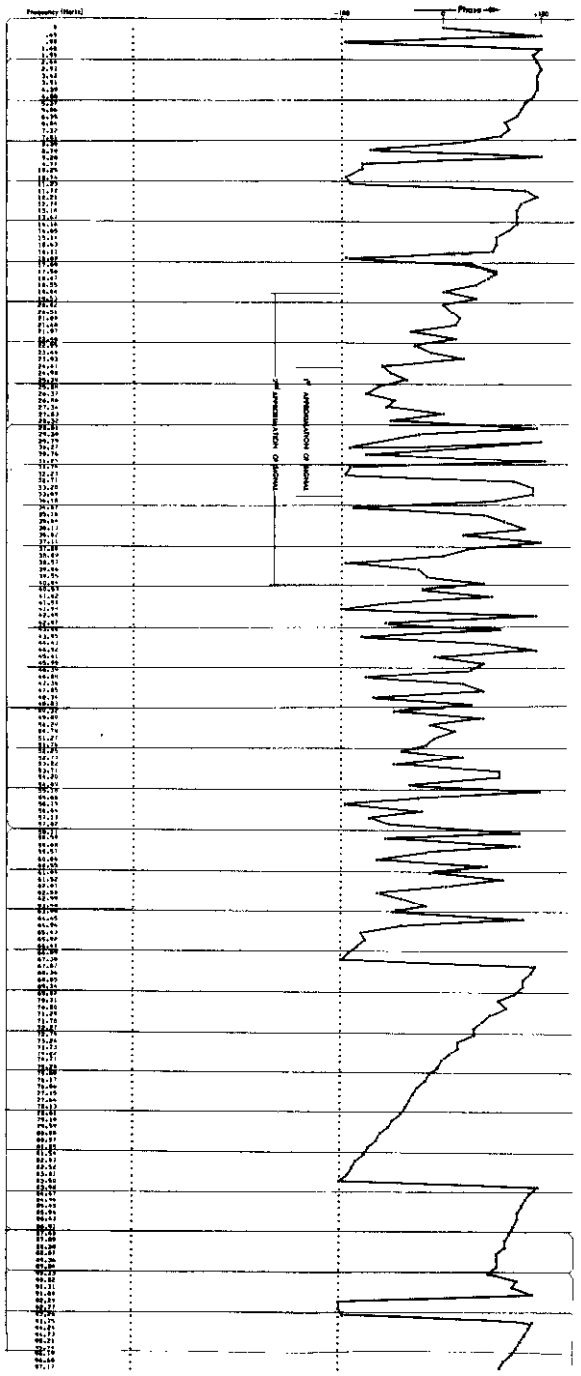


FIG. 19.—Phase spectrum of seismic trace of Fig. 14.

In standard seismic methods, the source of energy is dynamite. When ignited, dynamite burns so fast that the term 'explosion' is used to describe the process. If the energy released from the burning of dynamite is plotted as a function of time, the graph will appear similar to Fig. 20. The amplitude is zero at all times preceding the ignition. When the dynamite is ignited, there is an extremely rapid rise of energy to the maximum value. At this point all the material is oxidized, there is no more combustion and the energy drops again rapidly to zero.

Measurement of the absolute amount of energy released by the explosion is not required for production operations. The only requirement is to have a charge large enough to return sufficient reflected energy without over-driving the system. In such cases, the energy from the

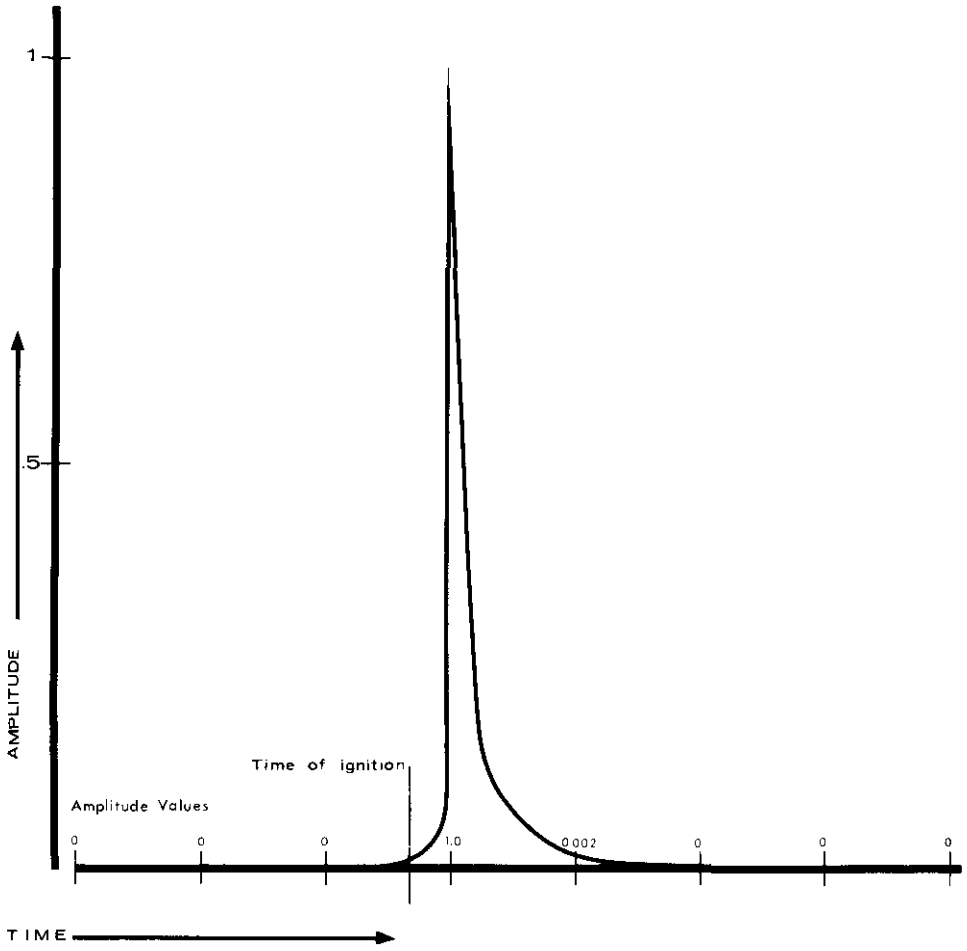


FIG. 20.—Theoretical shape of shot pulse.

normal charge size may be considered to be a single energy unit with a maximum amplitude of one, as indicated on the graph of energy versus time. If the shot impulse is sampled at the same time interval as is used in sampling the seismic trace, the resulting function will have values of 0, 0, 0, 1, 0, 0, 0, and so on. Thus, the shot pulse may be expressed by a time series function which has a value of zero at all points except one, at which it has a value of one.

This is a unique function which is termed a delta function, or a unit impulse.

Frequency Content of the Shot Pulse

The seismic signal can not contain a broad range of frequencies unless these frequencies are generated by the energy source. The shot pulse is a signal composed of all frequencies summed together in phase. To illustrate this, a number of cosine frequencies of equal amplitude are centered about a line passing through zero phase of each, as in Fig. 21, summed, and averaged to maintain the amplitude level. A poor approximation is obtained by summing only five frequencies. A greater number of frequency components makes a better approximation (Fig. 22). As more high frequency components are added, the resulting signal will approach the spiked shape of the unit impulse. Eventually, the spike becomes so narrow that if the resulting signal is sampled at the normal rate, the output values will be the same as the unit impulse.

UNIT IMPULSE RESPONSE

The reflection signal appearing on the seismic recording is quite different from the unit impulse developed by the dynamite shot. With the exception of the primary waves, or first breaks, the recorded signal is not the direct pulse of the dynamite, but is rather a modification of the unit impulse, which has been processed by travelling through the earth and the recording instruments. For a crude comparison, the shot pulse might be compared to striking a key on the piano. Energy is transmitted from the piano key to a string which responds by transmitting an oscillation through the air to the ear. The shot may transmit a unit impulse in essentially unaltered form to a reflecting horizon. The horizon responds by returning through the earth the typical reflection detected by a geophone and displayed on a seismic record.

This is a crude model of a much more complex chain of events. The seismic recording represents the total effect of the entire recording system, including the geophones, the galvanometers, the amplifiers, and the complex filtering effects of the earth. Thus, the recorded response is the interacting sum of many varied and complex system components.

It would be almost impossible to determine the individual contributions of each of the components, but this is not necessary. Fortunately, the contribution of each of the several components is nearly constant, so that if a shot is repeated, the resulting record is similar to that of the previous shot.

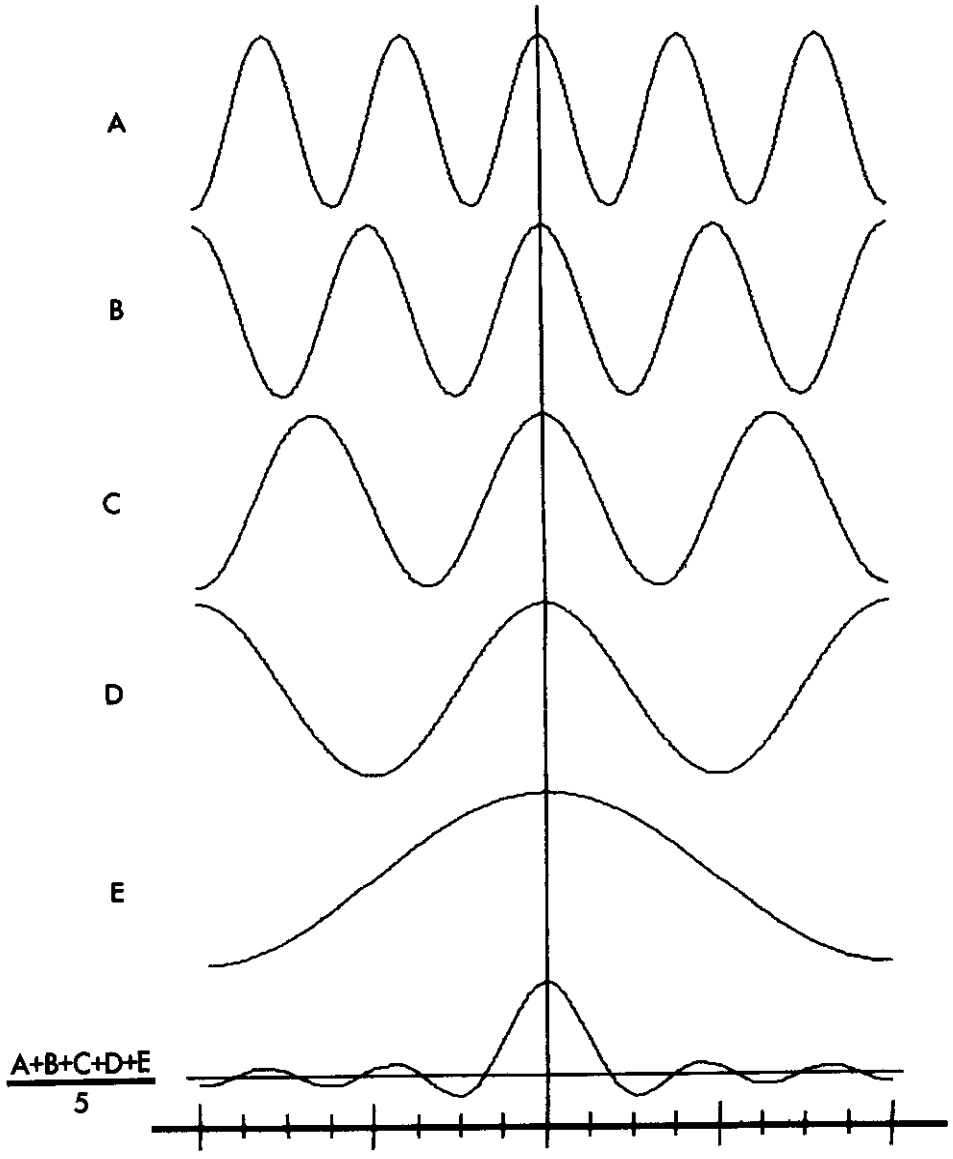


FIG. 21.—The bottom signal is the sum of the others.

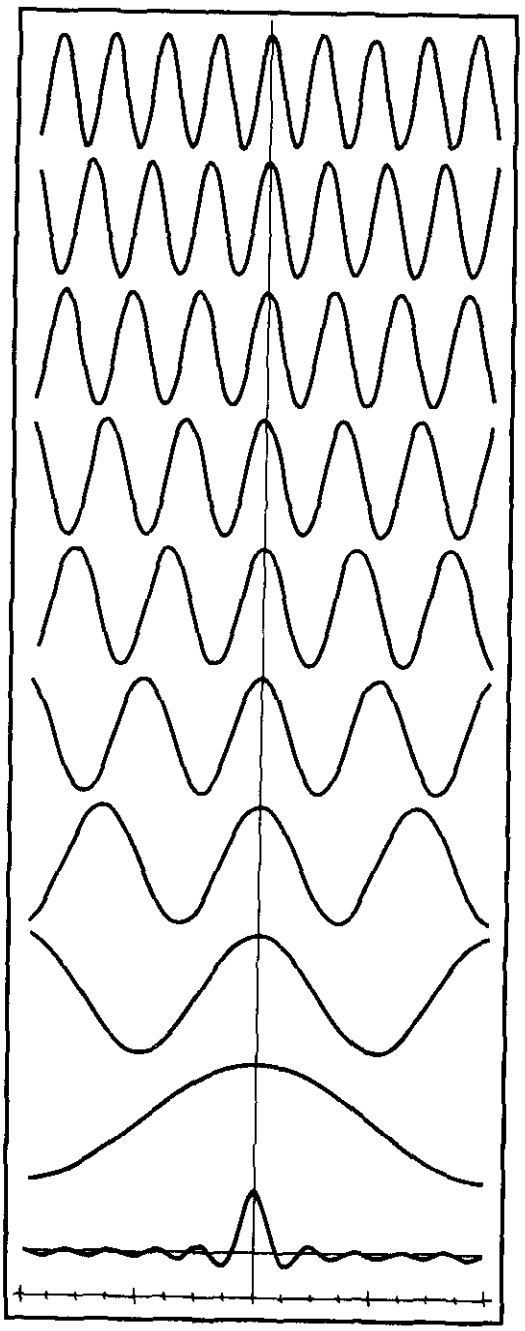


FIG. 22.—Signal Composition.
Increasing the number of frequencies sharpens the spike approximation.

The point of all this is that it is not necessary to define the component response of the earth and the instruments in order to synthesize or to analyze a seismic signal. It is sufficient to determine as closely as possible the response of the entire system to the unit impulse. In effect, it is possible to consolidate all of the components into a single term, so that unit impulses will always produce the unit impulse response. In this manner, a simple model of the entire system may be developed. Each time the unit impulse is introduced into the system model, the output will be the unit impulse response. The model serves very well for the purposes of analysis and synthesis of the seismic process.

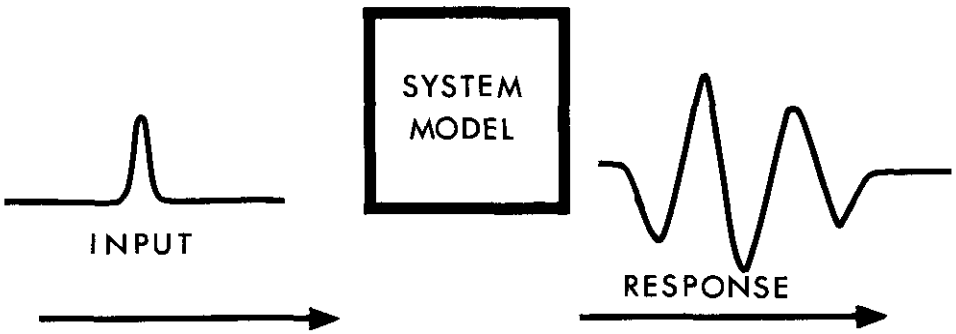


FIG. 23.—System model to produce a seismic reflection from a unit impulse.

The basic model (Fig. 23) consists of the following: a simple spike with a numerical value of one for a shot impulse input; the output is a wavelet which represents the recording of a reflection from a single formation contact; between these two signals is a box which represents the system. All of the intermediate effects of the process are thus lumped into one term. If a unit impulse is input, the unit impulse response is obtained as output.

THE TRANSFER FUNCTION

The two end signals are designated the unit impulse, and the unit impulse response. They may also be termed the input function and the output function. The process that occurs between these two extremes may then be defined as the transfer function. The relation between the input function, the transfer function, and the output function may be treated in a manner similar to that of a simple arithmetic equation. If two of the functions are known, the third may be computed.

The unit impulse input to the simple model is the number one. This is operated on by some transfer function to produce the output signal. The value of the transfer function may be determined by a form of division of the output by the input value, in this case one. The transfer function in this model then equals the output function. Therefore,

if the output response of any system to a single spike input can be determined, the transfer function of the system can be determined. Then, for any other input, the transfer function may be used as an operator to estimate the nature of the output signal and vice versa.

At this point, it should be clear that a system model using a reliable transfer function can be a powerful tool for seismic signal synthesis and analysis.

THE TRANSFER FUNCTION AS A FILTER

The synthesis of wave forms, including the shot pulse, indicates that high frequency components must be present if a sharp wave form is to be obtained. The spike input to the seismic model contains all frequencies, but the output is deficient in the higher frequencies. The system has acted as a filter to pass only the lower frequencies. Also, the wave forms of the output is not symmetrical. When frequency components are added together in phase, the resulting signal is symmetrical, as demonstrated by the construction of the input spike. A phase shift causes the wave form to be asymmetrical, similar to the response of the model to the unit impulse (Fig. 24).

It would appear, then, that the effect of the system has been to filter out many of the higher frequencies and to shift the phase of some of the remaining components. A phase shift is a time shift and, since the model represents a real time system, that is, nothing may come out of the system before something goes into it, any phase shift must be a time delay. A third effect may be some attenuation of amplitude of certain frequencies.

DECONVOLUTION

A simple wavelet of two frequencies in phase may be passed through the system. Assume these are frequencies that the system will not attenuate, but a phase shift or lag will be applied to one. The output will consist of the two frequencies separated by the amount of lag which was applied (Fig. 25a).

Now assume the output from the system was reversed in time, and again passed through the system. The two frequencies would again be passed unchanged, except for the lag. But since the signal went through in reverse order, the net effect of the lag should be to negate the lag that was applied in the original transit through the system. Therefore, the output of the second pass returns the signal back to its original condition (Fig. 25b).

If a spike passes through the system, some of the frequencies will be passed, but may be phase shifted. The remaining components are lost. If the output of this operation is reversed and again passed through the system, the frequency content of the output of the second pass will not change substantially, but all of the lags or phase shifts will be applied in the opposite direction. The effect of the reversed second

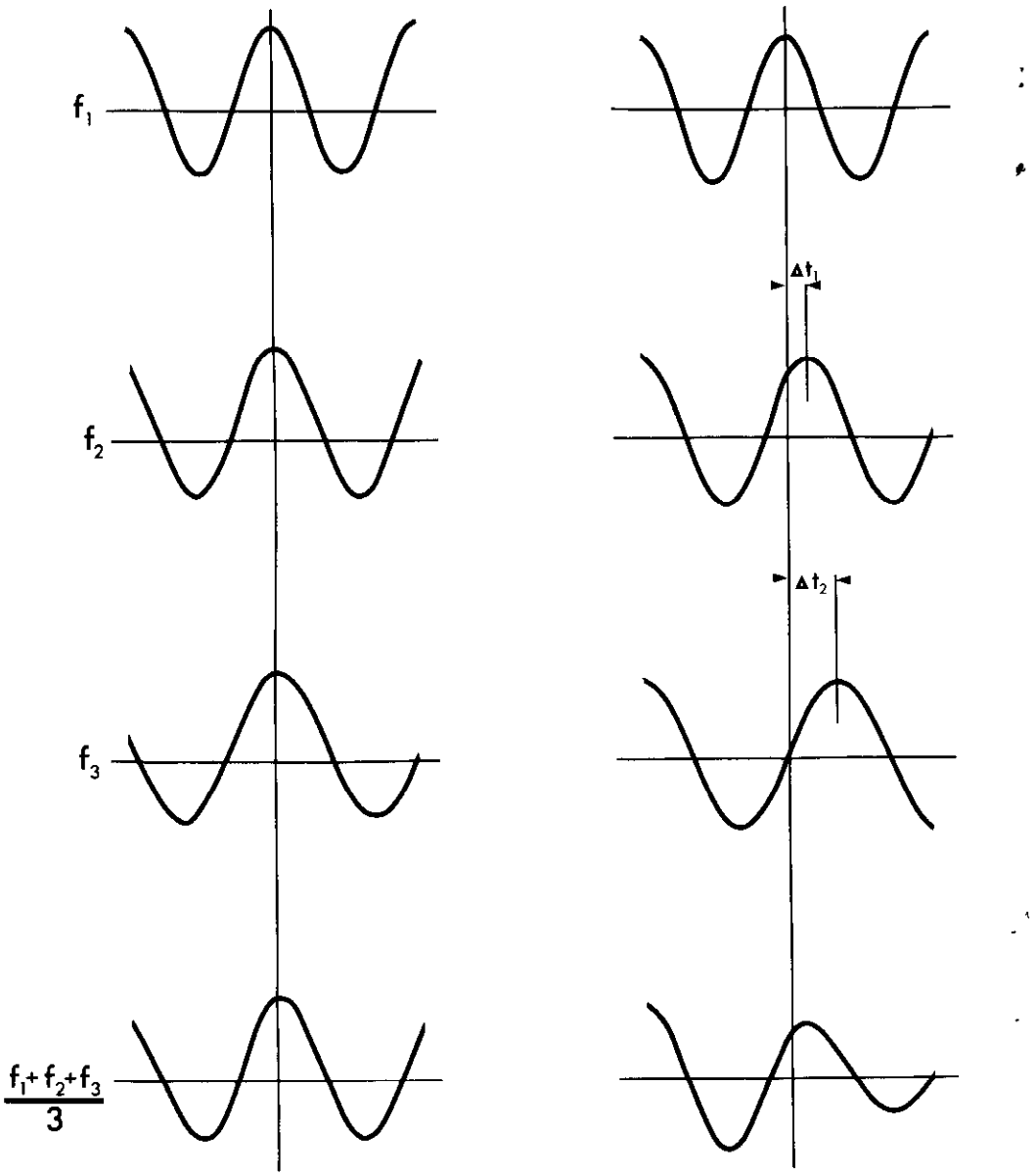


FIG. 24.—A slight phase shift or time lag of the frequency components causes the asymmetric appearance characteristic of a seismic reflection.

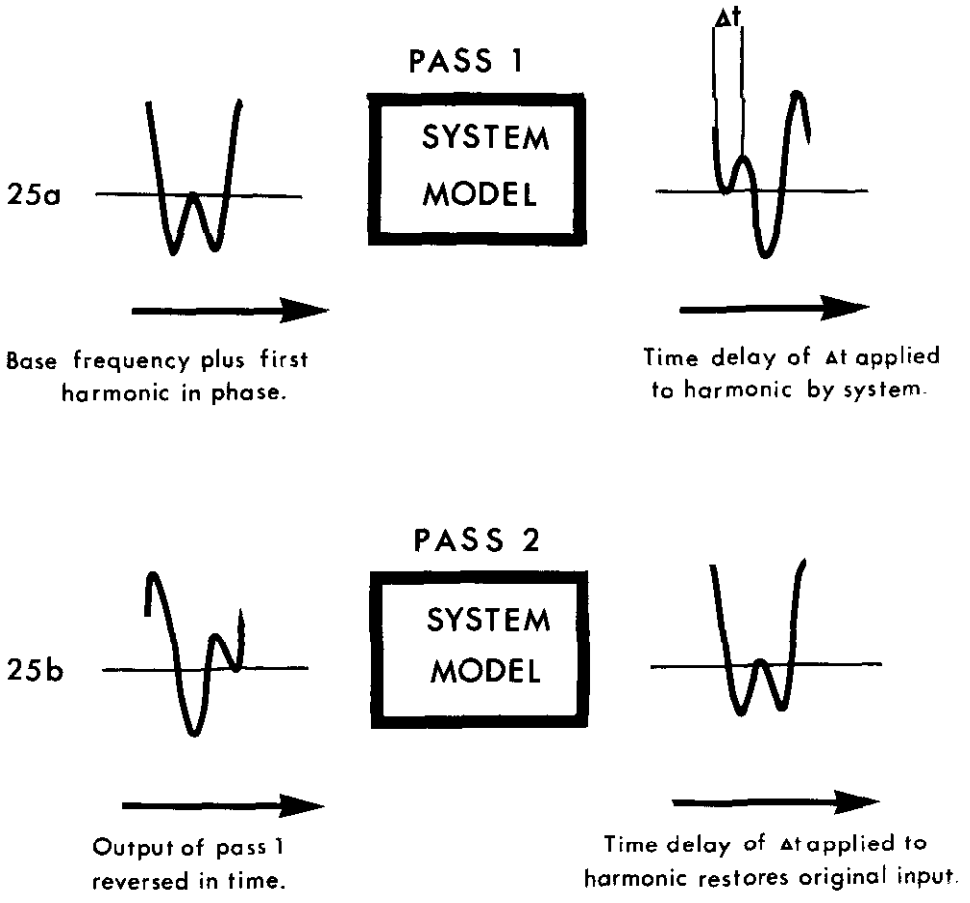


FIG. 25.—A time delay (phase shift) may be restored by reversing the output and passing it again through the system.

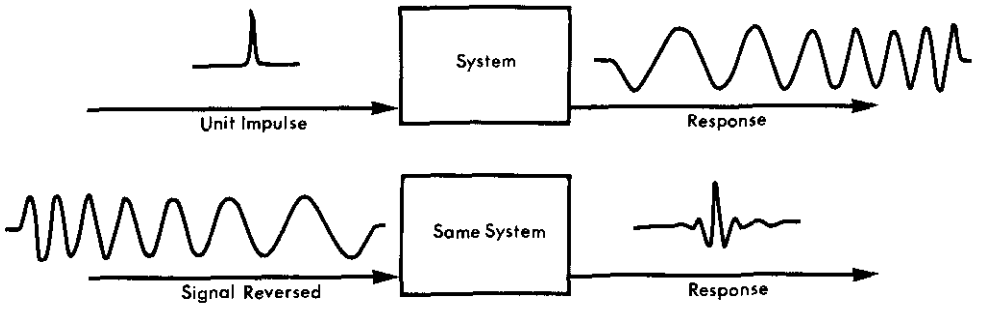


FIG. 26.—The response of a system to reversed output approximates the unit input.

pass will be to return the signal to the best approximation of the input spike that is possible to obtain with the limited frequency content present after the first pass (Fig. 26).

In summary, if the response of a system is reversed in time, and passed again through the system, the result will be the best approximation of the original input that can be attained with the limited number of frequency components present in the response.

This process is deconvolution. A completely deconvolved ideal seismic trace would contain a spike at each point on the trace where a reflection originated. The amplitude of the spike would be equal to the reflection coefficient. In practice, the process is never completely successful, inasmuch as a large portion of the frequency components required to produce a spike have been removed in the seismic process. The deconvolution can produce no better approximation of a spike than that permitted by summing the available frequencies.

To summarize the procedures used in the analysis and processing of seismic data, first endeavour to set up a model of the system which will respond to a unit impulse in the same manner as the recording system responds to the seismic shot. The transfer function can then be derived, which in turn may be used to reverse the effects of the field process and approximate the original string of reflection coefficients.

SYNTHETIC SEISMOGRAMS

A practical application of the system response model is the construction of synthetic seismograms.

Snell's Law provides a means for computing a reflection coefficient at each velocity interface within the geologic section. The approximate reflection coefficient at the contact between two velocity units may be computed by the formula $\frac{V_2 - V_1}{V_2 + V_1}$.

The reflection coefficient is computed for each velocity interface within the geologic section and properly located as a spike along a time series corresponding to reflection times.

The desired unit impulse response is used as a transfer function in our model. The series of reflection coefficients is then passed through the model system. Each reflection coefficient will in turn cause the system to respond in the same manner as a reflecting horizon will on a seismic recording. The resulting synthetic seismogram (Fig. 27) is usually a good predictor of the actual field recorded seismogram.

The system model used for the development of a synthetic seismogram may be further refined by introducing the effects of multiple reflections, transmission coefficients, and the gain of the instruments.

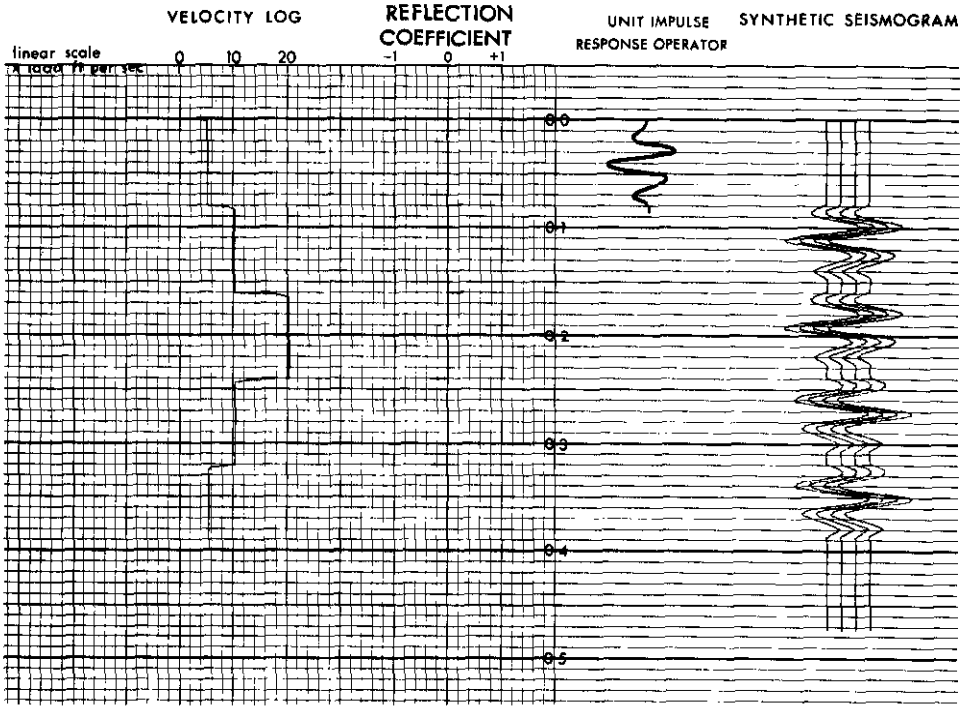


FIG. 27.—Synthetic seismogram.

CORRELATION PROCESSES

Cross Correlation

Cross correlation is merely a mathematical method of obtaining the best alignment between two signals. If two sine waves of identical period are to be aligned exactly, it would be quite easy to do this manually. One curve would be shifted in relation to the other until there was a point to point alignment of one curve with the other.

It would also be possible to sample the two curves at some uniform increment to obtain two tables of time series values (Fig. 28). These two series of values may then be placed in conjunction and corresponding points multiplied and summed. The resulting value is taken as the first point of the output signal. The second point is obtained by shifting one of the tables of values one position, and repeating the process (Fig. 29). In this operation, the signal which is shifted is termed the operator.

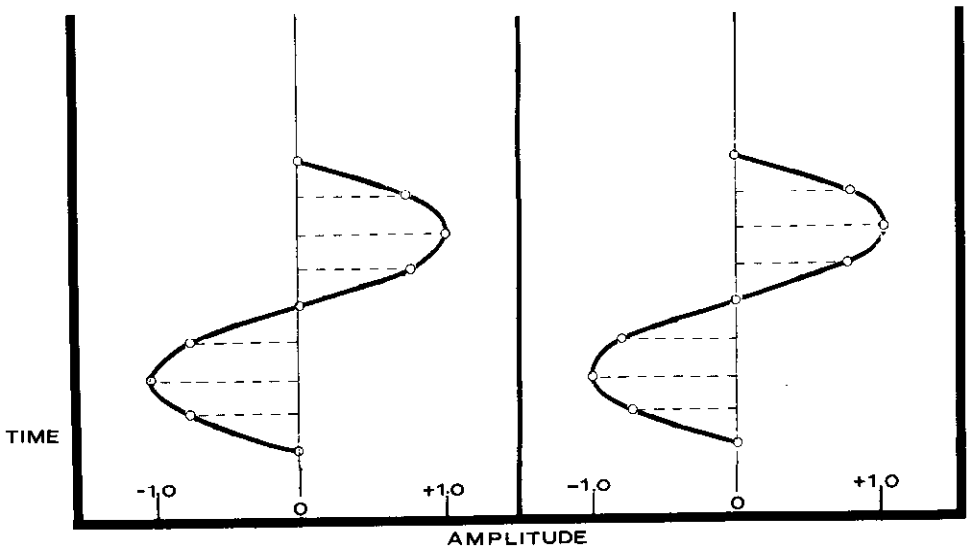


Table of Values

Curve 1	Curve 2
0.0	0.0
0.7	0.7
+1.0	+1.0
0.7	0.7
0.0	0.0
-0.7	-0.7
-1.0	-1.0
-0.7	-0.7
0.0	0.0

FIG. 28.—Table of values resulting from sampling of two identical sine waves.

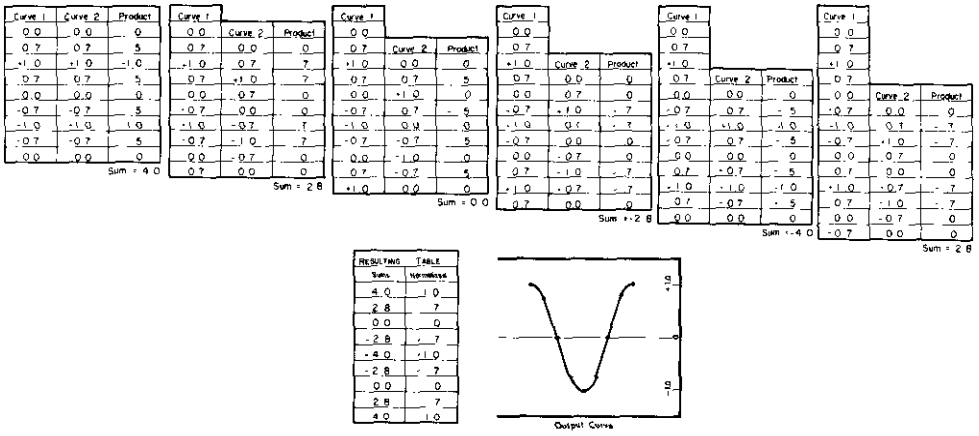


FIG. 29.—Mechanics of cross correlation.

The entire process may be repeated several times. The resulting output is then normalized by dividing all points by the maximum value. The results may then be plotted at the same scale as the input.

The reconstructed signal obtained from the output points will have a maximum value when the two input signals are exactly aligned, and a minimum value when the two signals are aligned with amplitude values of equal magnitude but opposite polarity. Thus, the mathematical cross correlation produces a maximum value at the point of optimum correlation and a minimum value at the point of the greatest difference.

Continuing the cross correlation process with sine or cosine waves will develop output which reaches a maximum value several times and, in fact, forms a wave having the same frequency as the input signals. Hence, the cross correlation of a single frequency with a like frequency produces output of the same frequency (Fig. 30).

Cross correlation of a single frequency with a signal will extract only that frequency from the signal. For this reason, cross correlation can be a very effective tool to extract a given frequency when it is buried in noise (Fig. 31). If a given frequency is distorted by the addition of a large amount of noise, and the resulting noisy signal is cross correlated with the noise-free frequency, the output will be the original signal minus the noise.

The length of the operator can be important when it is desired to extract some high frequency from a signal which has low frequency content (Fig. 32). A short operator, say equal to one period of the high frequency, will be able to extract that frequency, but the operator will, in effect, ride along the low frequency and this will be reflected in the output. To be sure of eliminating the low frequency, the length of the operator should equal the period of the lowest frequency to be eliminated from the output signal.

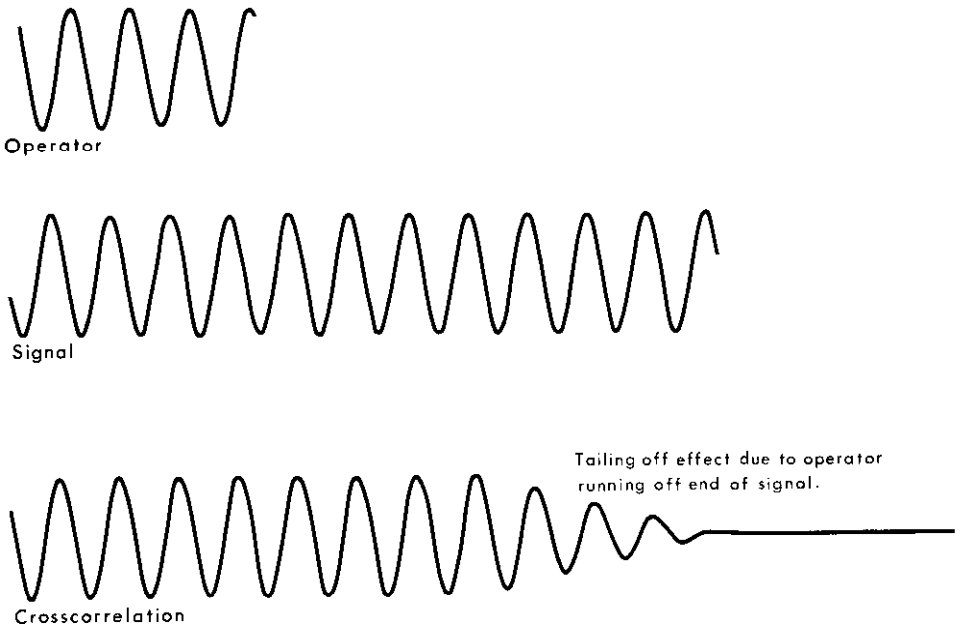


FIG. 30.—Cross correlation of like frequencies produces output of the same frequency.

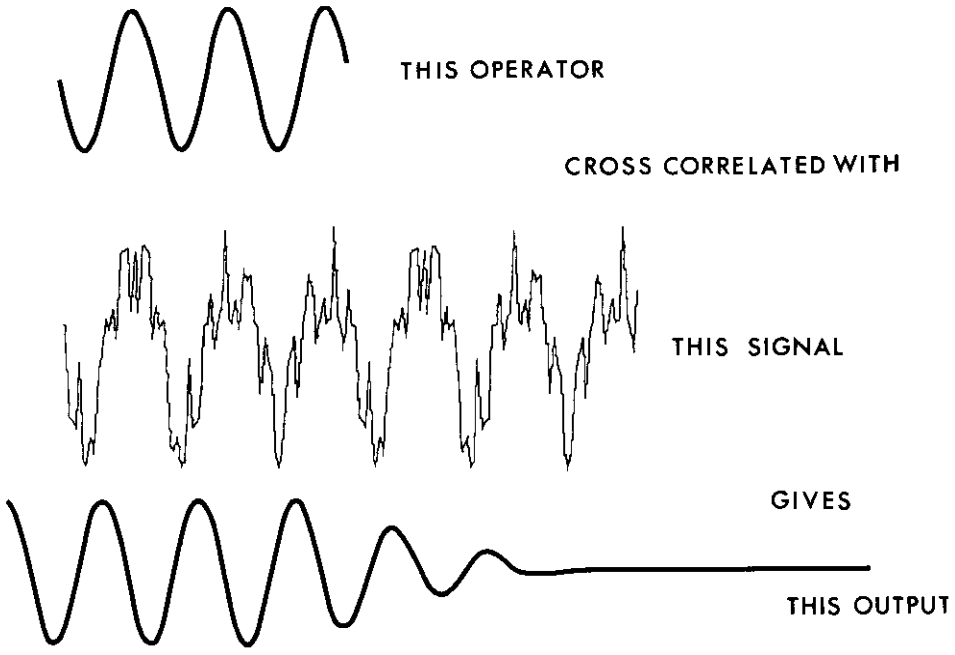


FIG. 31.—Cross correlation is a good method for removing noise.

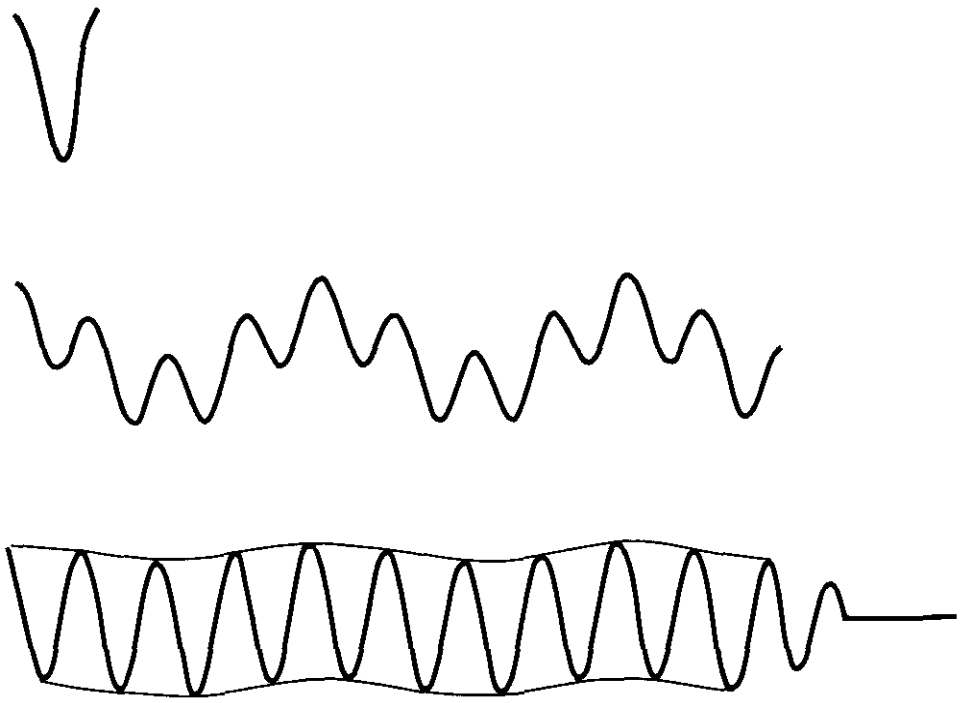


FIG. 32.—The short operator does not remove the long period component.

If the relative amplitude of a single frequency operator is fixed at some constant value, say a maximum of one, the output of any cross correlation will always have an amplitude proportional to the amount of that frequency which is buried in the input signal. Cross correlation may then be used to extract a given frequency from a signal and to determine the amplitude of that frequency within the given signal.

The amount of each of the frequency components present in a signal may be determined by cross correlation of the signal with an operator for each frequency selected.

Simultaneous extraction of a number of frequencies can be simplified by summing in phase the frequency components which are to be examined. In this manner, when it is desired to extract a band of frequencies, e.g., a band pass filter, all of the frequency components within that band may be added together in phase to produce a single operator. This operator may then be cross correlated with a signal to extract all frequencies within the range of the operator. This is the basis of band pass filtering.

When two complex signals are cross correlated, the resulting output will contain the frequencies which are common to both signals. The

output signal will reach a point of maximum amplitude when the greatest number of frequencies common to the two signals are in phase. In this manner, cross correlation will indicate the best match between two signals, even though both signals may be buried in noise to an extent that the match is not readily visible to the eye (Fig. 33).

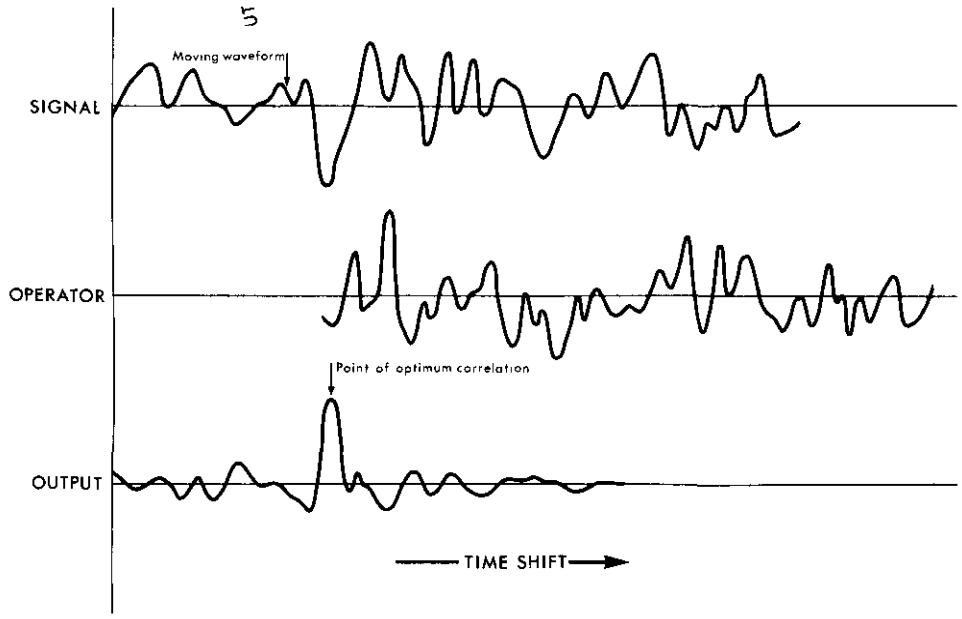


FIG. 33.—Crosscorrelation of two noisy signals.

Cross correlation between two identical frequencies will always produce that same frequency. However, the phase of the output may not be that of the input signal nor the operator. The phase of the output will represent the difference between the phase of the signal and the phase of the operator. Thus, if an operator at 90° is cross correlated with a signal of 180°, the output will have the phase of 180°—90°=90° (Fig. 34). For practical applications, it is customary to use a zero phase frequency operator so that the output will have the same phase as found in the signal.

Cross correlation works very well when the signals to be correlated have a very distinct character, or identity. This property is used to advantage in the Vibroseis System, registered by Continental Oil Company. The source of energy is a sweep signal generator. This generates a signal which changes frequency with time. The frequency content of the input signal may be adjusted to the normal frequency responses of the area. The signal is transmitted into the earth and reflected from successive reflectors. The resulting recording will contain a number

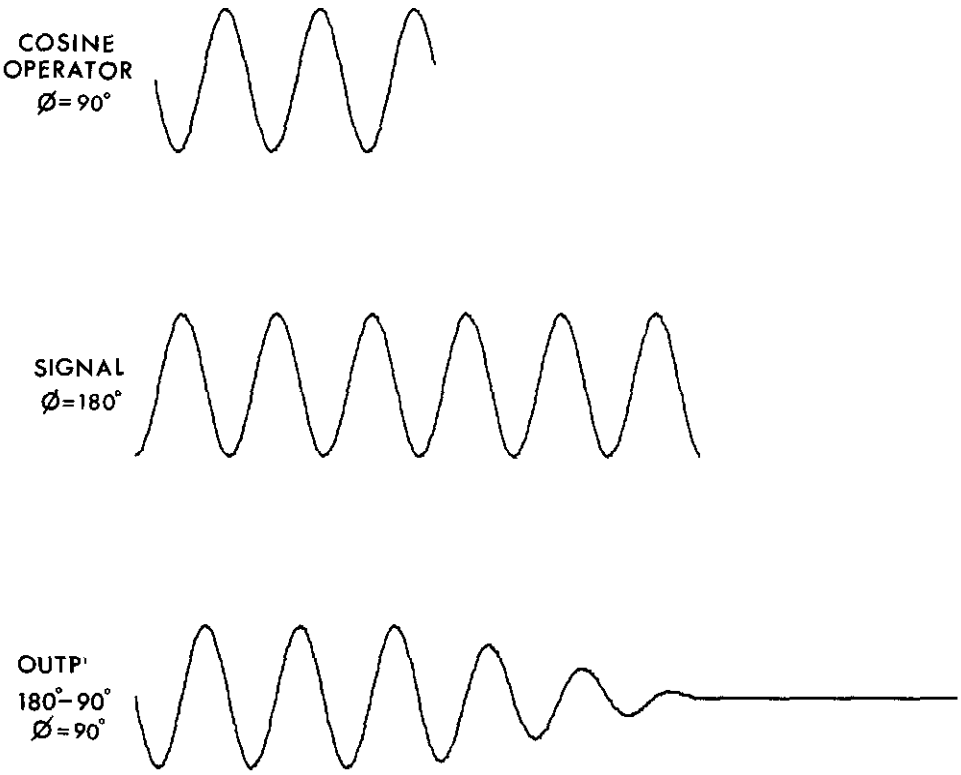


FIG. 34.—Cross correlation produces the difference in phase between operator and signal.

of reflected sweep signals. When the data are to be analyzed, the recorded signal is cross-correlated with the original input signal. The output, which appears very similar to a seismogram, is actually a cross-correlogram. Because of the unique character of the signal, a good correlation will be obtained only when the recorded reflected signal and the operator are aligned exactly. The output will contain events proportional to the strength of each reflected signal each time a correlation is obtained. The theoretical aspects of this method are very good.

The results of cross-correlation may not be so good where the operator is not distinct, particularly narrow band signals. This is the chief reason why cross-correlation techniques do not always work well on seismograms. The output may be somewhat sinusoidal, with several points of good correlation. One solution to this dilemma is to use a longer operator for cross-correlation. This, indeed, does work better, provided that the section is relatively free of multiples and a fairly uniform interval is maintained over the length of the cross-correlation. Should there be multiples present, it is quite possible that the correlation will be heavily weighted by the multiples.

Cross correlation techniques do not work well where the section changes substantially in thickness between points of correlation.

AUTO CORRELATION

The initial example of cross-correlation compared two signals of the same frequency and amplitude. Identical results could have been obtained by correlating a portion of the signal with itself. Correlation of a signal with itself is termed 'autocorrelation.' For any single frequency the results of autocorrelation will be exactly the same as cross-correlating two waves of identical frequency, phase, and amplitude.

The output of autocorrelation is a wave form symmetrical about the center point (Fig. 35). In practice, since the output is symmetrical, only one half is computed for normal applications. The process is always initiated with the operator in point for point contiguous relation with the signal.

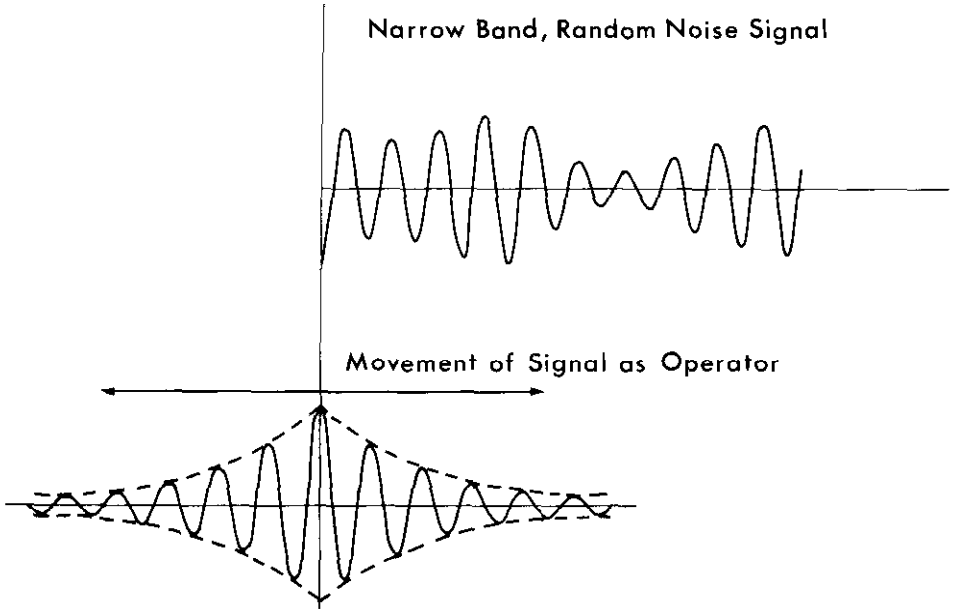


FIG. 35.—A complete autocorrelation function is obtained by sliding the signal past itself and cross correlating.

Regardless of the complexity of the signal to be autocorrelated the optimum correlation, and hence the maximum output value, will always be obtained at the initial point, for then the operator and the duplicate signal are perfectly aligned in all respects of amplitude, frequency, and phase. It is customary to divide all of the output by the initial value to normalize the output to a range within bounds of +1 to —1.

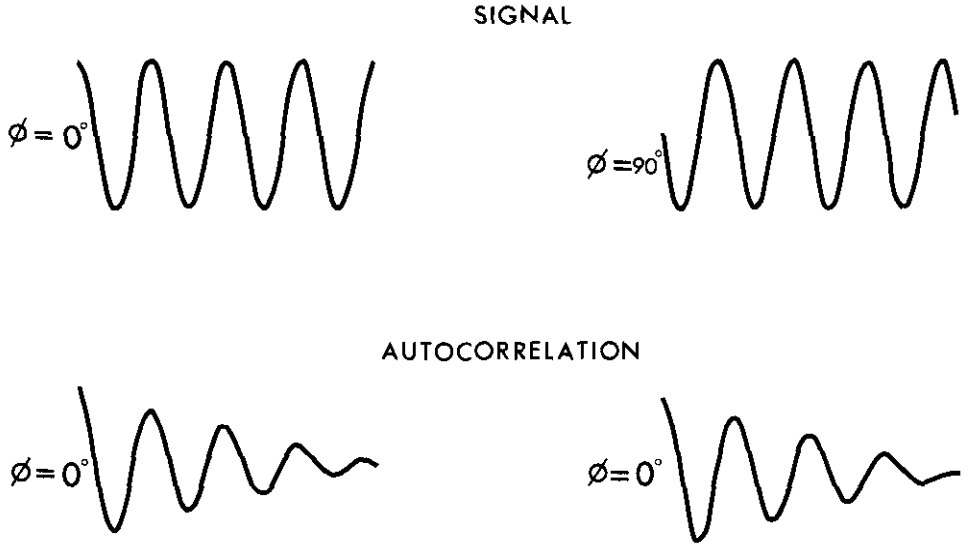


FIG. 36.—Phase information is lost in autocorrelation.

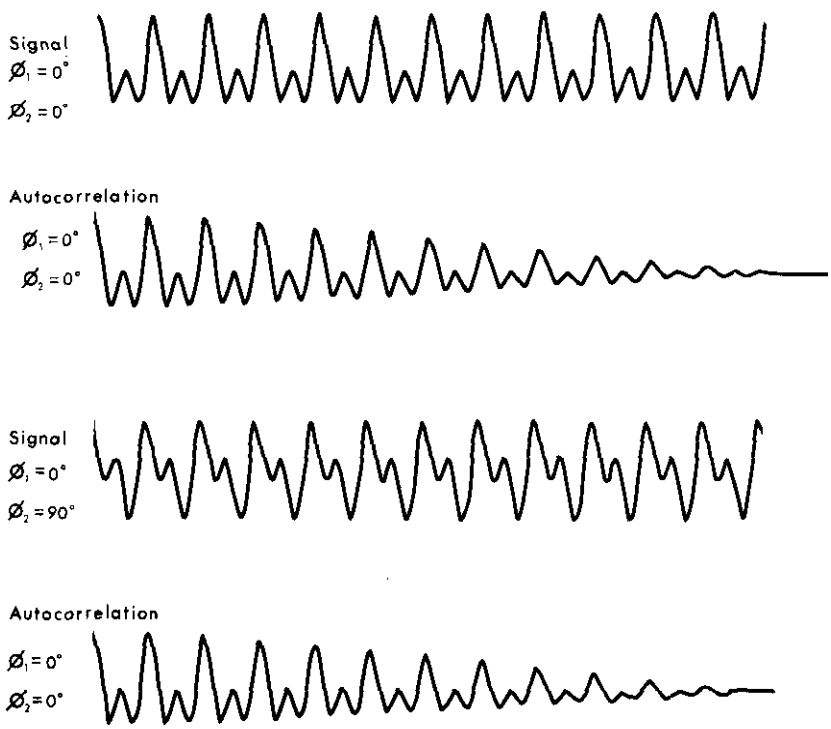


FIG. 37.—The output of autocorrelation is independent of the phase of the signal components.

The cross correlation process extracts the difference in phase between the operator and the signal. Since autocorrelation is merely cross-correlation of a signal with itself, the phase of the operator and the signal is always the same. Therefore, the difference in phase will always be zero, and the output will always be zero phase. Autocorrelation of a single frequency, illustrated in Fig. 36, produces exactly the same output regardless of the phase of the signal. If two frequencies are combined, in phase, the autocorrelation will be exactly equal to the input. However, if one of the two signals is shifted slightly in phase and the autocorrelation taken (Fig. 37) the output will again be equal to the sum of the two signals in phase. Furthermore, it is true that any combination of phase of a given set of frequencies and amplitudes will always provide exactly the same autocorrelation.

Any autocorrelation function may be duplicated by summing at zero phase a series of cosine waves each having the same frequency and amplitude as the components of the autocorrelated signal. This is similar to the sum of frequencies used to illustrate the approximation of the shot pulse (Fig. 22). The autocorrelation of an extremely broad range of frequencies, such as broad band random noise, will be a close approximation to a spike (Fig. 38). The frequency components of a seismic recording are restricted to the band limited by the recording

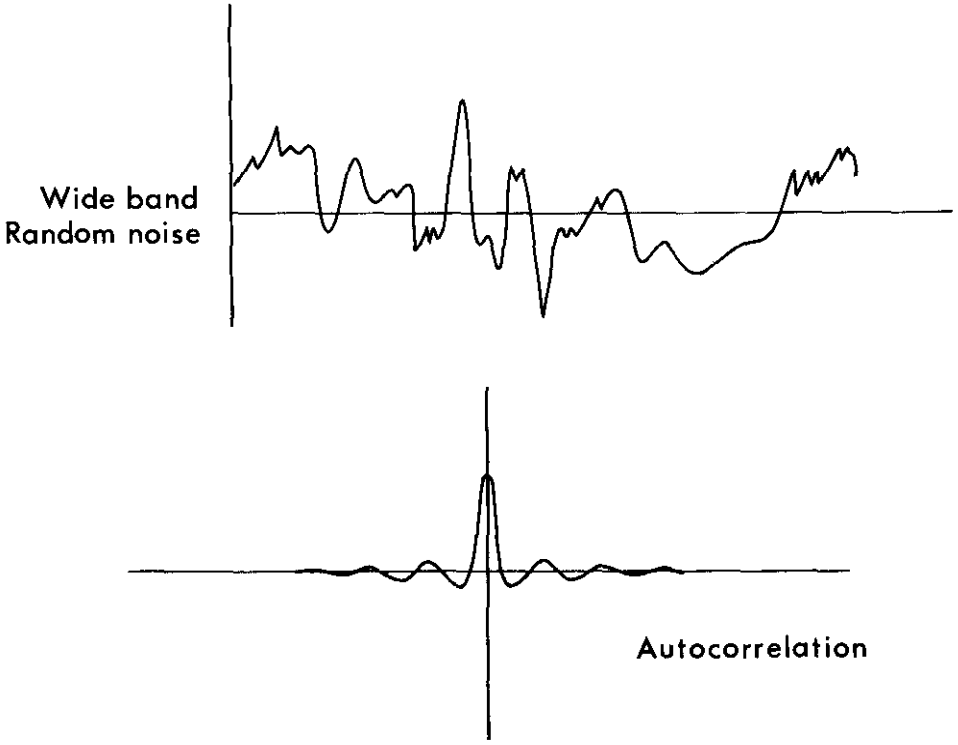


FIG. 38.—The autocorrelation of broad band noise approximates a spike.

band pass filters. The autocorrelation of the random noise components of the seismic signal will be the best approximation to a spike that can be obtained with that limited range of frequencies (Fig. 36).

The reflections on a seismic record are assumed to be a series of unit impulse responses to the reflection coefficient unit impulse. The response is considered to be the same for all impulses, except for amplitude and polarity. The autocorrelation of a noise-free seismic trace will be equal to the sum of all the components of the several unit impulse responses added together at zero phase. Each of the unit impulse responses will contain the same frequency components at a constant ratio. The sum, e.g., the autocorrelation, of all the responses will contain the total amplitude of all the individual responses.

The autocorrelation thus provides a method for consolidating all of the amplitude or energy of all frequency components of the unit impulse response into a short single wave form. The process converts all phase components to zero. Thus, frequency analysis, using the autocorrelation as a tool, maintains the frequency and amplitude relation but loses all of the phase information.

Autocorrelation of a seismic signal containing only reflections and random noise would equal the sum of the frequency components which make up the unit impulse response, plus the noise spike approximation added to the initial values of the autocorrelation (Fig. 39). The relative amplitude of the noise spike and the impulse response will be in relation to the relative amplitude of the two types of signals on the seismic trace. The effect of white noise is to add a uniform increment to each frequency over the recording range, including addition to those frequencies which make up the unit impulse response. This constant can be subtracted out, leaving the components which make up the unit impulse response.

The autocorrelation function is a useful tool for determining whether or not multiples may be present in the section. If regular multiples of any given reflecting band are repeated through the section, each time the reflecting band is shifted to the point where it lines up with the multiple of that reflection, a point of maximum correlation will be obtained.

In this case, the multiple events themselves are periodic and this information is thus extracted by the auto-correlogram.

Multiple reflections are inverted for all odd-numbered multiples and this holds true for the detection of the period of multiples on the autocorrelation function. The first, third, and successive odd-numbered multiple correlations will be of reversed polarity (Fig. 40).

Most multiple events are not single, sharp, repeated reflections, but rather, are broad bands of energy. This can be demonstrated by noting the envelope of the autocorrelation function. This will build up as a low frequency wave having the same period as that of the multiple.

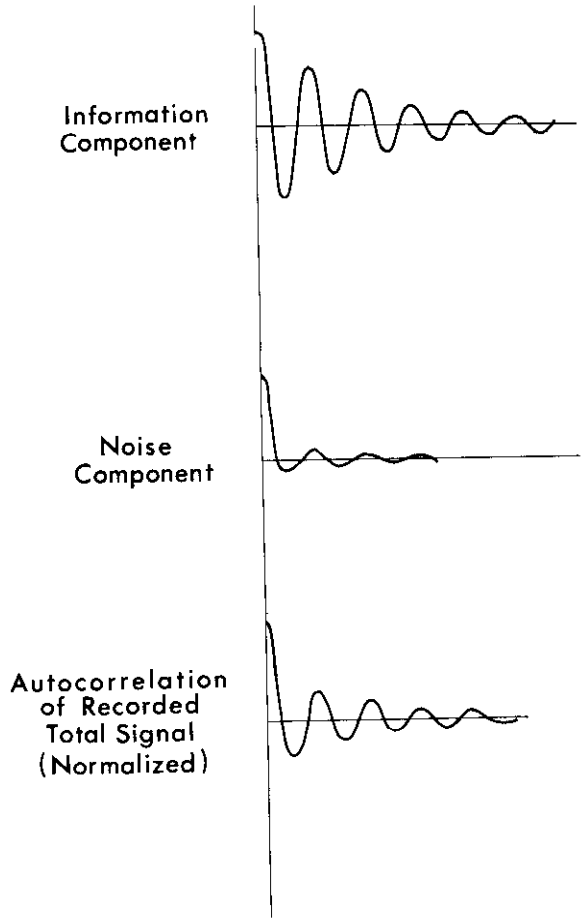


FIG. 39.—White noise added to a signal alters the relative height of the central lobe.

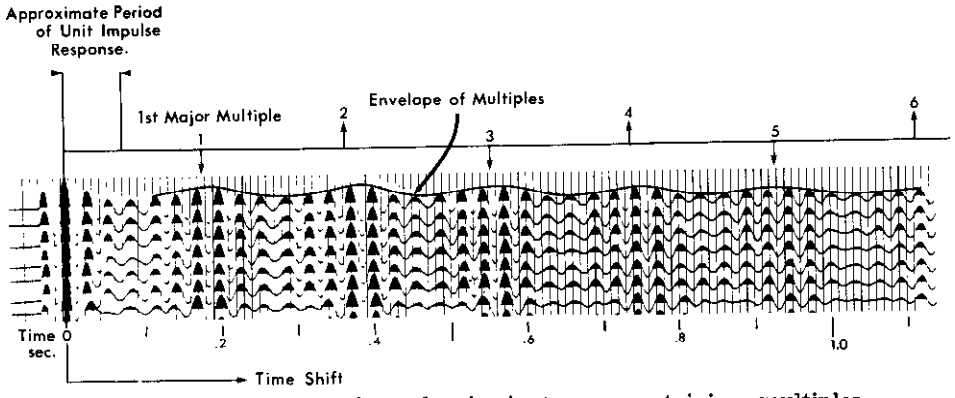


FIG. 40.—Autocorrelation of seismic traces containing multiples.

Although the autocorrelation function will indicate whether or not multiples are present, it will not indicate where they are, nor what kind of multiples they are. It is not possible to distinguish between an interbed multiple and one coming from the surface.

The parameters used for autocorrelation will control the range of periods of oscillation that can be examined. Multiples having a period greater than the maximum length of shift of the operator will not be detected. At the other end, the maximum frequency that can be determined will be the Nyquist Frequency.

Comparison of the autocorrelation of a signal before and after deconvolution will indicate to what extent the periodic components have been removed, but will not indicate how much, if any, of the random noise component has been removed.

In summary, the autocorrelation function of a seismic trace may be viewed as a consolidation of the total frequency content, including the unit impulse response to reflections and random impulse noise, the random noise components, and the periodic components such as reverberations and/or multiples. The random noise will be represented by increased amplitude on the first point or points of the auto correlation function and will be essentially a constant. The long-period multiples, say longer than 100 milliseconds, will be separated in time on the autocorrelation function from the impulse response components, and often may be separated out. Short period reverberations may merge with the unit impulse response components and may not readily be recognizable.

POWER SPECTRUM ESTIMATION

The autocorrelation function leads to a means of measuring the relative proportion of frequency components present in the signal. The autocorrelation function is a periodigram of amplitude versus time. All of the frequency components are centered at zero phase at time zero of the autocorrelation function.

Every frequency has its own unique period of oscillation and may be represented by a cosine wave. The relative amount of each frequency present in an autocorrelation function can be measured by a form of cross-correlation in the following manner.

Cosine Transform

For each frequency component to be tested, the corresponding cosine wave is developed and cross-multiplied with the autocorrelation. But instead of truly cross-correlating, no shifting is done. The autocorrelation process has converted the phase components of the signal to 0°, so it is necessary to merely take the single cross product with the cosine function of a given frequency at zero phase. The result is plotted on a chart of amplitude versus frequency, the next frequency is cross-multiplied and this process is repeated over the entire range of frequencies to be tested.

The values obtained from this operation are all positive because the operator and the signal are always in phase. The procedure has the effect of squaring or taking the first moment of the frequency component. The output of this operation is generally termed the power density estimate, or the power spectrum. The result (Fig. 18) will be a plot of relative amplitude versus frequency over the range of frequencies examined. A sharp high peak will indicate a predominance of the associated frequency. This is the most common procedure used in spectrum analysis to obtain the Fourier transform of the signal.

The transform is an estimate or approximation because a continuous range is sampled with finite sample spacing, and the range of frequencies which is examined is limited at both ends. Nothing is known about the presence of intermediate frequencies, nor any which are outside of the range.

Earlier, a basic assumption stated that any seismic signal can be approximated by summing the proper selection of frequency components. Practical work is limited to a very definite, limited set of frequencies. Hence, rarely is it possible to completely analyze any signal but a good approximation can be made.

The fact that the power spectrum is limited to an approximation means that the answer may not be exact. Some erratic values may appear in the output. For this reason, it is customary to smooth the power spectrum by applying a running average to the output. A number of smoothing methods exist; these differ primarily only in the relative values of the weights. Hamming, Hanning, and Daniell smoothing schemes are common methods named after the scientists who designed them. One of these, the Hamming method, smooths by adding approximately 23% of the values on either side of a given point to 54% of the value at the point.

The Fourier Transform is reversible. Hence, the autocorrelation may be reconstructed by summing together frequency components at the proper amplitude indicated on the power density estimate. But only the autocorrelation can be reconstructed from the power spectrum. Phase information would be required to reconstruct the original signal, and this has been lost.

The power spectrum estimate obtained from the autocorrelation function is a useful tool for determining the relative proportion of frequency components present in a given signal. In many cases, it is all the information that is required to effectively process the signal. It does not provide any information regarding the phase relationships which must be obtained for more exhaustive types of analysis and processing. The autocorelation method sacrifices knowledge of the phase for ease of computation and speed. The full transform must be derived directly from the original signal rather than from the consolidated form provided by the autocorrelation.

SINE-COSINE TRANSFORM

There are a number of computational procedures which develop both the amplitude and the phase spectra of the signal. The principle of operation may be illustrated in the following manner.

Cross correlation of a unique frequency with a signal extracts that frequency from the signal at the difference in phase between the operator and the signal component. To obtain the full transform, a seismic signal may be cross correlated by a series of frequency components, one operator for each of the frequencies to be measured. The frequency operator is a zero phase cosine wave. The output of each will be that frequency at the difference between zero phase and the phase of the frequency in the signal, e.g., the phase of the signal component. The amplitude output of each operator will be proportional in amplitude to the component.

Two output values are thus obtained for each cross correlation operation; the power and the phase. These may be plotted on separate charts, using the same frequency scale, but with the vertical scale indicating power for one (Fig. 18) and phase for the other (Fig. 19).

This is the full transform of the signal. It may be reversed to reconstruct the signal. The reconstruction is done by adding together a series of cosine waves, one for each frequency contained in the analysis. Each frequency component is summed together at the phase indicated on the phase spectrum, and at the relative amplitude indicated by the amplitude spectrum. The sum of these components will reconstruct the original signal within the limits of the conditions imposed by the parameters of the analysis (Fig. 14).

In actual practice, the Fourier Transform is computed as a series of complex number pairs, the real parts of which represent the products of the amplitude with the sine and cosine of each phase component. These may be expressed as a series of X and Y values. The amplitude is equal to $\sqrt{X^2 + Y^2}$. The tangent of the phase angle may be computed by X/Y. If the values of X and Y are plotted as coordinate points in space along an axis representing frequency, the transform may be visualized in three dimensions as a continuous curve winding about a central axis (Fig. 41). This three dimensional concept is extremely useful for certain types of analysis of a signal.

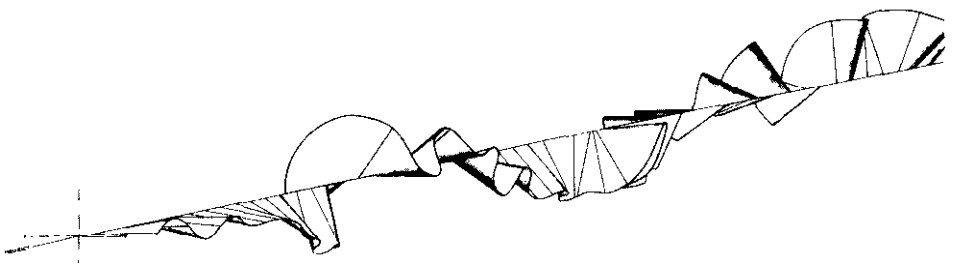


FIG. 41.—Spatial representation of Fourier transform.

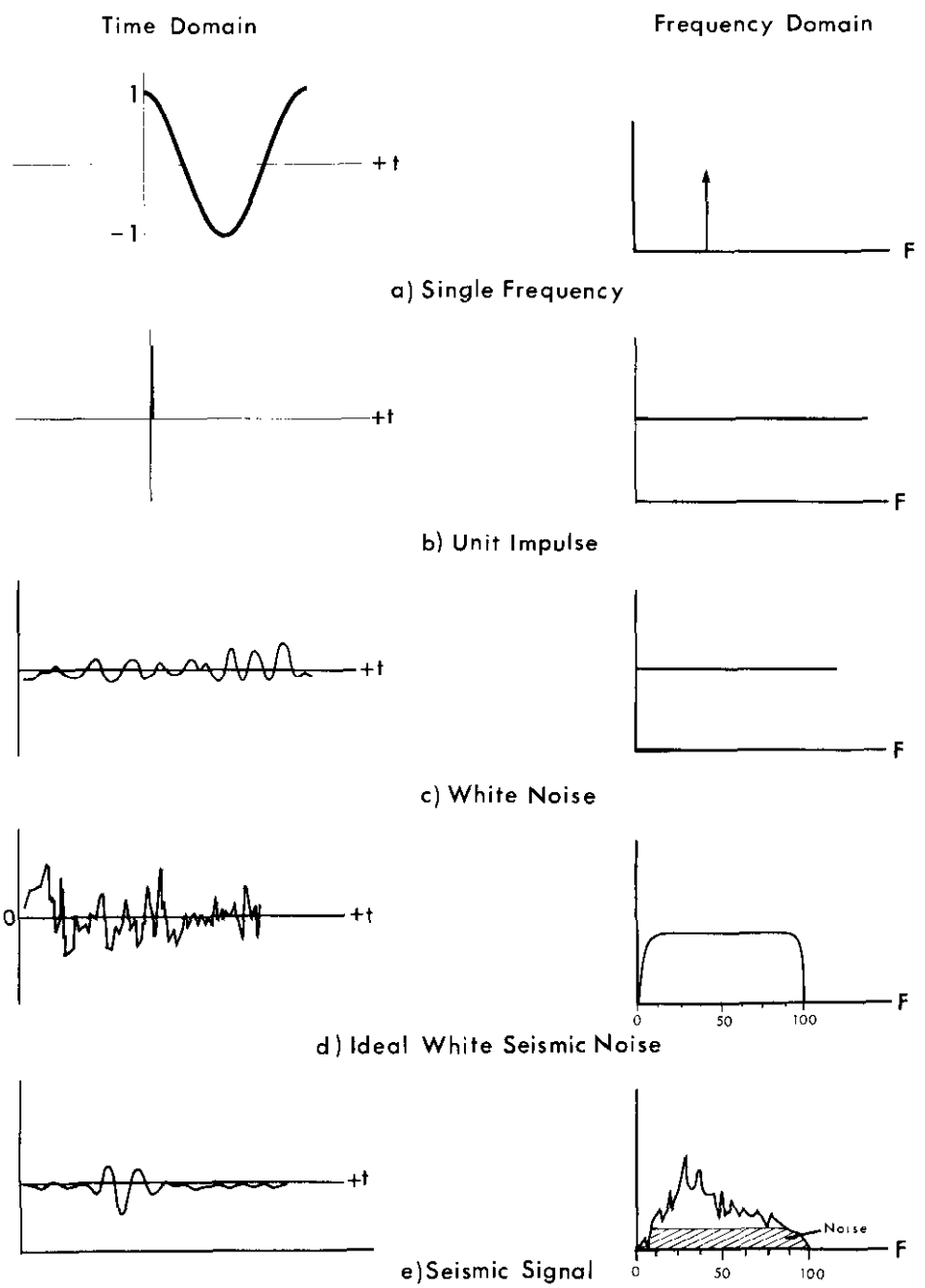


FIG. 42.—Representative transforms.

The power spectra of a few special types of signals are fairly easy to visualize directly. For instance, the transform of a single frequency will have a single output value plotted for that particular frequency, while all other values will be zero (Fig. 42a). Hence, a single frequency in the time domain may be visualized as a single point or spike in the frequency domain. Only the positive side is graphed in the illustrations.

A unit impulse is the zero phase sum of all frequencies at equal amplitudes. The transform of the spike then will consist of a uniform value for each of the frequencies examined. A single spike in the time domain is a straight line in the frequency domain (Fig. 42b).

Broad band random noise likewise contains equal amounts of all frequencies, so the transform of broad band or white noise is also a straight line in the frequency domain (Fig. 42c). Random noise and a unit impulse have the same power spectrum. They differ in the phase spectrum.

A seismic recording system is assumed to have uniform response over the normal range of seismic frequencies, say 10 to 100 cycles. Random seismic noise will be limited in frequency to the range of the instrument response. Hence, the power spectrum of ideal white seismic noise will be flat across the top, but cut off at both ends at the frequency response limits (Fig. 42d). A blocked spectrum of this sort is generally referred to as a 'box car.'

The spectrum of a seismic signal has an irregular shape which may be coloured by specific noise components, such as 60 cycle high line pickup. The irregular spectrum will be built up on the box car base of the random noise (Fig. 42e). The autocorrelation function contains random noise as a spike added to the initial point of the autocorrelation, whereas the power spectrum contains the noise as a flat base for the signal.

All computed power spectra are limited in the range of frequencies they can examine, and in the frequency increment within the range. In particular, the lowest frequency which can be satisfactorily represented on the power spectrum is the frequency having a period equal to the length of the cosine operator used to develop the transform. If the transform is taken from the autocorrelation, this will be equal to the length of the autocorrelation lag or shift. Very low frequency information is missing from an autocorrelation unless the lag has been made very long. If frequency content in the range of 8 to 10 cycles is to be measured, the autocorrelation must be lagged at least 100 to 125 milliseconds. Examination of a power spectrum should include checking the restrictions of the frequency content.

A single Fourier Transform may be used to adequately define any seismic trace, provided the sample increment is small enough and sufficient terms are used to develop the required approximation. The transform is a series and may be compared to using a polynomial to define a curve. If sufficient terms are used in the polynomial, almost any reasonable curve can be defined with a great deal of precision.

However, it is well known that the frequency content of a seismic trace tends to change somewhat with depth, and a Fourier Transform taken of a localized portion of the trace may indicate a quite different frequency content than would be indicated for the whole trace. In effect, a transform of the whole trace is an averaging process of all the frequency components over the length of the trace. It may be advantageous to analyze and to process the trace in sections. This procedure is discussed in greater detail in the section on time variant filters.

A true time variant or dynamic power spectrum analysis may be computed for the trace. The result of this process is a contour map of power levels plotted on a grid of frequency versus time. A section across the map at any time point will provide the power spectrum for that particular moment. The computation of the dynamic analysis is extremely long and tedious. Examination of short segments of the trace will necessarily be more limited in the number of frequency components which may be examined, particularly the low frequencies.

FILTERING OPERATIONS

Any operation which uses a moving operator in a multiplication summing process may be considered a filtering operation. A filter operator is merely a series of weights which are applied to an observed value and a number of adjacent values, in order to derive a new value for that point. A very simple filter which is often used to smooth out random variations in input data is a smoothing filter, or a running average. A simple running average operator is merely a series of equal weights equal to the reciprocal of the number of points to be used in the average. For instance, a 3-point running average would have an operator consisting of values of 1/3, 1/3, 1/3. When this operator is applied, the value of the observed point is modified by what is about to happen (the following value), and by what has already happened (the previous value) (Fig. 43b).

If the adjoining data were considered to have less than equal influence on the present point, then the operator values would be weighted so that the nearer points had more influence in determining the present value than those farther away. Such an operator might have coefficients of .1, .2, .4, .2, .1. The weights would then be a linear function of the distance away from the central observation (Fig. 43c).

The basic filtering techniques used for the digital processing of seismic records generally fall into two classes, band pass filtering and deconvolution—or inverse filtering. Band pass filtering attempts to eliminate certain frequencies and pass the others unaltered, while deconvolution filtering attempts to 'divide' the output function by the transfer function to obtain the input function of the total seismic system. In other words, deconvolution attempts to return the seismic signal to the series of impulse spikes corresponding to the reflection coefficients of the section.

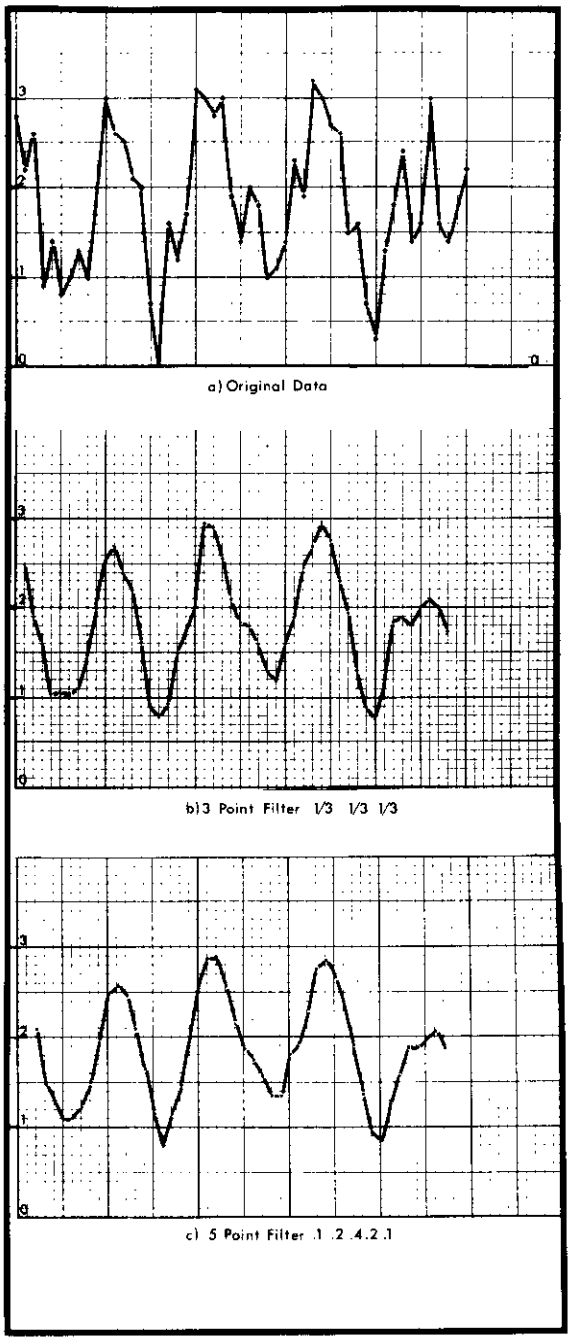


FIG. 43.—Effect of smoothing operators.

BAND PASS FILTERING

Band Pass filters can be designed to pass a band of frequencies ranging from the all pass filter which passes all frequencies, to a filter which will pass only one frequency and eliminate all others. Closely related to the latter is the Notch filter, which is designed to eliminate one specific frequency and pass all others.

Band Pass filters are symmetrical operators which cause no phase shift. Basically, they are simple to design, but can become complicated when certain restrictions are imposed, such as the sharpness of cut-off, the width of the band, and the length of the operator. Undesirable side effects may be produced which can nullify, to a large extent, the desired effect of the filter.

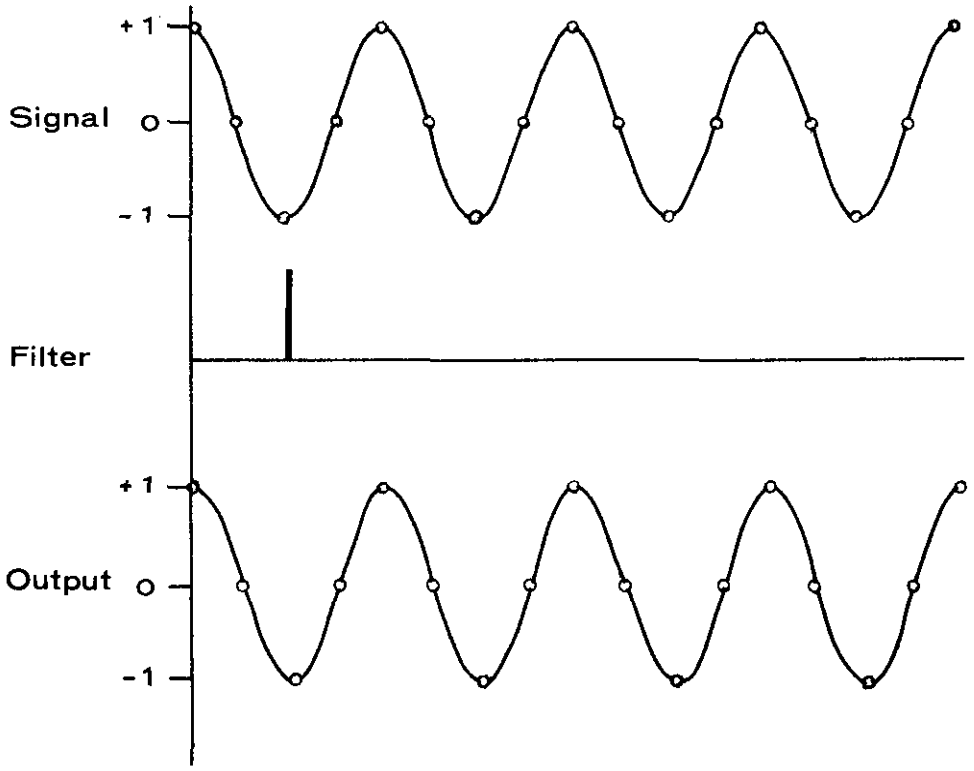


FIG. 44.—Filtering with a spike passes the signal unchanged.

The complete all frequency zero phase band pass filter is nothing more than a single spike with an amplitude of one, e.g., the unit impulse operator (Fig. 44). This operator will pass any signal unchanged. The

filter is truly broad band in that it passes all frequencies with equal ability, confirming the concept that a spike contains all frequencies.

The other extreme of a band pass filter passes only one specific frequency. This could be done with a single zero phase cosine wave operator. This will extract one frequency from a signal and reject all others.

Usually, it is desired to pass or reject a specific band of frequencies. Operators for this purpose can be developed without too much difficulty. Some filter design procedures may be subject to undesirable side effects which may be illustrated by the following extreme case.

For some reason, it is desired to design a filter which will pass a specific frequency but eliminate the first harmonic of that frequency. A simple 5-point operator may be developed with values of .5,0,0,0,—.5 over one half period of the basic frequency. This operator may then be moved past the signal, carrying out the appropriate cross product sum. The normalized output will be identical to the original signal (Fig. 45a). This operator has accomplished the purpose of passing the signal unchanged.

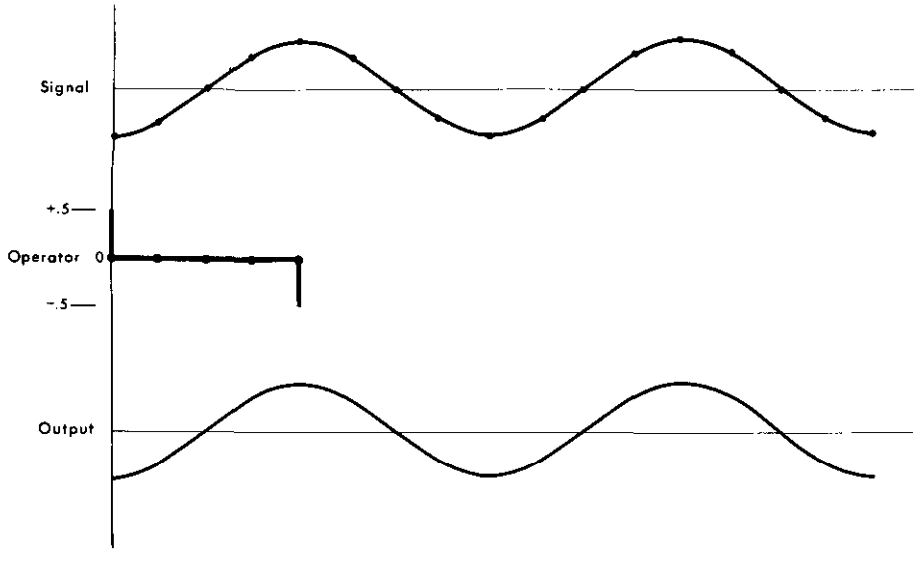


FIG. 45 (a).—The operator passes the signal unchanged.

The some operator may then be applied to the first harmonic of the basic frequency. This time the cross product is zero and the harmonic has been eliminated (Fig. 45b). The operator has apparently succeeded in passing one signal and rejecting the harmonic. However, the second harmonic will also be passed unchanged (Fig. 45c) and should higher frequencies be present, it will be found that successive harmonics are alternately passed and suppressed. This may not be the effect which

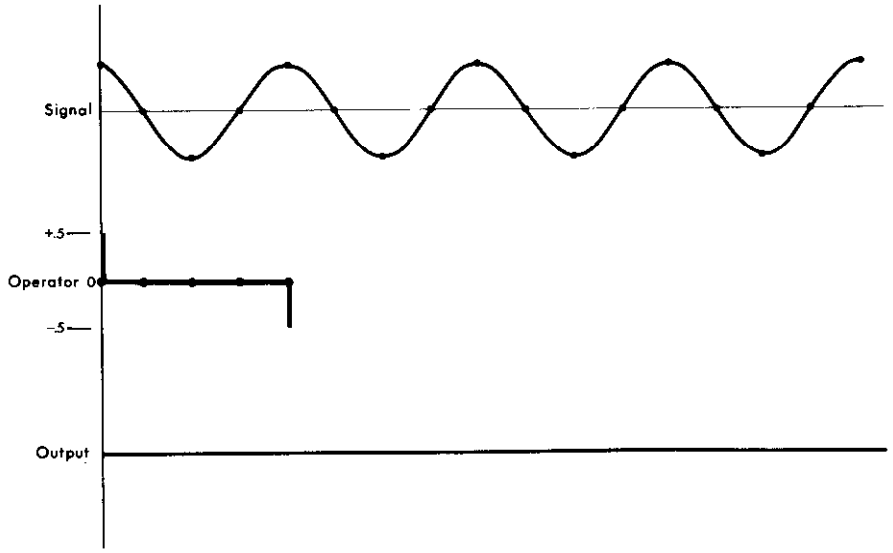


FIG. 45 (b).—The same operator cancels the first harmonic.

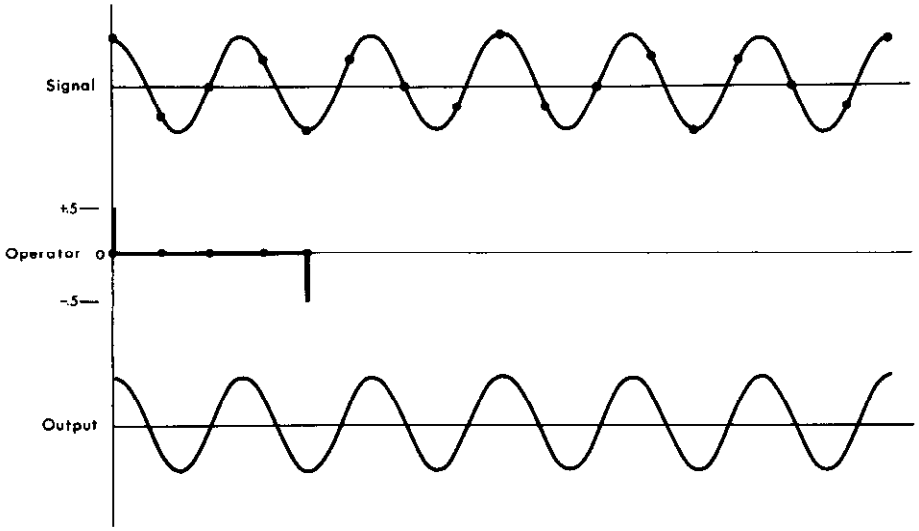


FIG. 45 (c).—The second harmonic is passed unchanged by the operator.

is desired from the original operator. The response of this operator (Fig. 46) indicates several side lobes of pass functions existing outside of the range in which the operator is expected to function. An extreme case such as this is not likely to be encountered in practice, but it illustrates the possibility of developing undesirable side lobes in the design of filters.

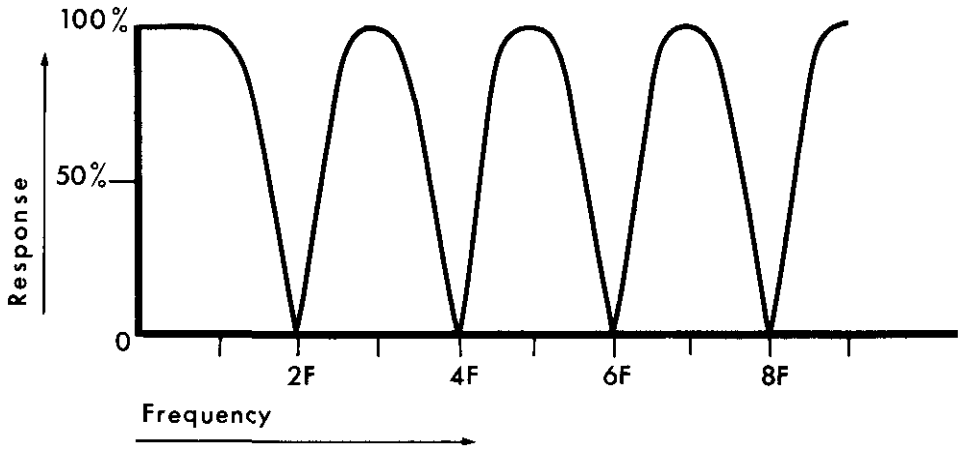


FIG. 46.—The response of the filter operator of Fig. 45 contains undesirable side lobes.

GIBB'S PHENOMENON

A single frequency may be extracted by using a cosine wave of that frequency as an operator. But a short operator will encounter difficulty in discriminating between its specific frequency, say 20 cycles per second and a nearby frequency of, say 20.5 cycles per second. In such case, the power spectrum of the filtered signal would not show a single spike at 20 cycles, but a wedge centered about 20 cycles, indicating that some portion of the nearby frequencies had been passed (Fig. 47). This effect may be minimized by use of a much longer operator which will discriminate between specific frequencies and provide a sharp cutoff. Short operators will have relatively flat slopes on the response curve, while long operators will have a relatively sharp cutoff.

A chart which illustrates the relative frequency response for various operator lengths is illustrated in Fig. 48. It is evident from the figure that increasing the length of the filter is effective in sharpening the slope. As the cutoff slope steepens, the length of the filter must be increased substantially to obtain a small increase in slope. Beyond a certain operator length, extending the length of the filter results in little improvement of the cutoff.

Long operators become very costly in terms of processing time. A further disadvantage arises from the fact that any filter operator must operate on a like number of data points to develop the proper output value. As the operator slides into the signal, and off the end, a length of signal equal to the length of the operator is not fully filtered. A substantial portion of each end of the output will thus be subject to different filtering than the remainder of the trace when a long operator is used.

Another undesirable effect of long operators is a distortion known as Gibb's Phenomenon. This is an oscillation or overshoot in the response

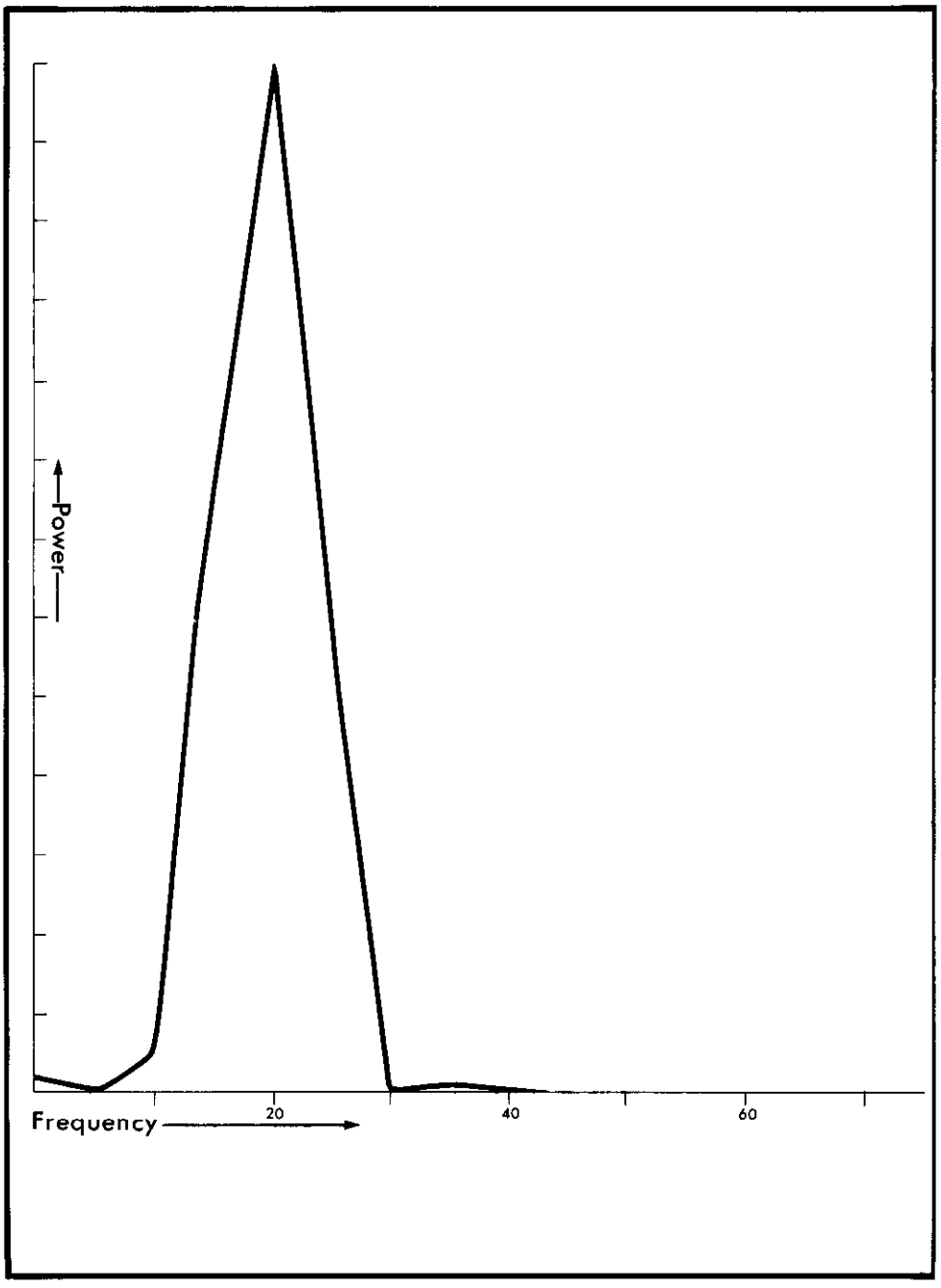


FIG. 47.—Short operators do not provide sharp response.

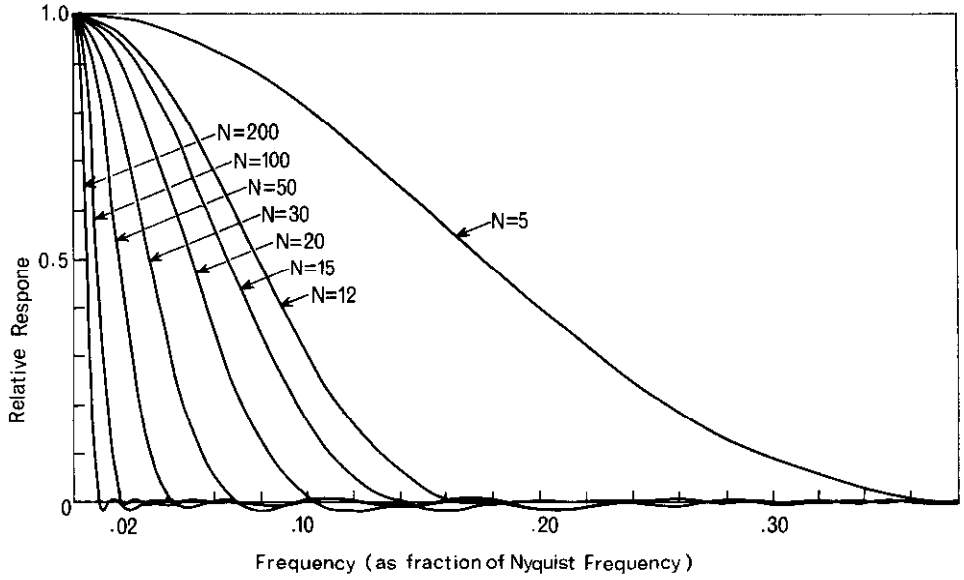


FIG. 48.—Increase in sharpness of a filter due to increased operator length.

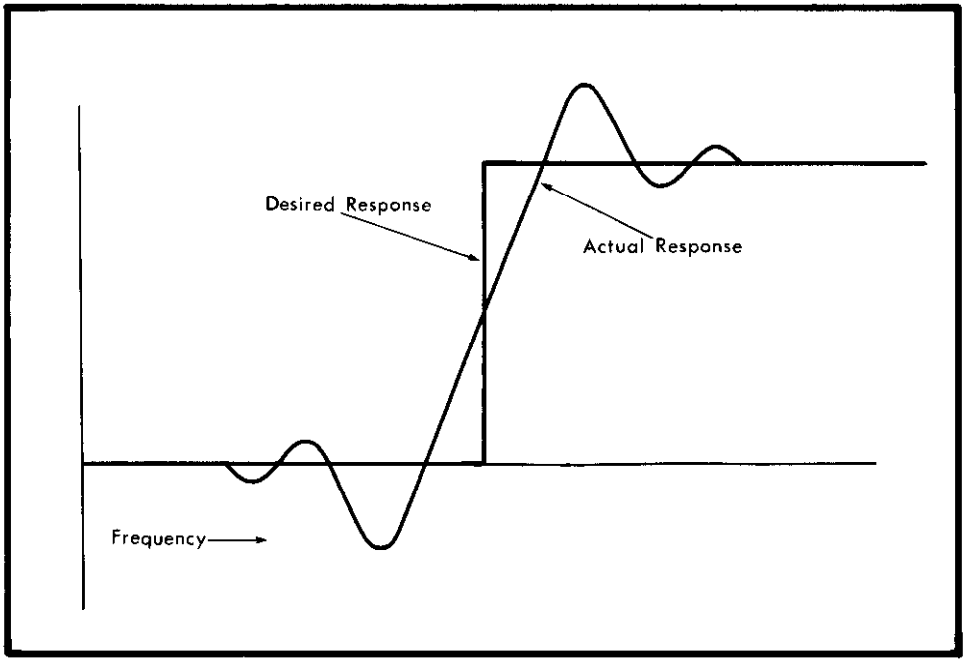


FIG. 49.—Gibb's Phenomenon.

of the filter at frequencies near the cutoff points (Fig. 49). The terminal oscillations have the effect of resonant response and can distort the output. As the sharpness of cutoff increases, the terminal oscillations tend to increase also.

A number of schemes exist to modify the basic filter operator in order to smooth the terminal oscillations and to increase frequency discrimination. One method applies a sine function to the terminal points of the operator. The weighting of the operator smooths the terminal oscil-

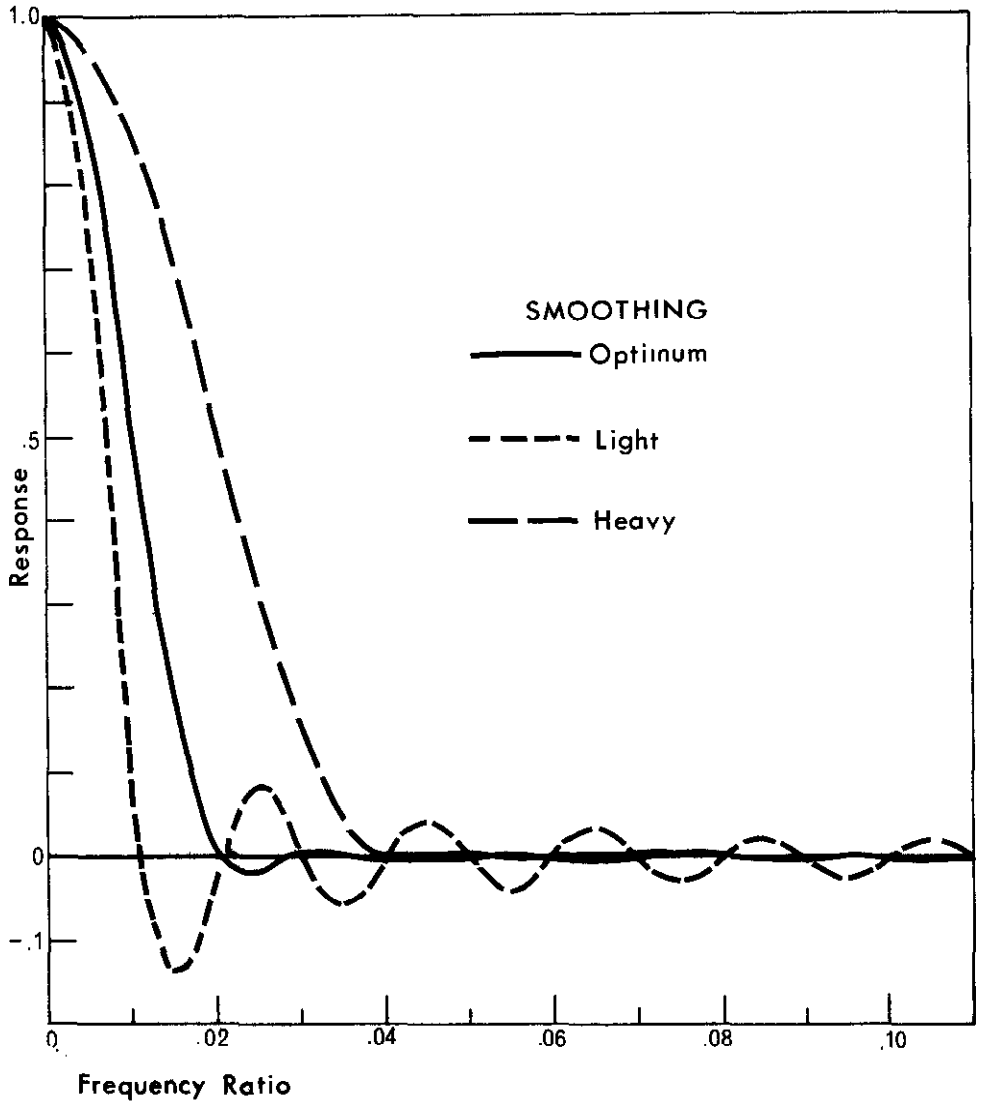


FIG. 50.—Effects of different smoothing techniques.

lations, but is done at the expense of some drop in the rate of cutoff. Discretion must be used in the design of the operator to find the best weight for a given combination of parameters in order to develop the optimum band pass filter operator. Fig. 50 illustrates the effect of optimum smoothing, light smoothing, and heavy smoothing of a typical band pass operator. For a given filter length there is an optimum smoothing weight. A lighter weight for that length will cause larger terminal oscillations, and a heavier weight will cause the slope of the response to be flat (Fig. 51).

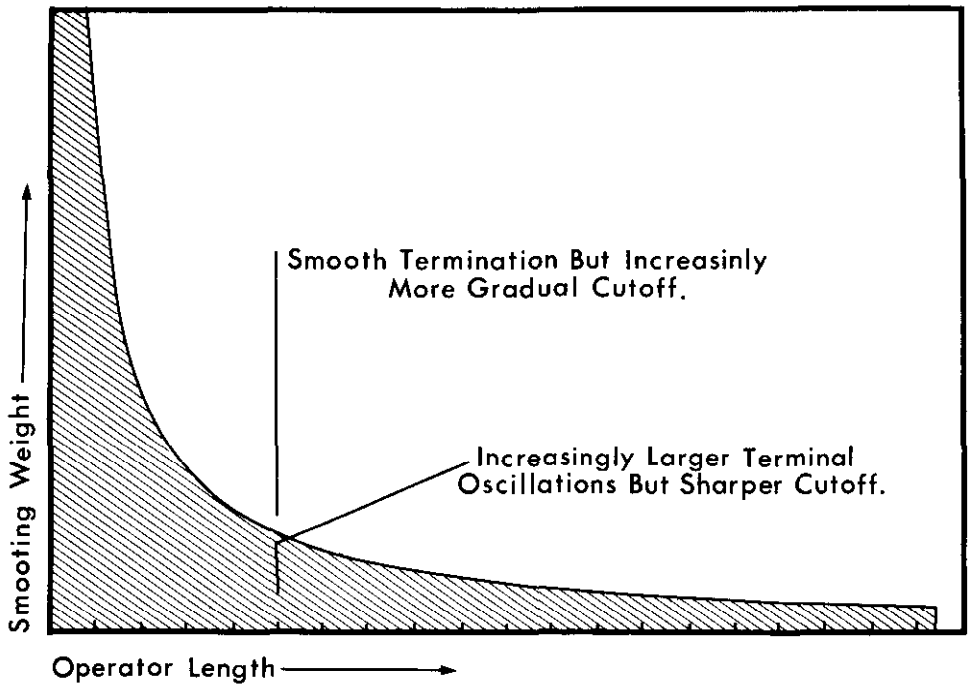


FIG. 51.—Smoothing weight selection.

Some filter design routines have fixed smoothing weights. More sophisticated routines endeavour to adjust the smoothing weight to the operator length to obtain the optimum filter length/weight combination.

MERITS OF DIGITAL FILTER PROCEDURES

Digital analysis procedures are, in many cases, not practical or may not even be possible with analog equipment. The same statement is true for the development and application of deconvolution operators.

Band pass filters are relatively common in analog equipment, and are somewhat easier to achieve. However, there is a substantial advantage in using the digital computer for band pass filtering as well as for the

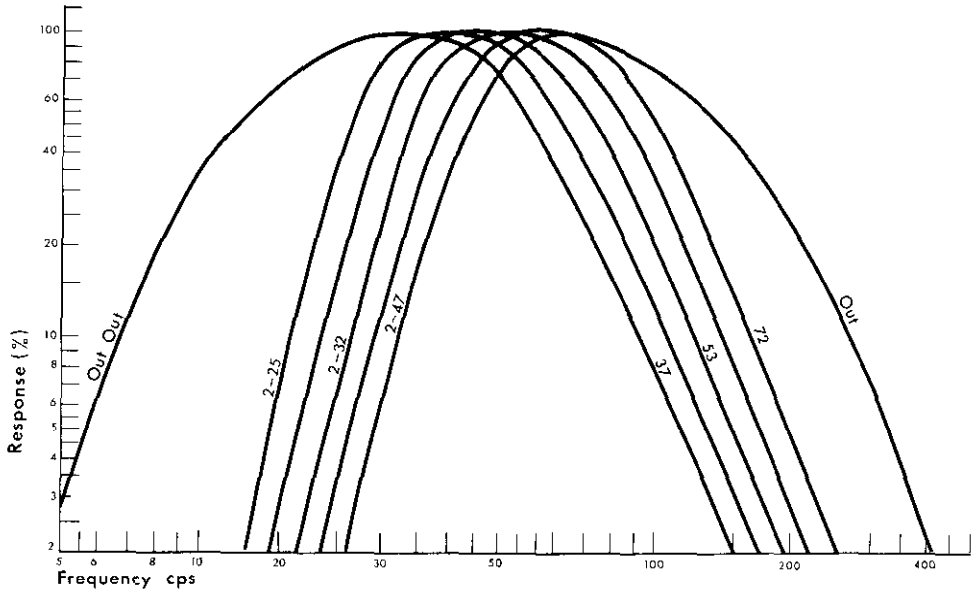


FIG. 52.—Response curves for typical analog seismic filters.

more sophisticated techniques. Figure 52 is a page from a catalogue of filter curves corresponding to a particular set of seismic amplifiers. Switches are used to connect electronic components providing combinations of high cut and low cut analog filters to pass a desired band. The rejection rate of the filter and the filter band width must be one of the catalogue combinations, and no other is permitted.

Digital band pass filters can be designed to meet almost any specification of band width, response, and rejection rate.

Perhaps of greater importance is the fact that digital band pass filters have zero phase shift characteristics. Figure 53 represents the amplitude and phase response for a typical analog seismic amplifier and filter. The amplitude response for this system is good with a relatively sharp rate of rejection and uniform attenuation across the top of the filter. The phase response curve is unfortunately not so good. Substantial phase shifts will be applied to the seismic frequencies. Furthermore, the phase shifts are not linear. A linear phase shift may be equated to a uniform time shift or static, which will not distort the appearance of the trace, but merely shift it in time. A non-linear phase shift will cause varying time shifts or lags to be applied to the frequency components which are passed by the system, resulting in the asymmetrical impulse response indicated on the diagram. The absolute amount of the phase shift can be substantial. For this curve, a phase shift of approximately 180 degrees is present at the twenty cycle frequency. This amounts to a time shift of 25 milliseconds.

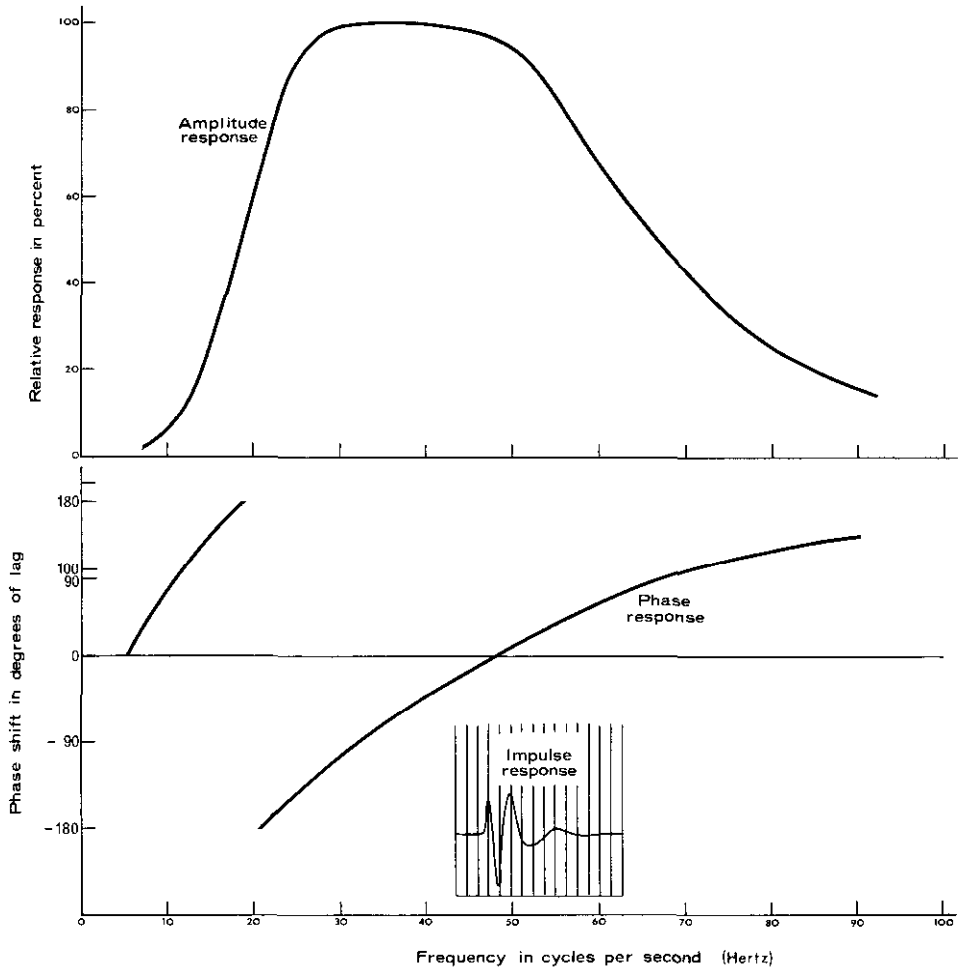


FIG. 53.—Response curves for a typical seismic amplifier and filter.

Occasionally, an attempt is made to economize in processing costs by combining analog band pass filtering with digital deconvolution. If the analog band pass filter is applied before deconvolution, it may cause sufficient phase shifting to make the deconvolution process less effective. Band pass filtering before deconvolution is generally not desirable, but if it must be done, then it is preferable to use a zero phase band pass operator.

The basic design of most digital band pass filters can be represented by summing in phase cosine waves over the band pass range. Specific design techniques modify the operator to obtain specific effects. The following three examples illustrate three typical design techniques, and some of the advantages and disadvantages of each.

Cosine Rejection Filter

The Cosine Rejection Filter is a zero phase cosine operator with terminal weighting. The design parameters allow the user to select four boundary points of the filter, namely the low cut, the low pass, the high pass, and the high cut, expressed as frequencies. These four points describe the approximate response curve of the filter which will be obtained, thus allowing the user to specify any desired rate of rejection slope. The operator length is the fifth parameter specified in this routine. The filter has the advantage of uniform response, effective weighting, and sharp cutoff capabilities.

Some skill is required on the part of the user to obtain the optimum combination of cutoff and operator length. The smoothing may not be effective under all combinations. In fact, certain combinations of operator length and rejection slope are incompatible and lead to a non-valid design.

Modified Ormsby Filter

The Ormsby Filter is a design which provides effective terminal smoothing. The filter parameters may be specified in the same manner as the Cosine Rejection Filter. It is effective for most combinations, but again, some discretion on the part of the user should be exercised in specifying the combination of slopes and operator length.

The improved smoothing of this routine is offset by a slight attenuation of the higher frequencies, which makes the response of the filter not completely uniform.

Optimum Self-Design Filter

Parameter specification for this routine is somewhat similar to analog filters, in that only the two 100% pass points need be specified. The routine then automatically generates an operator length and weight combination for the given frequency range specified. This has the advantage of providing an effective operator with little skill required on the part of the user.

In the basic form, there is no control of the rejection rate with this routine. The operator length is a function of frequency, and certain band pass specifications may produce operators which are longer or shorter than would normally be desired. For these reasons, the routine often carries an option to specify the operator length. Some control over the sharpness of cutoff may then be obtained by the operator length. When this option is used, the weighting function is adjusted to the length of operator specified.

It should be clear from the above discussion that while digital band pass filtering can be extremely flexible, it requires skill on the part of the interpreter to obtain the optimum results of the operation.

Notch Filters

The Notch Filter (Fig. 54) is the reverse of the band pass filter, in that all frequencies are passed except a specific frequency or narrow

band of frequencies. One example of Notch filter application is to remove 60 cycle interference from data recorded near high voltage transmission lines. Another useful application may be found in marine work where strong reverberations may be developed. However, other specific dereverberation filters may be better adapted for this work.

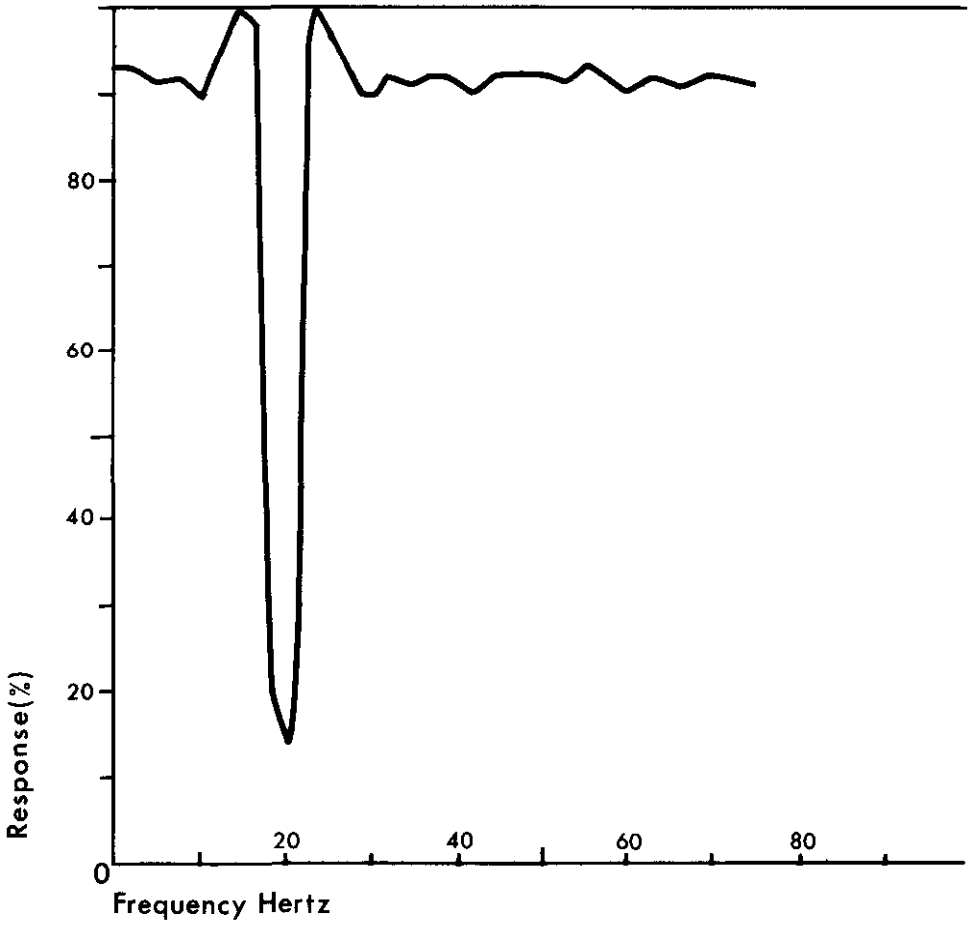


FIG. 54.—Notch filter for band rejection 18-22.

The considerations applicable to band pass filtering are generally valid for Notch filtering. It is wise not to attempt to make the notch very sharp, or distortion may occur at the cutoff frequency. For example, if it is desired to remove 60-cycle high line energy, a notch of 56 to 64 cycles is to be preferred over one of 59 to 61 cycles, unless the smoothing weight and operator length are carefully chosen.

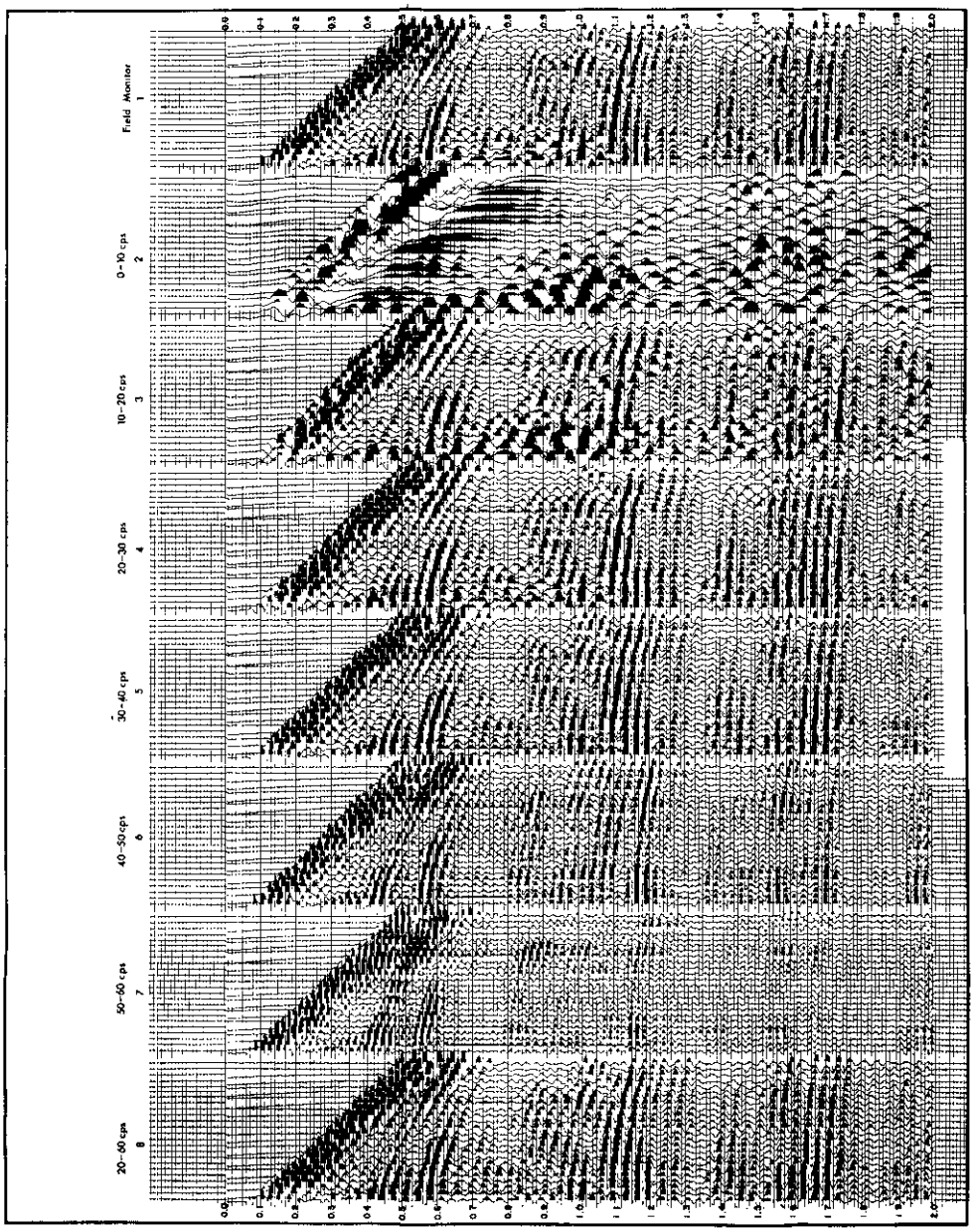


FIG. 55.—Bandpass filter display—time domain (41 pt. operator).

Band Width Analysis

The power spectrum will determine the approximate frequency components, but will not determine whether the energy in a given frequency range is noise or signal.

Such information may sometimes be obtained by filtering the seismic record with a sharp cut band pass filter over uniform frequency increments. This has the effect of passing the recorded signal through a 'prism' to break up the spectrum into primary bands (Fig. 55). The record is filtered in units of, say 10 cycles extending from 0-10, 10-20, 20-30, 30-40, and so on, over the normal frequency range. More precise divisions may be obtained by separating dominant component frequencies from the power spectrum. The results often demonstrate signal and noise separation, and the frequency components of the seismic signal.

CONVOLUTION

Band pass filters are symmetrical zero phase operators. They will produce the identical cross product if they are reversed end for end. This is not the case with other filters where the operator is not symmetrical.

The process of digital filtering is actually done by convolution. Although similar in procedure to cross-correlation, convolution possesses important differences in its mathematical properties.

Cross-correlation consists of sliding an operator past a signal in uniform incremental steps, each equal to the sample spacing. At each step the cross product is computed, and the result becomes one output value. A similar operation is used for convolution, with one important difference.

The difference may be illustrated by cross multiplication of a unit impulse response operator with a unit impulse. A unit impulse response function has been sampled to produce the values illustrated in Fig. 56. The series is to be passed through a system model containing a spike as a transfer function. The output desired is the unit impulse response. A practical application of this procedure would be in the development of a synthetic seismogram. With the operator and the transfer function reduced to two time series, the operation may be carried out as shown in Fig. 57. The values of the operator are successively shifted and cross multiplied with the unit impulses, to obtain a series of output values in sequence. These values may then be plotted. The output is surprisingly not the desired duplicate of the input, but is actually the mirror image. In order to reproduce the operator in its original form, the operator must be reversed, or folded back over itself before it is passed through the system.

The same effect may be obtained by altering the order of multiplication, so that the last point of the operator is multiplied by the first point of the signal, and so on. This is by definition convolution, and is the mathematical operation used for all digital filtering processes.

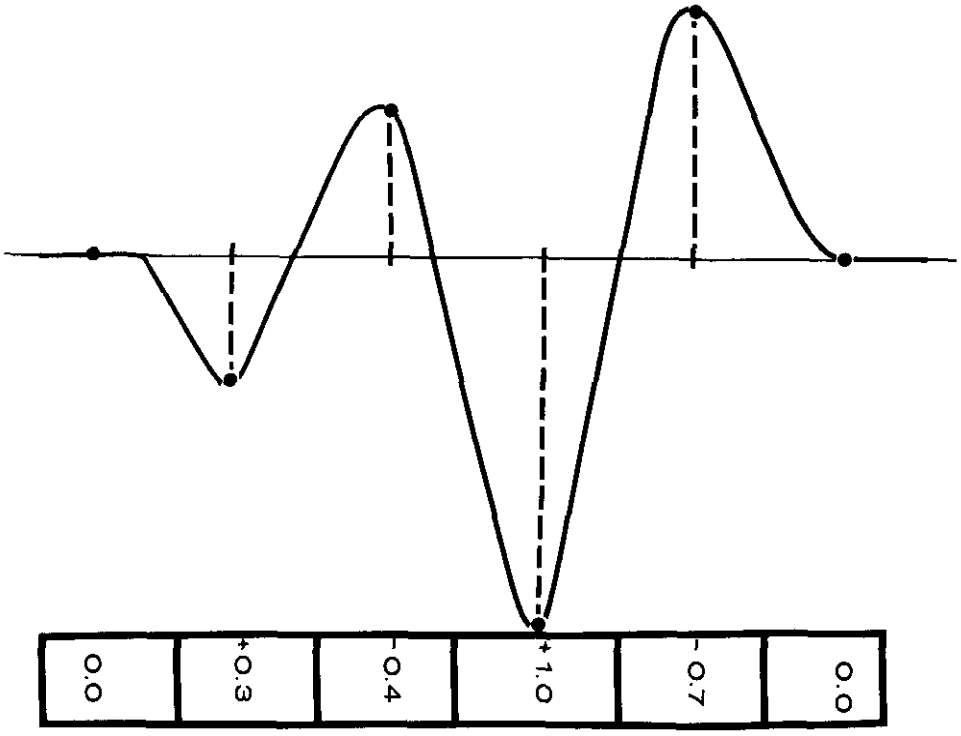


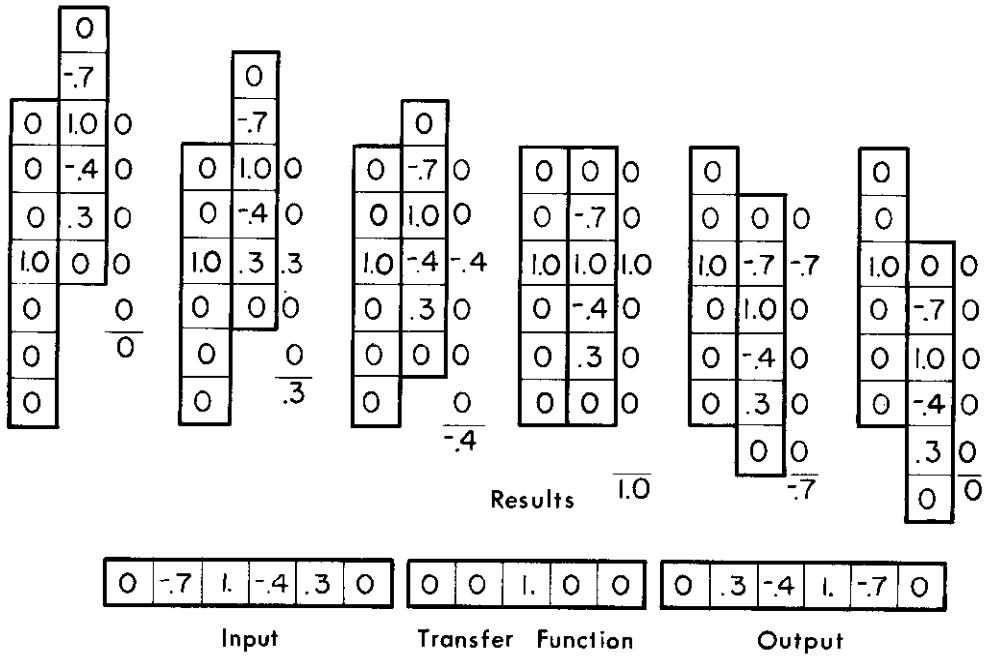
FIG. 56.—Sampled unit impulse response.

Mathematically, convolution has the same properties as multiplication, which is not true for cross-correlation. The deconvolution process is commutive, that is a*b equals b*a. (An asterisk is commonly used to signify convolution.) If the positions of the operator and the transfer function are interchanged in the model before convolution, the output will be identical. If the same procedure is followed for cross-correlation, the results will not be the same.

The commutive property has important practical applications, particularly when multiple convolution operations are to be carried out. Rather than convolve the signal in turn with each of the separate operators, all of the operators may first be convolved into a single time series and the result convolved with the entire trace. The final results will be identical either way.

The process of convolution is used in many applications other than seismic signal processing. A synthetic seismogram is constructed by convolving the unit impulse response with a time series of reflection coefficients in unit impulse form.

A simple running average is nothing more than the convolution of a series of weights with a series of data points. Numerical integration



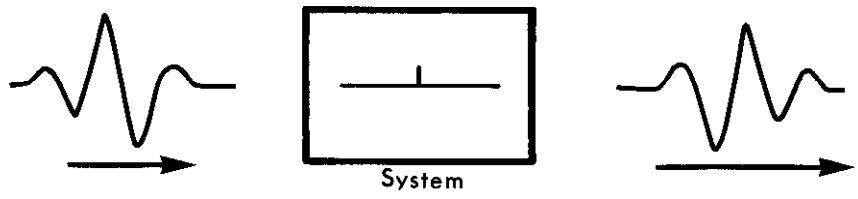


FIG. 57.—Convolution of impulse response with spike. Note that the output is the reverse of the input.

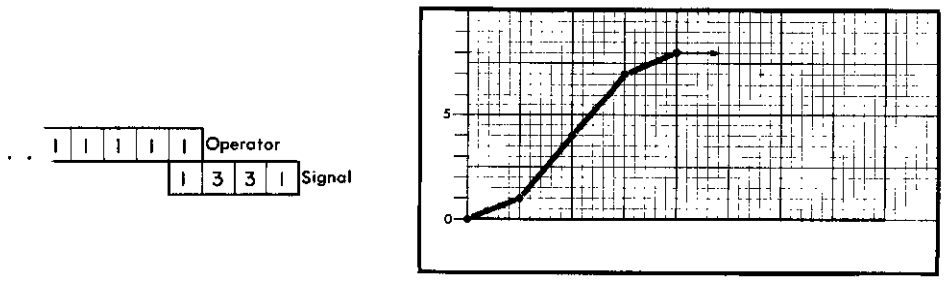


FIG. 58.—Approximation to integration by convolution with operator of 1's.

may be done by convolution. Any numerical series convolved with a non-terminating operator composed entirely of 1's will develop the series summation or the approximation to the integral (Fig. 58).

DECONVOLUTION

The convolution operation follows the rules for multiplication. Therefore, it should be possible to reverse the convolution process by an operation analogous to division. For example, convolution of the series 1, 3, 2, 1, by the operator 1, 2, 1, gives 1, 5, 9, 8, 4, 1. Note that convolution of an operator of length m with a signal of length n results in n+m—1 output points.

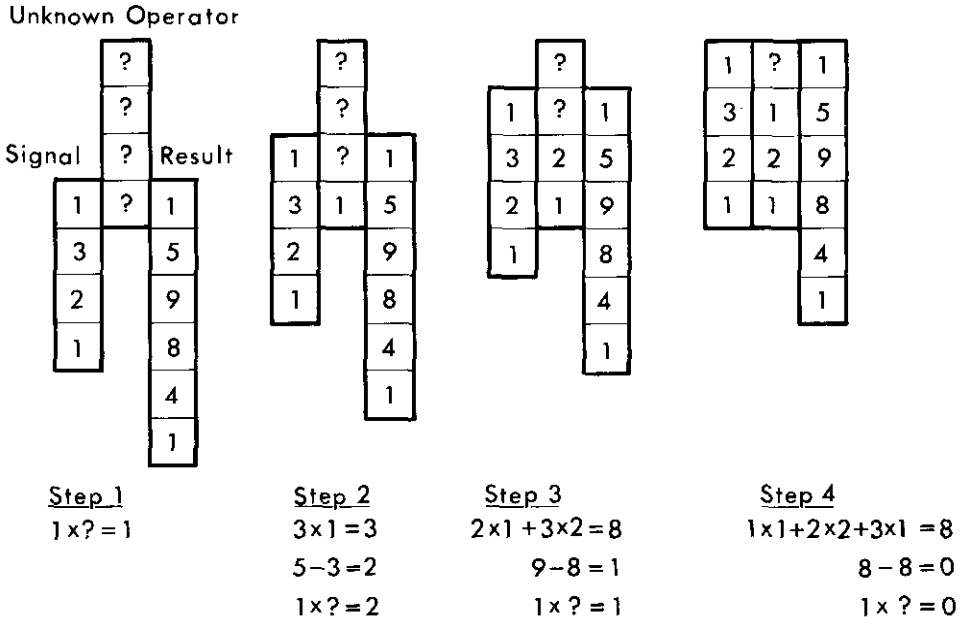


FIG. 59.—Procedure for division of convolution output to determine the operator.

The process may be reversed in the following manner (Fig. 59). It is assumed that the output and the original series are known and it is desired to determine the operator. The output and the series are placed in point to point alignment, and the first number of the output is divided by the first number of the original series. The result is the first point of the operator. The operator is then shifted one position, and the product of the known point of the operator and the second point of the series is obtained. This product is subtracted from the second point of the output. The remainder is divided by the first point of the series to obtain the second point of the operator. The operator function is again shifted right one point, and the whole process repeated until

the entire operator has been determined. The preceding operation bears a great deal of resemblance to polynomial division. Indeed, many operations in the realm of convolution can be shown to be similar to operations with polynomials.

The process of division just described is a form of deconvolution. If the unit impulse response corresponding to a given seismic trace can be well defined, the unit impulse response may be divided into the signal in order to obtain the series of unit responses corresponding to the reflection coefficients.

Deconvolution by division is arduous and time-consuming. An alternative and usually better method is provided by inverse operations.

Inverse Filtering

Mathematically, division may be replaced by multiplication with the inverse of the divisor. Therefore, for deconvolution, it should be possible to convolve the output with the inverse of the operator to obtain the signal, say in much the same manner as division by 4 can be accomplished by multiplying by ¼.

Convolution with a series of 1's provides a form of integration. For example 1, 3, 3, 1 * 1, 1, 1, 1, . . . equals 1, 4, 7, 8, 8, . . . (Fig. 58). It would be possible to use the division process to divide 1, 1, 1, 1, . . . into 1, 4, 7, 8, 8, . . . to obtain the original series 1, 3, 3, 1. An easier way would be to convolve the result with the inverse of the operator. The inverse in this case is 1, -1, which may be determined by dividing 1, 1, 1, 1, . . . into 1. The process desired is convolution; hence, the operator 1, -1 must be folded before the operation on the output is carried out. The result is 1, 3, 3, 1, which is the original signal and is the desired result. It will be noticed that the inverse operator is very short, only two points long, and as a result, the operation can be carried out much faster than would be possible with division.

INVERSE OPERATORS

Z—Inverse

Convolution with the inverse operator is the preferred method used for seismic deconvolution. One of the procedures used to determine the inverse operator is relatively simple. An autocorrelation is taken of a representative portion of the seismic signal, to consolidate the impulse response components at zero phase. The autocorrelation is then divided into the unit spike, i.e., the number 1, to determine the operator which, when convolved with the impulse response components, will produce the spike. The operator thus obtained is convolved with the recorded seismic signal to obtain an approximation to the unit impulse series. The suc cess of this operation depends on a number of factors.

The inverse function is rarely the short, neat operator illustrated by 1, -1. For instance, the inverse of 1, 2, 1 will be found to be 1, -2, 3, -4, 5, -6, . . . , a never-ending series with expanding terms. Practical considerations limit the length of the inverse operator which may be

used. If the operator is merely truncated at the desired number of terms, the resulting round-off error may cause meaningless output.

Least Squares Inverse

A better approach to the development of the inverse operator may be to use a least squares technique. This reduces truncation errors and ensures the development of the best possible operator within a designated operator length. However, the resulting operator will not necessarily be a good operator. The least squares method provides a best fit in the mathematical sense, and is to be preferred over a simple truncated operator but, like a least squares fit of a straight line to randomly scattered data points, the deviation can be extremely wide.

Development of the inverse operator is more likely to be a problem with land seismic work than water work. Marine operations are carried out under somewhat more uniform conditions. The main concern is often merely to remove a single frequency reverberation from the records. In such cases, often a very short operator is adequate. Land records tend to be more inconsistent, and plagued with a greater variety of noise.

Spectrum Whitening

The procedure of inverse filtering endeavours to replace the impulse response events of the seismic signal with unit impulses corresponding to the reflection coefficients. When viewed from the frequency domain, the operation is one of replacing the irregular 'coloured' frequency spectrum of the impulse response with the uniform white spectrum corresponding to the spike. If the deconvolution is complete, the output consists of nothing more than a series of spikes. The transform would show a uniform white amplitude spectrum over the range of the system response. For this reason, the deconvolution process is often referred to as a spectrum whitening process.

The autocorrelation of a white spectrum is a spike. An autocorrelation taken after the deconvolution process provides an indication of the effectiveness of the deconvolution. If the autocorrelation is sharp and relatively free from oscillations, the deconvolution has been effective (Fig. 60).

Several predominant peaks or spikes may have existed in the power spectrum before deconvolution was applied. These peaks will be lowered in the deconvolution process toward a flat line representing the white spectrum. The low amplitude elements may be effectively raised, which occasionally happens. In such case, which is particularly noticeable in the case of a long deconvolution operator, low amplitude, low frequency elements may be accentuated to an undesirable level. Some evidence of this may be seen in Fig. 60, in the early part of the autocorrelation after deconvolution.

Theoretically, inverse filtering should reduce the spectrum to a uniform white spectrum. In practice, if this operation were completely successful, it would result in a seismic trace consisting of nothing more

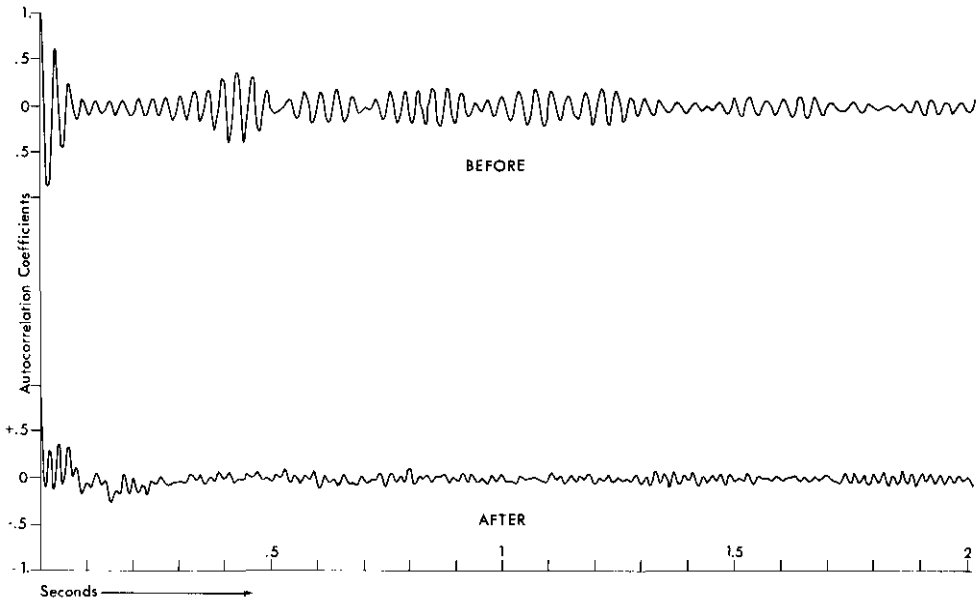


FIG. 60.—Autocorrelation of signal before and after deconvolution.

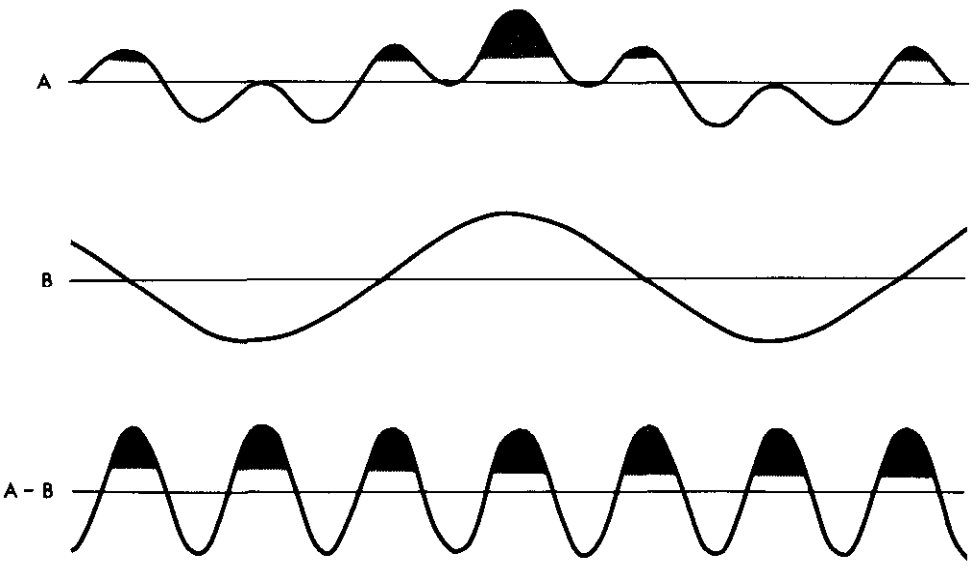


FIG. 61.—Filtering out low frequency components accentuates a ringing response.

than a series of spikes. Seismic information of this nature is alien to the interpreter, who bases much of his interpretation on the characteristic identity of reflection events. For these reasons, some band pass filtering is usually applied.

It is extremely important that band pass filtering be limited to a broad band. The sharpened definition of events can only be achieved by a broad range of frequencies, particularly the higher frequencies. However, the suppression of ringing is also dependent to a large extent upon the presence of low frequencies on the seismic record. The simple signal at the top of Fig. 61 is a model of a single narrow band spike. The character of a reflection of this nature could be recognized with ease over the length of a processed section. A low cut filter would remove the single low frequency component, leaving an oscillating sequence with no character at all. It is a fundamental rule that the broader the frequency band, the sharper the output that can be obtained. Some band pass filtering may be needed to remove excessive ground roll and/or high frequency chatter.

Development of sharp output requires the phase components to be shifted to zero phase in addition to whitening the amplitude spectrum. A large number of frequencies must fall in phase at each reflection interface. Unfortunately, the autocorrelation process carries no phase information, with the result that the inverse operator is developed from amplitude information only. Thus deconvolution, as applied to normal production data, is never completely successful in the theoretical sense. It is doubtful that complete spiking would be desirable until methods of interpretation, and handling of strongly spiked output, could be developed.

NOISE SUPPRESSION

Inverse filtering seeks to whiten the spectrum by converting the reflections to spikes. Autocorrelation is unable to distinguish white random noise from reflection coefficients, both of which will autocorrelate as a spike. Substantial amounts of random noise will therefore cause the autocorrelation to more closely resemble a spike than the autocorrelation of a signal containing limited amounts of random noise. In the extreme case of a signal which was all random noise, the autocorrelation would be a spike which would then result in an operator of 1, the 'do nothing' operator. The amount of random noise present in a signal, therefore, has a substantial bearing on the type of inverse operator which will be developed. The effect of the noise component may be suppressed by modification of the autocorrelation function to in turn modify the effectiveness of the inverse operator.

TRANSFORM RELATIONSHIPS

Many of the operations which are carried out in the time domain become simpler or easier to conduct in the frequency domain. Unfortunately, it has always been difficult to transform the signal from the

time domain into the frequency domain without spending a great deal of processing time. New computing equipment and methods promise to simplify the operation and it is likely that in the future, many of the techniques will operate in the frequency domain rather than in the time domain.

Some of the basic time/frequency relationships have been demonstrated in an earlier section. The relationships may be extended to cover operations with signals. A given frequency component may be extracted from a complex signal in the time domain by cross-correlating a single frequency operator with the signal. Cross-correlation of a single frequency with a noisy signal will produce as output a single frequency which differs from the operator only in amplitude and phase. The amplitude of the output signal is proportional to the amplitude of that frequency component in the noisy signal.

A single frequency in the time domain is a spike on the amplitude spectrum in the frequency domain. Random white noise in the time domain bcomes a flat line on the amplitude spectrum. (See Fig. 42.) The transform of the output would also be a spike on the amplitude spectrum, with the amplitude equal to the amplitude of that frequency in the noisy signal.

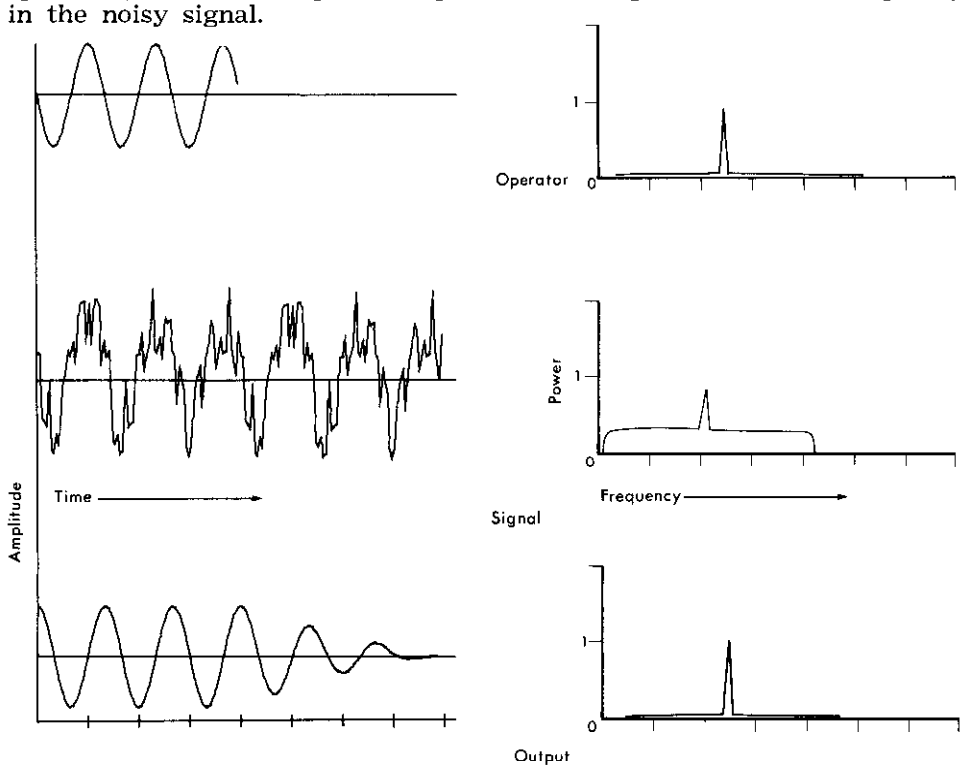


FIG. 62.—Comparison of cross correlation in the time domain and the frequency domain.

The identical results of cross-correlation in the time domain may be achieved in the frequency domain by merely multiplying corresponding points of the amplitude spectra of the operator and the signal (Fig. 62). For instance, in the above example, the amplitude spectrum of the unit frequency operator will be zero at all points except that of the designated frequency, which will have a value of one. Thus, cross-multiplication of corresponding points on the amplitude spectra of the operator and signal will result in multiplication by zero for all frequencies except the designated frequency, which will be multiplied by one. Note that the amplitude of the output is equal to the sum of the signal plus the noise component of that frequency.

A band pass filter is merely an extension of this operation. A band pass is a box car block of unit frequencies on the amplitude spectrum multiplied with the corresponding points on the signal amplitude spectrum.

It is true for all cases that cross-correlation in the time domain is equivalent to multiplication of the amplitude spectra in the frequency domain.

Convolution is an operation identical in procedure to cross-correlation, except that the operator is folded. Therefore, it may be stated that both cross correlation and convolution in the time domain are equivalent to multiplication of the amplitude spectra in the frequency domain.

The difference between the two operations lies in the effects on the phase. Cross-correlation produces the difference between the phase of the operator and the phase of the signal, whereas convolution develops the sum of the phase of the operator and the phase of the signal. This is true in both the time domain and the frequency domain.

Many operations which are extremely lengthy in the time domain, say a full autocorrelation of a signal, become relatively simple in the frequency domain. Autocorrelation in the frequency domain is equivalent to squaring the amplitude spectrum and setting the phase spectrum to zero.

A band pass operation in the frequency domain consists of simply chopping off the ends of the amplitude spectrum at the desired cutoff frequencies. If a sloping cutoff response is desired, this is also easily obtained in the frequency domain.

Development of the inverse filter may be done in the frequency domain by merely taking the inverse of the amplitude spectrum. Deconvolution by spectrum whitening attempts to level out the amplitude spectrum and bring all values to a constant value, say one. Each value on the amplitude spectrum of a signal may be multiplied by its reciprocal to bring all values to a constant. Thus, the inverse filter in the frequency domain is the inverse or reciprocal of the amplitude spectrum of the signal. The phase components may be treated separately, whereas control of the phase is lost in the time domain. If deconvolution were

completely successful, it would develop an identical flat amplitude spectrum for each trace. Character and interval timing of the traces would then be completely dependent upon the phase spectrum for each trace.

Thus, phase is more important than amplitude in defining a seismic trace. Substantial changes in the amplitude spectrum may be made without destroying the characteristics which identify the seismic signal. Relatively small changes in phase may produce serious results, particularly in the reflection timing. A phase shift is a time shift, and modification of the phase spectrum will result in a shift in the time domain.

The inverse operator developed in the time domain is not a zero phase operator. It is a minimum phase operator in which the phase shift is held to a minimum for the particular conditions, but under certain conditions the minimum amount may still be relatively large in the absolute sense. Some phase shift is desirable in convolution to place the unit impulse components in phase in order to develop the approximation to the spike.

A thorough understanding of the basic relation between operations in the time domain to those in the frequency domain is essential if the interpreter is to determine the nature and effect of the processing operations and to evaluate the results.

SPECIAL FILTERING TECHNIQUES

This discussion has concentrated on the fundamental operations of seismic data processing, namely data analysis, band pass filtering and inverse filtering. Many other filtering procedures exist, but these are often merely variations of the methods of application or special design techniques for basic band pass filtering or inverse filtering.

Time Variant Filters

In many areas, the dominant frequencies of a seismic recording vary as a function of time, generally becoming lower with increasing time. It would be desirable to develop an adaptive filter which would itself be modified as a function of time in order to provide the optimum inverse for all points. This is a time variant filter.

A true continuous time variant filter is difficult to implement and may be costly to apply. A compromise is often made by segmenting the seismic trace and treating each piece as a separate signal. A filter is then designed for each segment, applied, and the segmented results are then joined together to form a complete trace.

This is the procedure used in most so-called 'time variant' filters. A better, more correct description might be modified step or block filter.

A serious flaw in the block technique occurs when the phase shift characteristics developed by the inverse operators vary from segment to segment. The phase shift will appear as a time shift on the processed signal and if it is different in each segment the time relationship between reflectors may be altered. The problem is accentuated at the ends of the

segments, where pronounced discontinuities may occur. These are usually camouflaged by providing an overlap zone which is faded from one segment into the next. This procedure smooths the output, but the data within the overlap is distorted. Care should be taken to ensure that the breaks fall in some unimportant part of the section.

In spite of the limitations the technique is useful for signals which can not be satisfactorily processed with a single operator.

Recursive Filters

Long filter operators are expensive to develop and to apply, and may be associated with undesirable side effects. Time varying filters are difficult to compute and the application of the operator may be more expensive than the fixed operator. These disadvantages are more than offset by the capabilities of a long and/or time varying operator on occasion.

A technique which contains elements of both the long operator and the time varying operator is known as recursive filtering. A recursive filter operator differs from the standard operator in the form of application. The current output value of a recursive operator is computed by applying weights to a selected number of points on the signal, as well as to some of the previous output. Thus, like compound interest, each new output value contains a certain percentage of the previous value which, in turn, contains a certain percentage of the value before it and so on. In a sense, the recursive filter may be considered to have unlimited length. Each output point will contain some element obtained from every previous point back to the first. The procedure may also be considered time variant, in that the information contained in each successive output point is a mathematical series containing one additional term for each shift of the operator along the signal.

The recursive technique may be used with any filter type, whether band pass, inverse, or some other process. It is not widely used because of certain problems in the design of the operator. The difficulties lie in the fact that the output is the product of a series. It is possible for round-off errors to occur in an iterative process of this nature such that the value of the output becomes progressively more unreliable with the length of the application. Mathematical tests may be used to determine when the operator will be unstable, but there may be no effective remedy to correct an unstable condition.

A true recursive filter should not be confused with use of recursion techniques to design a filter. Several filter design programs use a recursive method to build up the filter operator point by point until it meets some prescribed specification. Each recursion cycle of the filter design program develops a single additional filter point. A test is then made for some prescribed tolerance. If the filter does not meet the test, one more point is designed and the test repeated. In this manner, a minimum operator length consistent with some desired tolerance is obtained.

Multi-Channel Filters

The filters described to this point are single channel filters. They use a single operator to operate on a single signal and produce a single output. Multi-channel filters add the dimension of spatial relationships between several similar signals. They endeavour to establish the coherent or correlative components that may exist between several similar channels. The filter output from a multi-channel input is generally a single channel, but frequently the operation is carried out in the manner of a running average. One channel of a multi-channel group is dropped and a new channel picked up to develop each output trace.

In the simplest terms, the multi-channel filter endeavours to find points of similarity occurring at the same time on each of several input channels. Such a relation will cause the maximum output to be obtained. A design variation may be used to enhance or to attenuate, as the case may be, events which are similar on the several channels, but which are displaced by some uniform time increment in each channel. An angular event of this sort may be considered as one moving across the detectors at some velocity. For this reason, these filters are sometimes termed 'velocity' filters.

Multi-channel, or velocity filters may be of particular value when using the common depth point method of recording. Assuming the traces to be stacked have been properly adjusted for both static and dynamic corrections, all of the events which represent true reflections on the recordings should be in phase. Undesirable events, such as multiples, ghosting, and energy having a strong horizontal component will be out of phase. The multi-channel filter discriminates between the in-phase and out-of-phase events to accentuate the energy ratios between the two types of signal.

Theoretical calculations, verified by actual observations, show that multi-channel filters may improve the signal to noise ratio by a factor of two to one over conventional stacking methods. It should be possible to replace a 6-fold stack with a 3-fold multi-channel filtered stack to obtain equivalent results.

Correlation Techniques and Shaping Filters

A number of techniques adapted from telephony and radar operations assume that the required signal is known, and that the problem is merely one of extracting the known signal from a background of noise. In seismic work, this rarely occurs. The rare exception exists when a record is shot near a well. A synthetic seismogram developed from the acoustic log may then be used for the model signal. A filter can then be developed which will convert the actual recording to a close approximation of the model. However, when the recording location is moved away from the well, the lithology changes and the model signal at the well is no longer valid. There are several variations to this technique, but all are characterized by the use of a model trace.

Automatic Statics

An interesting modification of the preceding filter is a procedure sometimes used to apply automatically static corrections to common depth point recorded traces. The process exploints the phase relationships of cross-correlation and convolution to transfer the phase of one trace to that of another.

The basic procedure is as follows. A model trace or signal is set up which has the desired phase characteristics. Each of the traces to be stacked is cross-correlated in turn with the model trace. Cross-correlation is equal to taking the product of the amplitude spectra of the signal and the operator, and the difference in phase between the signal (in this case, the model trace) and the operator. Thus, letting A_m stand for the amplitude spectrum of the model and Φ_m the phase spectrum of the model, the product of the first operation may be represented by:

$$\begin{array}{ll} A_m \cdot A_1 & \Phi_m - \Phi_1 \\ A_m \cdot A_2 & \Phi_m - \Phi_2 \\ A_m \cdot A_3 & \Phi_m - \Phi_3 \end{array}$$

This completes the first stage of the process. The product of the first operation is then convolved with the model trace. This time, the output is again equal to multiplication of the amplitude spectrum, but the resulting phase spectrum is the sum of the two phase spectra. Thus, the second operation results in the following output:

$$\begin{array}{ll} (A_m \cdot A_1) \cdot A_1 & (\Phi_m - \Phi_1) + \Phi_1 \\ (A_m \cdot A_2) \cdot A_2 & (\Phi_m - \Phi_2) + \Phi_2 \\ (A_m \cdot A_3) \cdot A_3 & (\Phi_m - \Phi_3) + \Phi_3 \end{array}$$

These terms may be consolidated and terms of the phase spectra cancelled out to equal the following:

$$\begin{array}{ll} A_m \cdot A^2_1 & \Phi_m \\ A_m \cdot A^2_2 & \Phi_m \\ A_m \cdot A^2_3 & \Phi_m \end{array}$$

Thus for each trace, the resulting amplitude spectrum consists of the product of the amplitude of the model times the amplitude squared of the original trace. The phase for each is the phase of the model. All of the traces now have the same phase so they will be correctly aligned.

Unfortunately, the effectiveness of this process is illusory because it eliminated all of the advantages which are theoretically gained by stacking common depth point traces. The reason for this follows.

Any seismic trace is primarily defined by the phase components. The basic premise of common depth point shooting is that the desired reflected events will be in phase, while the multiples and noise will be out of phase. When the traces making up the gather are summed, the in-phase components will be enhanced and the out-of-phase components will cancel. If, however, all of the traces making up the gather have

exactly the same phase, the sum of the component traces will be no different than that obtained by adding any one trace to itself a number of times.

The amplitude components ordinarily cause some variation in the component traces, but in this operation the resulting amplitude spectrum is so badly distorted that the signal must be deconvolved. Deconvolution seeks to whiten the amplitude spectrum of each trace to a common level so the net result of the example is a set of three traces having the same phase spectrum and similar amplitude characteristics, in other words, three very similar traces.

A synthetic trace could be used for the model trace. However, since the model trace is substituted for the recorded trace in every case, there would be little point in performing the operation, as the entire line would be similar.

One of the traces from each gather group may be used as the model for that group, but this merely ensures that the phase components of the single trace, including those defining both reflections and multiples, are substituted for the others in the gather group, largely negating any value of the common depth point operation.

SUMMARY AND CONCLUSIONS

The object of this paper has been to describe in elementary terms the basic operations which are used for digital processing of seismic traces. Secondary objectives have been to point out some of the reasons why the digital process is superior to the analog method and, to a greater extent, to point out that the advantages gained by each of the various processes can be offset in part by disadvantages, or undesirable side effects. In some cases, the detrimental side effects are minor and, with reasonable care, they may be compensated for and can be ignored. In other cases, the side effects may have a strong influence on the quality of the final product.

The great number of digital techniques which can be used to improve the data requires, in general, much more skill on the part of the interpreter than was ever required with analog equipment. He can, using more or less standard routine techniques, obtain appreciable improvement. However, only with substantial experience and careful attention to the results of processing techniques, can he specify the right combination of procedures and parameters to produce the truly optimum output.

Occasionally there is a strong inclination on the part of the seismologist or interpreter to leave the selection of procedures to the digital analyst. The analyst, with his close contact with processing data under a broad range of conditions, often becomes very adept in producing the optimum product. But rarely will he be as well acquainted with the particular problems of an area, and the objectives of the study, as will be the interpreter. In some cases, the analyst may even tend to become

oriented more toward the processing itself than toward the geology and lithology of the section being processed.

Therefore, the geophysicist who wishes to be truly adept in methods of interpretation and processing, will strive to become proficient in the determination of the digital processing parameters, and in the recognition of the cause and effect relationship that exists when sophisticated processing techniques are applied. For it is true in digital processing as in standard interpretation that only long experience with numerous types and conditions of seismic recording can develop the skill of the interpreter.

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